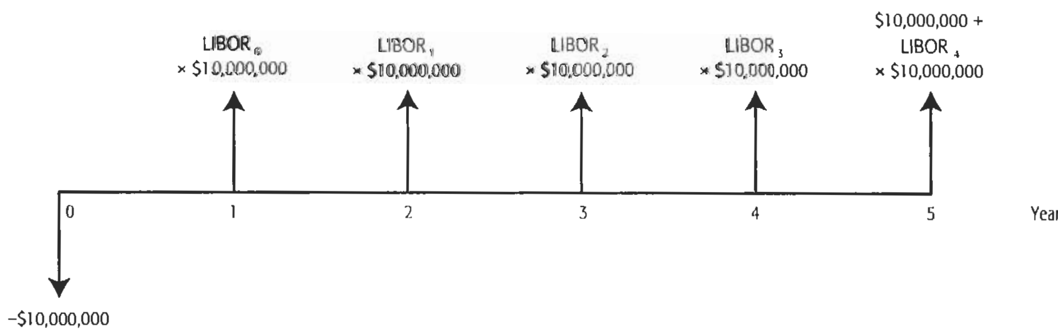
21 Swaps: Economic Analysis and Pricing

Answers to Questions and Problems

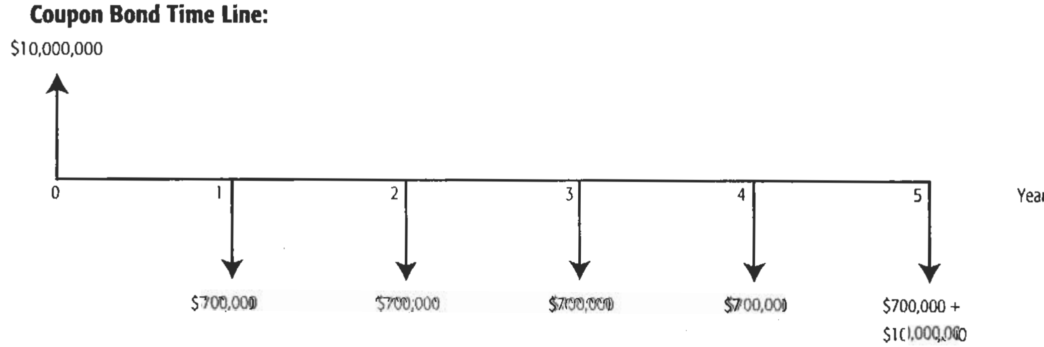
1. A trader buys a bond that pays an annual coupon based on LIBOR with a principal amount of $10 million. The actual payment in each year depends on the level of LIBOR observed one year previously. The same trader simultaneously sells a $10 million dollar bond that pays a fixed rate of 7 percent interest and also has an annual coupon. Both bonds have a maturity of five years and are priced at par. The trader also enters a pay-fixed interest rate swap with annual payments, a tenor of five years, and a notional principal of $10 million. For the swap, the fixed rate is 7 percent and the floating rate is LIBOR. The swap also pays in arrears.

A. Construct a time line for each bond showing the payments associated with the bond.

FRN Time Line:

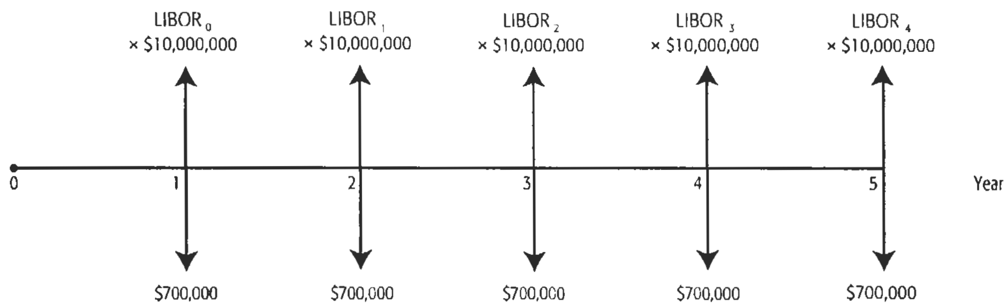


Coupon Bond Time Line:



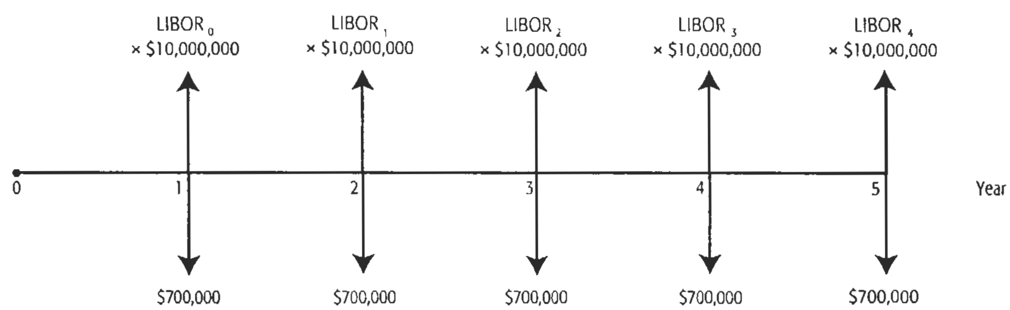
B. Construct a time line showing the net payments resulting from the two bonds.

Bond Portfolio Time Line:



C. Construct a time line showing the payments for the swap.

Pay-Fixed Swap Time Line:



D. Based on the two time lines from parts B and C, what conclusion can you draw regarding the relationship between swaps and bond portfolios?

A plain vanilla swap can be constructed or synthesized by a bond portfolio. This is the case, because the bond portfolio and the swap have identical cash flows. This suggests that the prices of bonds can be used to value swaps, and that swaps can be used to transform cash flow patterns on bond portfolios.

2. Assume that today is December 17, 2003. A firm enters a plain vanilla interest rate swap as the receive-fixed party on a swap with a tenor of one year, quarterly payments at the end of the next four quarters, and a notional principal of $25 million. At the same time, this firm buys a strip of Eurodollar futures for the next four contracts, with 25 contracts per expiration. (Ignore daily settlement; in other words, assume that all futures-related cash flows occur at the expiration of the futures contract, which occurs at the end of each quarter.) At present, $t = 0$, the LIBOR yield curve is flat at 8 percent, and the fixed rate on the swap is also 8 percent.

A. Complete the following table using our familiar $LIBOR_t$ notation. Assume that the Eurodollar futures rate converges to LIBOR at expiration.

Quarter	Net Receive-Fixed Swap Cash Flow	Net Long Futures Cash Flow
0	0	0
1	$(0.08 - LIBOR_0)/4 \times \$25{,}000{,}000$	$(0.08 - LIBOR_1)/4 \times \$25{,}000{,}000$
2	$(0.08 - LIBOR_1)/4 \times \$25{,}000{,}000$	$(0.08 - LIBOR_2)/4 \times \$25{,}000{,}000$
3	$(0.08 - LIBOR_2)/4 \times \$25{,}000{,}000$	$(0.08 - LIBOR_3)/4 \times \$25{,}000{,}000$
4	$(0.08 - LIBOR_3)/4 \times \$25{,}000{,}000$	$(0.08 - LIBOR_4)/4 \times \$25{,}000{,}000$

All of the futures contracts should be entered at 8 percent, because the yield curve is flat. The payoff on each futures depends on the deviation of spot LIBOR at expiration from the initial futures rate of 8 percent. With daily settlement being considered, the actual flows would have been the same on the futures, but they would have been incurred piecemeal as the futures approached expiration.

B. If this swap had been a determined-in-advance/paid-in-advance swap, what would be the payment at $t = 3$ on the swap?

The payment would be:

$$\frac{\$500{,}000 - \frac{LIBOR_3}{4} \times \$25{,}000{,}000}{1 + \frac{LIBOR_3}{4}}$$

C. What conclusion can you draw regarding the relationship between a plain vanilla interest rate swap and a strip of Eurodollar futures?

A strip of Eurodollar futures is very similar to an interest rate swap. However, while they may be conceptually very close, there are important differences in the timing of cash flows, as the table in part A shows. As the answer to part B shows, the Eurodollar strip is even closer to a determined-in-advance/paid-in-advance swap. Even then, the cash flow for the futures and swap are not quite identical. When we consider daily settlement on the futures, the equivalence is degraded even more. Still, a Eurodollar strip and a plain vanilla swap are very similar.

3. Consider a plain vanilla swap from the point of view of the pay-fixed counterparty. The swap has a tenor of five years with annual payments, a notional principal of $50 million, payments in arrears, a floating rate of LIBOR, and a fixed rate of 7 percent. Assume that the pay-fixed counterparty buys a call option with an expiration date of two years. If the call is in-the-money, it pays the observed LIBOR on that date minus 7 percent on a notional principal of $50 million. The pay-fixed party also sells a similar put option: two-year expiration, $50 million notional principal. If the put is in-the-money, the payoff equals 7 percent minus observed LIBOR times the notional principal. In each case, the rate is observed in two years, with the payment date actually occurring one year later. Bearing in mind that the rates observed at year 2 determine the actual cash flow at year 3 on the swap, put, and call:

A. Complete the following table showing the pay-fixed side of the swap, the long call, and the short put for the payment at year 3 as a function of LIBOR observed at year 2.

	Payment at Year 3		
$LIBOR_2$	**Pay-Fixed Net Swap Payment**	**Long Call Payoff**	**Short Put Payoff**
0.05	– \$3,500,000 + 0.05 × \$50,000,000 = – \$1,000,000	0	– (0.07 – 0.05) × \$50,000,000 = – \$1,000,000
0.06	– \$3,500,000 + 0.06 × \$50,000,000 = – \$500,000	0	– (0.06 – 0.05) × \$50,000,000 = – \$500,000
0.07	– \$3,500,000 + 0.07 × \$50,000,000 = \$0	0	0
0.08	– \$3,500,000 + 0.08 × \$50,000,000 = \$500,000	(0.08 – 0.07) × \$50,000,000 = \$500,000	0
0.09	– \$3,500,000 + 0.09 × \$50,000,000 = \$1,000,000	(0.09 – 0.07) × \$50,000,000 = \$1,000,000	0

B. What does this table show about the relationship between a single swap payment and the call/put portfolio of options?

A single payment on a plain vanilla interest rate swap is equivalent to a call/put portfolio suitably adjusted as to timing and notional principal. A pay-fixed payment is equivalent to a long call/short put portfolio, while a receive-fixed payment is equivalent to a short call/long put portfolio.

C. What does the table show about an entire swap and a possible portfolio of options?

A plain vanilla interest rate swap can be replicated as a strip of call/put portfolios, with each portfolio corresponding to a single payment. This foreshadows the next chapter, which discusses caps (strips of interest rate calls), floors (strips of interest rate puts), and collars (strips of put/call portfolios).

D. If the swap had been a receive-fixed swap, what option position would have replicated the swap payment at year 3?

The same options, but with a short call/long put portfolio, would be equivalent to the receive-fixed payment.

4. Explain how an interest rate swap can be analyzed as a strip of futures. What are some limitations of this analysis?

We have already noted that a swap may be regarded as a portfolio of forward contracts. A swap may also be thought of as a portfolio of forward contracts, or FRAs. For example, a swap agreement with quarterly payments based on Eurodollar deposit rates is essentially similar to a strip of Eurodollar futures contracts in which the futures maturities match the payment dates on the swap. However, a strip of FRAs can exactly replicate the cash flows of a swap, while a strip of futures cannot. The futures contracts involve daily settlement cash flows, for instance, but an FRA and a swap do not. A strip of futures is unlikely to have expiration dates that exactly match the payment dates of a swap, particularly since Eurodollar futures have just four expiration dates per year for the main contracts on the March, June, September, December cycle. Also, swaps are generally paid in arrears, whereas futures are paid as they approach expiration through daily settlement.

5. Today the following rates may be observed: $FRA_{0,3} = 0.0600$; $FRA_{3,6} = 0.0595$; $FRA_{6,9} = 0.0592$; $FRA_{9,12} = 0.0590$; $FRA_{12,15} = 0.0590$; $FRA_{15,18} = 0.0588$; $FRA_{18,21} = 0.0587$; $FRA_{21,24} = 0.0586$, where the subscripts pertain to months. Consider a plain vanilla swap with a tenor of two years, quarterly payments, and a notional principal of $10 million.

A. Compute the discount rates for each payment date on the swap.

$$Z_{0,3} = 1 + FRA_{0,3}/4 = 1 + 0.0600/4 = 1.01500$$

$$Z_{0,6} = Z_{0,3}(1 + FRA_{3,6}/4) = 1.015(1 + 0.0595/4) = 1.030098$$

$$Z_{0,9} = Z_{0,6}(1 + FRA_{6,9}/4) = 1.030098(1 + 0.0592/4) = 1.045343$$

$$Z_{0,12} = Z_{0,9}(1 + FRA_{9,12}/4) = 1.045343(1 + 0.0590/4) = 1.060762$$

$$Z_{0,15} = Z_{0,12}(1 + FRA_{12,15}/4) = 1.060762(1 + 0.0590/4) = 1.076409$$

$$Z_{0,18} = Z_{0,15}(1 + FRA_{15,18}/4) = 1.076409(1 + 0.0588/4) = 1.092232$$

$$Z_{0,21} = Z_{0,18}(1 + FRA_{18,21}/4) = 1.092232(1 + 0.0587/4) = 1.108260$$

$$Z_{0,24} = Z_{0,21}(1 + FRA_{21,24}/4) = 1.108260(1 + 0.0586/7) = 1.124496$$

B. Find the SFR for this swap.

$$SFR = \frac{\sum_{n=1}^{N} \frac{FRA_{(n-1)\times MON,\, n\times MON}}{Z_{0,\, n\times MON}}}{\sum_{n=1}^{N} \frac{1}{Z_{0,\, n\times MON}}}$$

For our swap, $MON = 3$. Dealing with the numerator and denominator separately, we have:

$$NUMERATOR = \frac{0.0600}{1.01500} + \frac{0.0595}{1.030098} + \frac{0.0592}{1.045343} + \frac{0.0590}{1.060762}$$

$$+\frac{0.0590}{1.076409}+\frac{0.0588}{1.092232}+\frac{0.0587}{1.108260}+\frac{0.0586}{1.124496}$$

$$= 0.059113 + 0.057761 + 0.056632 + 0.055620$$

$$+ 0.054812 + 0.053835 + 0.052966 + 0.052112$$

$$= 0.442851$$

$$DENOMINATOR = \frac{1}{1.01500}+\frac{1}{1.030098}+\frac{1}{1.045343}+\frac{1}{1.060762}$$

$$+\frac{1}{1.076409}+\frac{1}{1.092232}+\frac{1}{1.108260}+\frac{1}{1.124496}$$

$$= 0.985222 + 0.970781 + 0.956624 + 0.942719$$

$$+ 0.929015 + 0.915556 + 0.902315 + 0.889287$$

$$= 7.491519$$

$$SFR = \frac{NUMERATOR}{DENOMINATOR} = \frac{0.442851}{7.491519} = 0.059114$$

C. Find the *APPROXSFR* for this swap.

$$(1 + APPROXSFR)^T = \prod_{n=1}^{N} (1 + FRA_{(n-1)\times MON,\, n\times MON})^{PART}$$

$$=(1.0600^{0.25})(1.0595^{0.25})(1.0592^{0.25})(1.0590^{0.25})$$

$$(1.0590^{0.25})(1.0588^{0.25})(1.0587^{0.25})(1.0586^{0.25})$$

$$= 1.121693$$

$$APPROXSFR = 0.059100$$

D. Assume that the same swap is to be negotiated as an off-market swap in which the receive-fixed party will receive 7 percent. What payment at the initiation of the swap will make the transaction a fair deal?

The swap will be a "fair deal" if the present values of the two sides of the swap are equal. We have just found that the *SFR* for the plain vanilla swap is less than 6 percent. If the swap is to be negotiated with a fixed rate of 7 percent, the fixed rate payer will be paying too much. To make the off-market swap a fair deal, the receive-fixed party must make, and the pay-fixed party must receive, a payment of X. The present values of the payments made by the receive-fixed party, including our payment X, are:

$$X + PART \times NP \times \left(\frac{FRA_{0,3}}{Z_{0,3}}+\frac{FRA_{3,6}}{Z_{0,6}}+\frac{FRA_{6,9}}{Z_{0,12}}+\frac{FRA_{9,12}}{Z_{0,12}}+\frac{FRA_{12,15}}{Z_{0,15}}+\frac{Z_{15,18}}{Z_{0,18}}+\frac{FRA_{18,21}}{Z_{0,21}}+\frac{FRA_{21,24}}{Z_{0,24}}\right)$$

For the pay-fixed party, now paying 7 percent, the present value will be:

$$PART \times NP \times 0.07 \times \left(\frac{1}{Z_{0,3}}+\frac{1}{Z_{0,6}}+\frac{1}{Z_{0,9}}+\frac{1}{Z_{0,12}}+\frac{1}{Z_{0,15}}+\frac{1}{Z_{0,18}}+\frac{1}{Z_{0,21}}+\frac{1}{Z_{0,24}}\right)$$

For both of these terms, we have already found the quantities in parentheses in part B of the question, where we found the *SFR* for the plain vanilla swap. They are equal to the *NUMERATOR* and *DENOMINATOR* computed there. Therefore, the fair deal will meet the condition:

$$X + PART \times NP \times 0.442851 = PART \times NP \times 0.07 \times 7.491519$$

From the statement of the question, we know: $PART = 0.25$ and $NP = \$10,000,000$. Therefore, we have:

$$X = 0.25 \times \$10,000,000\,[\,0.07\,(7.491519) - 0.442851] = \$203,888$$

The receive-fixed party must pay the pay-fixed party $203,888 at the outset of the swap to make the off-market swap a fair deal. This $203,888 compensates the pay-fixed party for paying about 1 percent per annum more than the *SFR* on a plain vanilla swap. This makes sense intuitively, because the pay-fixed party is paying about 1 percent over the *SFR* on $10,000,000 for each of two years, or something over $100,000 per year.

6. Today in the market you observe the following discount rates, where the subscripts indicate months: $Z_{0,3} = 1.0240$; $Z_{0,6} = 1.0475$; $Z_{0,9} = 1.0735$; $Z_{0,12} = 1.0976$; $Z_{0,15} = 1.1193$; $Z_{0,18} = 1.1472$; $Z_{0,21} = 1.1655$; $Z_{0,24} = 1.1872$. Compute all possible three-month FRA rates.

$$1.0240 = 1 + FRA_{0,3}/4;\ FRA_{0,3} = 4 \times 0.0240 = 0.09600$$

In general, for FRAs to cover a fraction of the year equal to *PART*, where $a > 0$:

$$FRA_{a,b} = \left(\frac{Z_{0,b}}{Z_{0,a}} - 1\right) PART$$

Therefore,

$$FRA_{3,6} = (Z_{0,6}/Z_{0,3} - 1) \times 4 = (1.0475/1.0240 - 1) \times 4 = 0.091797$$

$$FRA_{6,9} = (Z_{0,9}/Z_{0,6} - 1) \times 4 = (1.0735/1.0475 - 1) \times 4 = 0.099284$$

$$FRA_{9,12} = (Z_{0,12}/Z_{0,9} - 1) \times 4 = (1.0976/1.0735 - 1) \times 4 = 0.089800$$

$$FRA_{12,15} = (Z_{0,15}/Z_{0,12} - 1) \times 4 = (1.1193/1.0976 - 1) \times 4 = 0.079082$$

$$FRA_{15,18} = (Z_{0,18}/Z_{0,15} - 1) \times 4 = (1.1472/1.1193 - 1) \times 4 = 0.099705$$

$$FRA_{18,21} = (Z_{0,21}/Z_{0,18} - 1) \times 4 = (1.1655/1.1472 - 1) \times 4 = 0.063808$$

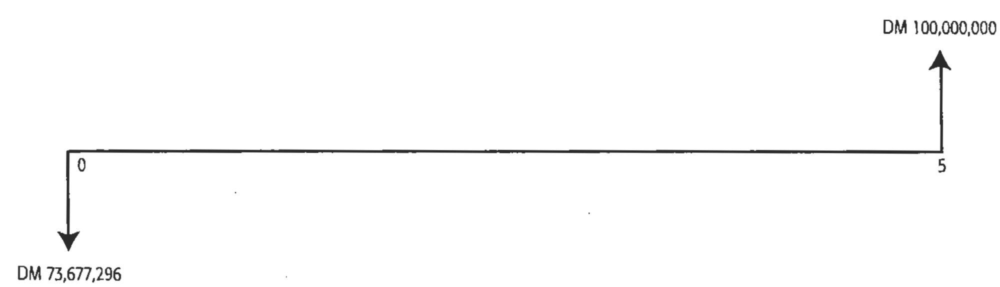
$$FRA_{21,24} = (Z_{0,24}/Z_{0,21} - 1) \times 4 = (1.1872/1.1655 - 1) \times 4 = 0.074474$$

7. A zero-coupon swap is a swap in which the fixed rate is zero. Instead of making periodic coupon payments, the fixed rate payer makes a single large payment at the termination of the swap. Zero-coupon swaps may be either interest rate swaps in any currency (with no exchange of principal) or foreign currency swaps (with the customary exchange of principal). Using one zero-coupon swap (either an interest rate or a foreign currency swap), and one bond of any type, construct a synthetic zero-coupon bond that pays German marks, and has a principal amount of DM 100 million and a maturity of five years. The German mark yield curve is flat at 6.3 percent. Assume annual payments throughout.

A. Find the cash flows associated with the German zero and draw a time line for this instrument.

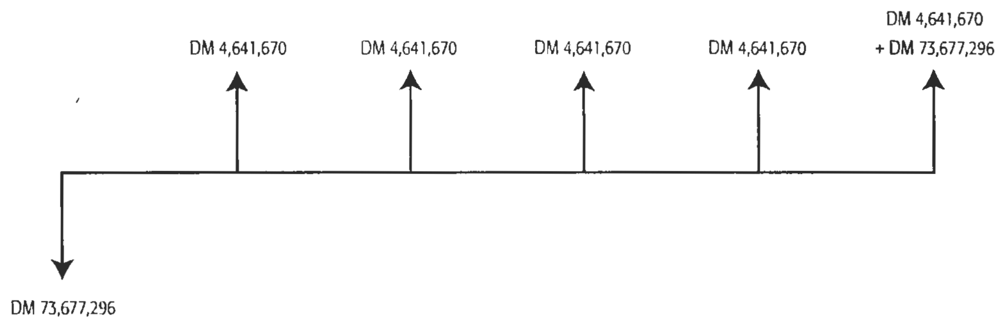
With a flat yield curve at 6.3 percent and annual compounding, the five-year zero-coupon factor is $(1.063)^5 = 1.357270$. A zero-coupon bond to pay DM 100,000,000 in five years would cost DM 73,677,296.

Zero-Coupon DM Bond:

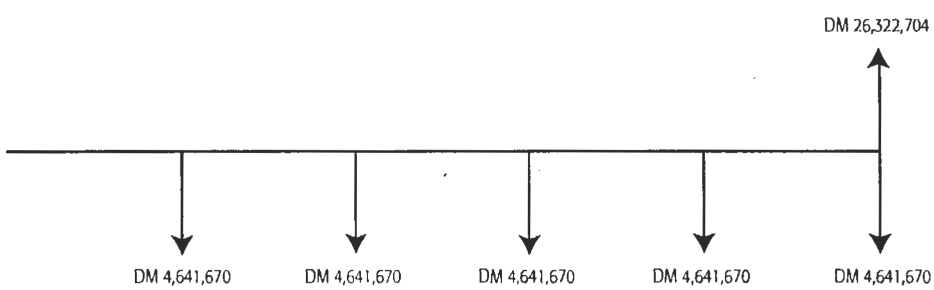


B. Draw time lines for the two instruments that will replicate the German zero, showing the cash flow amounts at each time, under the assumption of constant interest rates.

DM FRN:



Zero-Coupon Receive-Fixed DM Interest Rate Swap:



C. State the transactions necessary to replicate the German zero.

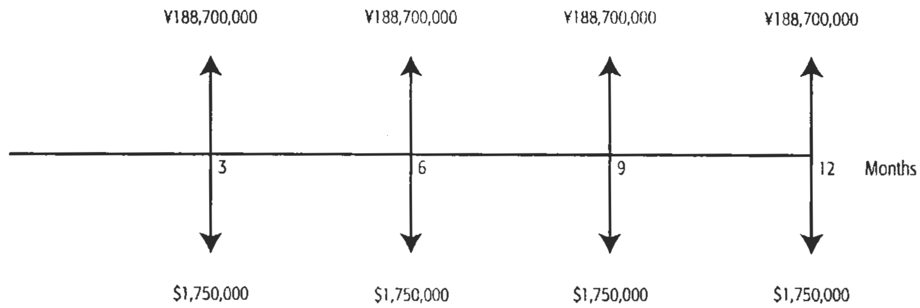
Replicating the DM zero-coupon bond can be accomplished by buying a DM FRN with a maturity of five years and with an initial outlay of DM 73,677,296. At present rates, the annual coupon payment would be DM 4,641,670. Also, one would enter a DM zero-coupon interest rate swap to receive a fixed payment at the termination of the swap in the amount of DM 26,322,704 and to make five floating rate payments at annual intervals. At present rates, the annual floating payment would be DM 4,641,670.

D. Explain how the replication you have created still works if interest rates change.

If rates change, the floating payments will vary on both the FRN and the zero-coupon interest rate swap. However, they will still be the same quantity on the two instruments and will still exactly offset each other. Therefore, the replication will be maintained with a net outflow at $t = 0$ of DM 73,677,296 and a net inflow at $t = 5$ of DM 100,000,000.

8. Consider an already existing fixed-for-fixed foreign currency swap that was negotiated at an exchange rate of \$1 = ¥111 with a notional principal of \$100 million = ¥11.1 billion. The swap has quarterly payments and a remaining tenor of one year. The dollar fixed rate is 7 percent, and the yen fixed rate is 6.8 percent. Assume that the following foreign exchange rates are observed now, where ${}_{\$,¥}FX_{x,y}$ is the value of the dollar in terms of yen for a contract initiated at month x with delivery at month y: ${}_{\$,¥}FX_{0,3} = 115$; ${}_{\$,¥}FX_{0,6} = 117$; ${}_{\$,¥}FX_{0,9} = 119$; ${}_{\$,¥}FX_{0,12} = 120$.

A. Prepare a time line showing the remaining payments on the swap from the point of view of the dollar payer.



B. Calculate the present value of the swap from the point of view of the dollar payer if the following US interest rates hold: $FRA_{0,3} = 0.0635$; $FRA_{3,6} = 0.0640$; $FRA_{6,9} = 0.0642$; $FRA_{9,12} = 0.0645$.

Given the forward exchange rates stated above, the dollar value of each of the four yen payments is:

Payment 1: ¥188,700,000/115 = $1,640,870
Payment 2: ¥188,700,000/117 = $1,612,821
Payment 3: ¥188,700,000/119 = $1,585,714
Payment 4: ¥188,700,000/120 = $1,572,500

The net dollar payments from the point of view of the dollar payer are:

Net Payment 1: = $1,640,870 − $1,750,000 = −$109,130
Net Payment 2: = $1,612,821 − $1,750,000 = −$137,179
Net Payment 3: = $1,585,714 − $1,750,000 = −$164,286
Net Payment 4: = $1,572,500 − $1,750,000 = −$177,500

The discount rates are as follows:

$Z_{0,3} = 1 + 0.0635/4 = 1.015875$
$Z_{0,6} = 1.015875\,(1 + 0.0640/4) = 1.032129$
$Z_{0,9} = 1.032129\,(1 + 0.0642/4) = 1.048695$
$Z_{0,12} = 1.048695\,(1 + 0.0645/4) = 1.065605$

From the point of view of the dollar payer, the present value (PV) of the remaining swap commitment is:

$$PV = \frac{-\$109{,}130}{1.015875} - \frac{\$137{,}179}{1.032129} - \frac{\$164{,}286}{1.048695} - \frac{\$177{,}500}{1.065605}$$
$$= -\$107{,}425 - \$132{,}909 - \$156{,}658 - \$166{,}572$$
$$= -\$563{,}564$$

C. How does the solution of this problem relate to the interest rate parity theorem?

The solution evaluated the swap from the point of view of the US dollar payer. In doing so, each yen cash flow was converted into dollars at the foreign exchange forward rate corresponding to the timing of the yen flow. These converted dollars were then discounted at the US dollar rate. In essence, the combination of the US interest rate with the forward foreign exchange rate takes account of the interest rate parity between the dollar and the yen. The solution did not need to confront the yen interest rates directly, because they were effectively considered by using the US interest rate and the forward foreign exchange rates.

9. Today you observe the following US interest rates: $FRA_{0,3} = 0.0650$; $FRA_{3,6} = 0.0655$; $FRA_{6,9} = 0.0659$; $FRA_{9,12} = 0.0661$, where the subscripts pertain to months. Consider a plain vanilla interest rate swap with

quarterly payments and a tenor of one year with a notional principal of $50 million and a zero-cost collar that parallels this swap, having four payments and a remaining life of one year, with a notional principal of $50 million. The collar has a common strike rate of 6.5 percent for the put (floor) and the call (cap).

A. Without computation, determine whether the collar is fairly priced. Explain.

The collar cannot be fairly priced, since the strike rate is 6.50 percent, the price of the collar is zero, and the term structure lies above that rate. Because rates start at 6.5 percent and rise from there, the *SFR* must exceed 6.50 percent, and the zero-cost collar should have a strike rate that equals the *SFR*.

B. Find the *SFR* for a plain vanilla swap.

The discount rates are as follows:

$$Z_{0,3} = 1 + 0.0650/4 = 1.016250$$

$$Z_{0,6} = 1.016250\,(1 + 0.0655/4) = 1.032891$$

$$Z_{0,9} = 1.032891\,(1 + 0.0659/4) = 1.049908$$

$$Z_{0,12} = 1.049908\,(1 + 0.0661/4) = 1.067258$$

Next, we apply Equation 20.5, treating the numerator and denominator separately for convenience:

$$NUMERATOR = \frac{0.0650}{1.016250} + \frac{0.0655}{1.032891} + \frac{0.0659}{1.049908} + \frac{0.0661}{1.067258} = 0.252076$$

$$DENOMINATOR = \frac{1}{1.016250} + \frac{1}{1.032891} + \frac{1}{1.049908} + \frac{1}{1.067258} = 3.841611$$

$$SFR = \frac{0.252076}{3.841611} = 0.065617$$

C. Explain an arbitrage strategy based on the facts presented, being sure to state the transactions you would make to exploit the arbitrage.

A long call/short put option portfolio with both options having a common strike rate pays (LIBOR − strike rate) × notional principal. A plain vanilla pay-fixed interest rate swap pays (LIBOR − *SFR*) × notional principal. These values would require adjustment for swaps with payment intervals other than one year. In our problem, the common strike rate is 6.5 percent, the *SFR* is 6.5617 percent, and payments occur quarterly.

If we buy the zero-cost collar with a strike rate of 6.5 percent and sell the pay-fixed swap with an *SFR* of 6.5617 percent, our cost will be zero. (Selling a pay-fixed swap is equivalent to entering a receive-fixed swap.) The payoff on a single date will be as follows:

Collar: (LIBOR/4 − 0.065/4) × $50,000,000
Receive-Fixed Swap: (0.065617/4 − LIBOR/4) × $50,000,000
Combined Position: (0.065617/4 − 0.065/4) × $50,000,000 = $7,712.50

Therefore, this arbitrage strategy will yield four inflows of $7,712.50 with no outflows.

D. Compute the present value of your arbitrage transactions.

$$PV = \frac{\$7,712.50}{1.016250} + \frac{\$7,712.50}{1.032891} + \frac{\$7,712.50}{1.049908} + \frac{\$7,712.50}{1.067258} = \$29,628.43$$

10. For a fairly priced plain vanilla swap with annual payments and a tenor of five years, the fixed rate is 6.44 percent. The yield curve is flat. You also are aware of two bonds available in the marketplace, each with annual coupon payments and maturities of five years. The first is a coupon bond with a coupon rate of 8.25 percent. The second is an FRN paying LIBOR + 2 percent. Assume zero default risk for all instruments.

A. Without computation, but by inspecting the information given, which bond should be worth more?

With a flat yield curve, all FRA rates and discount rates will be equal to the fairly priced swap fixed rate of 6.44 percent. The FRN will, therefore, pay 8.44 percent, so it should be worth more than the coupon bond paying 8.25 percent.

B. Compute the no-arbitrage price difference between the two bonds.

Assuming a par value of 100 on both bonds, the FRN will pay 8.44 − 8.25 = 0.19 more each period. Therefore, the price differential should be equal to the present value of those flows, discounted at the rate of 6.44 percent.

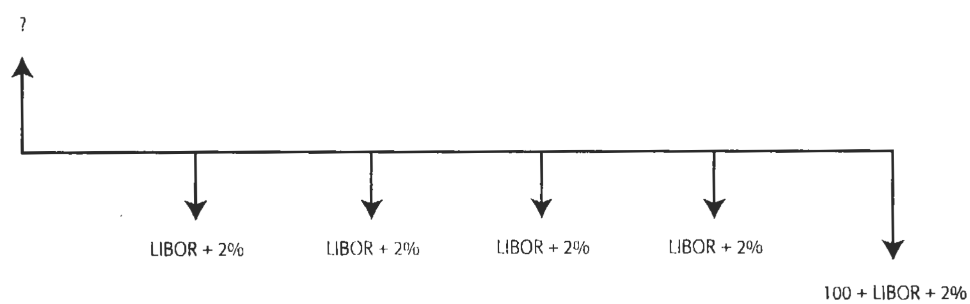
$$PV = \frac{0.19}{1.0644} + \frac{0.19}{1.0644^2} + \frac{0.19}{1.0644^3} + \frac{0.19}{1.0644^4} + \frac{0.19}{1.0644^5} = 0.790859$$

C. Explain what arbitrage transactions you would enter if the price difference between the bonds were 6 percent of par. (Assume that the truly more valuable bond is priced higher than the truly less valuable bond, but the price difference is 6 percent instead of the no-arbitrage price difference you found in part B of this question.) Draw a time line for each instrument assuming a par value of 100. Hint: You will need a third instrument, some kind of interest rate swap, to ensure that the transaction is an arbitrage opportunity rather than a speculation.

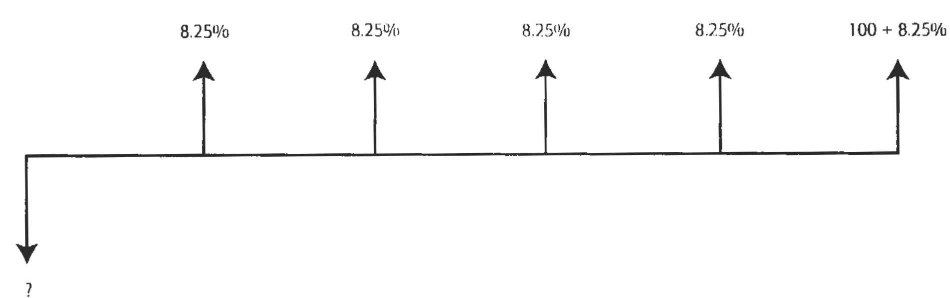
The price difference should be 0.79 percent of par, not 6 percent. Therefore, if the FRN is priced so much higher than the coupon bond, we can create an arbitrage opportunity by selling the overpriced instrument and buying the underpriced one. This requires selling the FRN and buying the coupon bond. Doing only this leaves a risk exposure and is not arbitrage, however.

Instead, the arbitrageur should: sell the FRN, buy the coupon bond, and initiate a pay-fixed plain vanilla interest rate swap, as shown below, in a diagram assuming par values of 100 on each instrument:

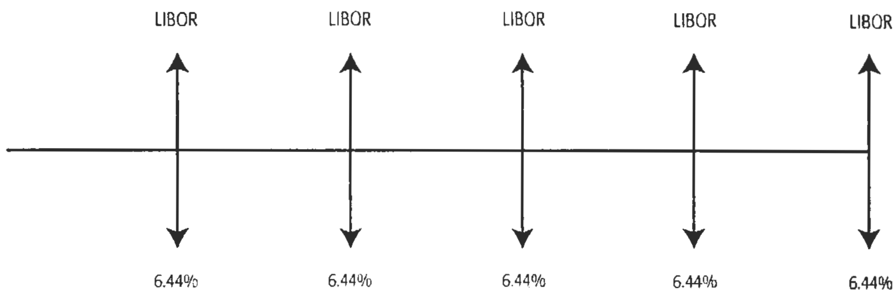
Sell FRN:



Buy Coupon Bond:



Initiate Pay-Fixed Swap:



The prices of the bonds are unknown, but the price difference is 6 percent of par by assumption, so there is a net inflow of 6 percent of par at $t = 0$. At each coupon date, the net cash flow is:

$$-\text{LIBOR} - 2\% + 8.25\% + \text{LIBOR} - 6.44\% = -0.19\%$$

As we have already seen, the present value of those five coupon flows is 0.790859 percent of par. At maturity of the bonds, 100 percent of par is due on the FRN and 100 percent of par is received on the coupon bond for a net zero payment. Therefore, the arbitrage has a present value of 6 percent of par from the price differential of the bonds, less 0.790859 percent of par for the coupon outflows. Also, this is an arbitrage because all uncertainty has been eliminated with respect to interest rates.

11. Consider two term structure environments, one in which the yield curve rises and one in which the yield curve falls by the same amount, as shown in the table below.

Maturity (subscripts pertain to years)	**FRA Rates Environment #1**	**FRA Rates Environment #2**
$FRA_{0,1}$	0.0600	0.0600
$FRA_{1,2}$	0.0610	0.0590
$FRA_{2,3}$	0.0620	0.0580
$FRA_{3,4}$	0.0630	0.0570
$FRA_{4,5}$	0.0640	0.0560

A. For a plain vanilla interest rate swap with a five-year tenor, annual payments made in arrears, and a notional principal of $30 million, find the *SFR* for this swap in each yield curve environment.

The discount rates are as follows:

Environment #1
$Z_{0,1} = 1.06$
$Z_{0,2} = 1.06 \times 1.0610 = 1.124660$
$Z_{0,3} = 1.124660 \times 1.0620 = 1.194389$
$Z_{0,4} = 1.194389 \times 1.0630 = 1.269635$
$Z_{0,5} = 1.269635 \times 1.0640 = 1.350892$

Environment #2
$Z_{0,1} = 1.06$
$Z_{0,2} = 1.06 \times 1.0590 = 1.122540$
$Z_{0,3} = 1.122540 \times 1.0580 = 1.187647$
$Z_{0,4} = 1.187647 \times 1.0570 = 1.255343$
$Z_{0,5} = 1.255343 \times 1.0560 = 1.325642$

For the two environments, the *SFR*s are:

$$SFR_1 = \frac{\frac{0.0600}{1.06} + \frac{0.0610}{1.124660} + \frac{0.0620}{1.194389} + \frac{0.0630}{1.269635} + \frac{0.0640}{1.350892}}{\frac{1}{1.06} + \frac{1}{1.124660} + \frac{1}{1.194389} + \frac{1}{1.269635} + \frac{1}{1.350892}} = 0.061879$$

$$SFR_2 = \frac{\frac{0.0600}{1.06} + \frac{0.0590}{1.122540} + \frac{0.0580}{1.187647} + \frac{0.0570}{1.255343} + \frac{0.0560}{1.325642}}{\frac{1}{1.06} + \frac{1}{1.122540} + \frac{1}{1.187647} + \frac{1}{1.255343} + \frac{1}{1.325642}} = 0.058112$$

B. Find the present value of each payment from the point of view of the pay-fixed party in each environment. Find the sum of the present value of the payments in each environment.

	Present Value of Pay-Fixed Cash Flows—Environment #1		
Payment #	**Amount**	**Zero-Coupon Factor**	**Present Value**
1	(−0.061879 + 0.06) × $30,000,000 = −$56,370.72	1.06	-$53,179.92
2	(−0.061879 + 0.0610) × $30,000,000 = −$26,370.72	1.124660	-$23,447.72
3	(−0.061879 + 0.0620) × $30,000,000 = $3,630.00	1.194389	$3,038.61
4	(−0.061879 + 0.0630) × $30,000,000 = $33,629.28	1.269635	$26,487.35
5	(−0.061879 + 0.0640) × $30,000,000 = $63,629.28	1.350892	$47,101.68

	Present Value of Pay-Fixed Cash Flows—Environment #2		
Payment #	**Amount**	**Zero-Coupon Factor**	**Present Value**
1	(-0.058112 + 0.06) × $30,000,000 = $56,647.89	1.06	$53,441.41
2	(-0.058112 + 0.0590) × $30,000,000 = $26,647.89	1.12254	$23,738.93
3	(-0.058112 + 0.0580) × $30,000,000 = -$3,352.11	1.187647	-$2,822.48
4	(-0.058112 + 0.0570) × $30,000,000 = -$33,352.11	1.255343	-$26,568.12
5	(-0.058112 + 0.0560) × $30,000,000 = -$63,352.11	1.325642	-$47,789.74

The present values necessarily sum to zero in each environment.

C. Evaluate the default risk on these two swaps from the point of view of a swap bank in which the swap bank pays fixed, assuming a relatively stable yield curve environment.

As with any fairly priced plain vanilla swap, the present value of all the payments is zero when the swap is initiated. However, the pattern of positive and negative present value payments is quite different. From the point of view of the pay-fixed party, the positive present value payments occur later in Environment #1 and earlier in Environment #2. This suggests that Environment #1 is riskier for a pay-fixed party, because the pay-fixed party expects to lose on the early payments and make it up on the later. If the pay-fixed party suffers a default after two payments in Environment #1, for example, he never gets any positive present value payments. This contrasts markedly with Environment #2.

D. Explain how any observed difference in default risk from the point of view of the bank might affect the bank's pricing of the swap.

A rising yield curve (Environment #1) exposes a pay-fixed party to greater default risk than a falling yield curve environment, because of the timing of positive and negative present value cash flows in each situation. Therefore, we might expect a swap dealer to demand slightly more favorable terms to take the pay-fixed side of a swap in a rising yield curve environment.

Note: The remaining questions all use the same interest rate data, the same foreign exchange data, the same notional principal, and the same tenor.

Interest Rate Data: Problem 12
Foreign Exchange Data: Problem 13
Swap Terms: Semiannual payments; $100,000,000 = ¥13,350,000,000 notional principal
Tenor: 5 years

12. The following data pertain to US and Japanese interest rates over the next five years for semiannual periods. Complete the following tables.

	US Rates		
Maturity (Semiannual Periods)	**Annualized Par Yield**	**Zero-Coupon Factor**	**Forward Rate Factor**
1	0.067700	1.03385000	1.03385000
2	0.068504	1.06969145	1.03466794
3	0.069367	1.10776466	1.03559271
4	0.070146	1.14800662	1.03632717
5	0.070294	1.18872682	1.03547035
6	0.071231	1.23416869	1.03822735
7	0.071893	1.28132127	1.03820594
8	0.071988	1.32793480	1.03637926
9	0.072105	1.37654384	1.03660499
10	0.072293	1.42770275	1.03716475

	Japanese Rates		
Maturity (Semiannual Periods)	**Annualized Par Yield**	**Zero-Coupon Factor**	**Forward Rate Factor**
1	0.048327	1.02416350	1.02416350
2	0.049302	1.04992199	1.02515076
3	0.049428	1.07600705	1.02484475
4	0.050119	1.10414448	1.02614986
5	0.050766	1.13369515	1.02676341
6	0.051651	1.16569027	1.02822197
7	0.052546	1.19976103	1.02922797
8	0.053157	1.23451641	1.02896859
9	0.053170	1.26740872	1.02664388
10	0.053356	1.30243902	1.02763931

Values for these tables were found by applying the bootstrapping method.

13. Find the foreign exchange rates between the US dollar and the Japanese yen for the next ten semiannual periods consistent with the interest rates of problem 12, and complete the following table.

	Dollar/Yen Exchange Rates	
Maturity (Semiannual Periods)	**$ per Yen**	**Yen per $**
0	0.00749064	133.500000
1	0.00756148	132.249192
2	0.00763168	131.032725
3	0.00771172	129.672796
4	0.00778820	128.399336
5	0.00785425	127.319667
6	0.00793067	126.092691
7	0.00799985	125.002294
8	0.00805747	124.108458
9	0.00813565	122.915856
10	0.00821106	121.786982

Given the spot exchange rates and the term structures for the two countries, the forward exchange rates of the dollar versus the yen can be found by applying the interest rate parity theorem. As an example, we compute the value of the dollar in terms of yen for a horizon of 3.5 years (7 semiannual periods). From Equation 21.1, we have:

$$ {}_{x}Z_{t,\,T} = {}_{x,\,y}FX_{t,\,t} \times {}_{y}Z_{t,\,T} \times {}_{y,\,x}FX_{t,\,T} $$

In terms of our example, we have:

${}_{x}Z_{t,\,T}$ = the 7-period zero-coupon factor for the United States = 1.28132127
${}_{x,\,y}FX_{t,\,t}$ = the spot exchange rate = 133.5
${}_{y}Z_{t,\,T}$ = the 7-period zero-coupon factor for Japan = 1.19976103
${}_{y,\,x}FX_{t,\,T}$ = the unknown foreign exchange forward rate = 125.0022935

The table shows all of the resulting calculations.

14. Based on the US rates of problem 12, find the SFR for a plain vanilla US interest rate swap with a tenor of five years, semiannual payments, and a notional principal of $100,000,000.

For a plain vanilla interest rate swap, the SFR is given by Equation 21.13:

$$ SFR = \frac{\sum_{n=1}^{N} \frac{FRA_{(n-1)\times MON,\, n\times MON}}{Z_{0,\, n\times MON}}}{\sum_{n=1}^{N} \frac{1}{Z_{0,\, n\times MON}}} $$

The following table shows the intermediate computations:

	US Rates			
Maturity (Semiannual Periods)	**Zero-Coupon Factor**	**$1/(Z_{0,t})$**	**Forward Rate $FRA_{t-1,t}$**	**$FRA_{t-1,t}/Z_{0,t}$**
1	1.03385000	0.96725831	0.03385000	0.03274169
2	1.06969145	0.93484902	0.03466794	0.03240929
3	1.10776466	0.90271881	0.03559271	0.03213021
4	1.14800662	0.87107512	0.03632717	0.03164369
5	1.18872682	0.84123617	0.03547035	0.02983894
6	1.23416869	0.81026201	0.03822735	0.03097417
7	1.28132127	0.78044439	0.03820594	0.02981761
8	1.32793480	0.75304902	0.03637926	0.02739537
9	1.37654384	0.72645707	0.03660499	0.02659195
10	1.42770275	0.70042591	0.03716475	0.02603115
SUMS		8.28777582		0.29957408

For the plain vanilla swap, the semiannual $SFR = 0.29957408/0.828777582 = 0.03614650$, so the annualized SFR equals $2 \times 0.03614650 = 0.07229300$. Note also that the SFR on the swap will equal the coupon rate on a fixed coupon bond of the same maturity that trades at par. From the completed table of problem 12, that rate is 0.072293.

15. Based on the Japanese rates of problem 12, find the SFR for a plain vanilla Japanese interest rate swap with a tenor of five years, a notional principal of ¥13,350,000,000, and semiannual payments.

The SFR on the swap will equal the coupon rate on a fixed coupon bond of the same maturity that trades at par. From the completed table of problem 12, that rate is 0.053356. It can also be computed directly as illustrated for the US swap in the preceding problem.

16. Complete the following tables, based on the US and Japanese interest rates of problem 12, the foreign exchange rates of question 13, and the SFRs computed in questions 14 and 15. The following letters correspond to column labels in the tables.

A. Cash flows on a US semiannual coupon bond with a five-year maturity, a par value of $100,000,000, which trades at par.

B. The cash flows consistent with the term structure of problem 12 for a semiannual US dollar floating rate bond with a par value of $100,000,000 and a maturity of five years.

C. The cash flows on the receive-fixed side of the US dollar interest rate swap computed in question 14.

D. The cash flows consistent with the term structure on the pay-floating side of the US dollar interest rate swap computed in question 14.

E. The cash flows on a Japanese semiannual coupon bond with a five-year maturity, a par value of ¥13,350,000,000, which trades at par.

F. The cash flows consistent with the term structure of problem 12 on a semiannual Japanese yen floating rate bond with a par value of ¥13,350,000,000 and a maturity of five years.

G. The cash flows on the pay-fixed side of the Japanese yen interest rate swap computed in question 15.

H. The cash flows on the pay-floating side of the Japanese yen interest rate swap computed in question 15.

	US Instruments			
Maturity (Semiannual Periods)	**A Buy Semiannual Bond**	**B Sell FRN**	**C Receive-Fixed Interest Rate Swap**	**D Pay-Floating Interest Rate Swap**
0	−$100,000,000	$100,000,000	$0	$0
1	3,614,650	−3,385,000	3,614,650	−3,385,000
2	3,614,650	−3,466,794	3,614,650	−3,466,750
3	3,614,650	−3,559,271	3,614,650	−3,559,346
4	3,614,650	−3,632,717	3,614,650	−3,632,720
5	3,614,650	−3,547,035	3,614,650	−3,547,017
6	3,614,650	−3,822,735	3,614,650	−3,822,745
7	3,614,650	−3,820,594	3,614,650	−3,820,546
8	3,614,650	−3,637,926	3,614,650	−3,637,964
9	3,614,650	−3,660,499	3,614,650	−3,660,495
10	103,614,650	−103,716,475	3,614,650	−3,716,481
Present Value of Cash Flows	0	0	$29,957,408	−$29,957,408

	Japanese Instruments			
Maturity (Semiannual Periods)	**E Buy Semiannual Bond**	**F Sell FRN**	**G Receive-Fixed Interest Rate Swap**	**H Pay-Floating Interest Rate Swap**
0	−¥13,350,000,000	¥13,350,000,000	¥0	¥0
1	356,151,300	−322,589,400	356,151,300	−322,589,400
2	356,151,300	−335,765,850	356,151,300	−335,765,850
3	356,151,300	−331,680,750	356,151,300	−331,680,750
4	356,151,300	−349,102,500	356,151,300	−349,102,500
5	356,151,300	−357,286,050	356,151,300	−357,286,050
6	356,151,300	−376,763,700	356,151,300	−376,763,700
7	356,151,300	−390,193,800	356,151,300	−390,193,800
8	356,151,300	−386,736,150	356,151,300	−386,736,150
9	356,151,300	−355,697,400	356,151,300	−355,697,400
10	13,706,151,300	−13,718,980,650	356,151,300	−368,980,650
Present Value of Cash Flows	0	0	¥3,099,999,852	¥3,099,999,852

For the floating instruments, the cash flow consistent with the term structure equals the forward rate times the notional principal, being careful to adjust the floating rate for the periodicity of the swap payments. We already have the semiannual *FRF*s in the table for problem 12. So, using those forward rates and the notional principal of $100,000,000, the following table gives the payments on the US dollar FRN:

	US Rates	
Maturity (Semiannual Periods)	**Forward Rate $FRA_{t-1,t}$**	**Floating Payment $FRA_{t-1,t} \times NP$**
1	0.03385000	3,385,000
2	0.03466794	3,466,794
3	0.03559271	3,559,271
4	0.03632717	3,632,717
5	0.03547035	3,547,035
6	0.03822735	3,822,735
7	0.03820594	3,820,594
8	0.03637926	3,637,926
9	0.03660499	3,660,499
10	0.03716475	3,716,475

For the floating rate payments on the Japanese bond, the Japanese forward rates are used, along with the notional principal of ¥13,350,000,000.

	Japanese Rates	
Maturity (Semiannual Periods)	**Forward Rate $FRA_{t-1,t}$**	**Floating Payment $FRA_{t-1,t} \times NP$**
1	0.02416350	322,582,725
2	0.02515076	335,762,646
3	0.02484475	331,677,412
4	0.02614986	349,100,631
5	0.02676341	357,291,524
6	0.02822197	376,763,299
7	0.02922797	390,193,399
8	0.02896859	386,730,677
9	0.02664388	355,695,798
10	0.02763931	368,984,789

Note that the present values of the two sides of the swap are the same, as they must be for fairly priced swaps.

17. Based on the tables of the preceding question, explain how a US dollar plain vanilla receive-fixed interest rate swap is equivalent to a portfolio of bonds. Show how to replicate the swap position with a bond portfolio.

    As the first table in question 16 shows, if we buy the semiannual bond and sell the FRN, the resulting cash flows are identical to the cash flows on the receive-fixed swap.

18. Explain the transactions necessary to replicate a US dollar plain vanilla receive-fixed swap as a portfolio of FRAs.

    An interest rate swap is a portfolio of off-market FRAs. To replicate the receive-fixed swap, one would enter 10 off-market FRAs. In each, one would contract to receive $3,614,650 and to pay the forward rate times the half-year periodicity of the swap payments, times the notional principal. The floating rate payments implied by the term structure appear in column D of the table in problem 16.

19. Based on the previous calculations, complete the following tables detailing the cash flows for a fixed-for-fixed currency swap. What is the expected present value benefit or loss on the periodic payments for the dollar payer? What is the expected present value benefit or loss on the reexchange of principal for the dollar

payer? Explain the portfolio of capital market instruments that would replicate this fixed-for-fixed swap from the point of view of the dollar payer.

	Fixed-for-Fixed Currency Swap–Dollar Payer Perspective			
	Actual Cash Flows		Dollar Value of Cash Flows	
Date (Semiannual period)	Receipts	Payments	Receipts	Payments
0	$100,000,000	¥13,350,000,000	$100,000,000	$100,000,000
1	¥356,151,300	$3,614,650	2,693,032	3,614,650
2	¥356,151,300	$3,614,650	2,718,033	3,614,650
3	¥356,151,300	$3,614,650	2,746,538	3,614,650
4	¥356,151,300	$3,614,650	2,773,778	3,614,650
5	¥356,151,300	$3,614,650	2,797,300	3,614,650
6	¥356,151,300	$3,614,650	2,824,520	3,614,650
7	¥356,151,300	$3,614,650	2,849,158	3,614,650
8	¥356,151,300	$3,614,650	2,869,678	3,614,650
9	¥356,151,300	$3,614,650	2,897,521	3,614,650
10	¥356,151,300	$3,614,650	2,924,379	3,614,650
10	¥13,350,000,000	$100,000,000	$109,617,627	$100,000,000

	Fixed-for-Fixed Currency Swap–Dollar Payer Perspective		
	Present Value of Cash Flows		
Date (Semiannual Periods)	Receipts	Payments	Net
0	$100,000,000	$100,000,000	0
1	2,604,858	3,496,300	-891,443
2	2,540,951	3,379,152	-838,201
3	2,479,352	3,263,013	-783,661
4	2,416,169	3,148,632	-732,463
5	2,353,190	3,040,774	687,584
6	2,288,601	2,928,814	-640,212
7	2,223,609	2,821,033	-597,424
8	2,161,008	2,722,009	-561,000
9	2,104,925	2,625,888	-520,963
10	2,048,311	2,531,795	-483,484
10	76,779,026	70,042,591	6,736,435
SUMS	$200,000,000	$200,000,000	0

The dollar payer expects to lose a total of $6,736,436 on the periodic payments in present value terms (the sum of the first 11 elements in the final column of the table) and make up the same amount on the reexchange of principal (the last element of the final column). The dollar payer could replicate her swap by buying a yen-denominated coupon bond and by issuing a dollar-denominated coupon bond, both of the appropriate principal and timing.

20. Consider again the fixed-for-fixed currency swap of the preceding question. Explain how this swap could be replicated in the foreign exchange forward market, detailing the FOREX forward contracts necessary to replicate the swap.

A fixed-for-fixed currency swap can be replicated by a portfolio of foreign exchange transactions. First, in the spot market, the dollar payer would exchange ¥13,350,000,000 for $100,000,000 at the prevailing spot rate. Next, the dollar payer would require a sequence or strip of 10 off-market FOREX forward transactions to pay $3,614,650 and receive ¥356,151,300 each six months for the next five years. Finally, the dollar payer would need a forward contract to pay ¥13,350,000,000 and receive $100,000,000 at the end of the five years. This would also be an off-market transaction. The cumbersome nature of these transactions, and the fact that the replication requires so many off-market FOREX forwards, indicates the operational advantage of the swap contract.

21. Explain how a plain vanilla dollar-pay currency swap can be replicated using capital market instruments as the key elements of the replicating portfolio. Illustrate the replicating cash flows in a table.

To synthesize a plain vanilla dollar-pay currency swap one buys a Japanese coupon bond and sells a dollar-denominated FRN. At initiation, the yen bond costs ¥13,350,000,000 and the sale of the dollar bond yields an inflow of $100,000,000. Each semi-annual period, the trader receives ¥356,151,300 and pays US six-month LIBOR × 0.5 × $100,000,000. At period 10, in addition to the periodic payment, the trader receives ¥13,350,000,000 and pays $100,000,000. As the table shows, these are exactly the cash flows on a plain vanilla dollar-pay currency swap.

	Synthesizing Instruments		Synthesized Currency Swap	
Semiannual Periods	Buy Japanese Coupon Bond	Sell FRN $ Bond	Receipts	Payments
0	– ¥13,350,000,000	$100,000,000	$100,000,000	– ¥13,350,000,000
1	356,151,300	– 3,385,000	¥356,151,300	– $3,385,000
2	356,151,300	– 3,466,750	¥356,151,300	– $3,466,750
3	356,151,300	– 3,559,346	¥356,151,300	– $3,559,346
4	356,151,300	– 3,632,720	¥356,151,300	– $3,632,720
5	356,151,300	– 3,547,017	¥356,151,300	– $3,547,017
6	356,151,300	– 3,822,745	¥356,151,300	– $3,822,745
7	356,151,300	– 3,820,546	¥356,151,300	– $3,820,546
8	356,151,300	– 3,637,964	¥356,151,300	– $3,637,964
9	356,151,300	– 3,660,495	¥356,151,300	– $3,660,495
10	356,151,300	– 3,716,481	¥356,151,300	– $3,716,481
10	¥13,350,000,000	– $100,000,000	¥13,350,000,000	– $100,000,000

22. Explain how a yen fixed-pay plain vanilla currency swap could be replicated by combining two other swaps. Prepare a table showing the replicating cash flows.

A plain vanilla currency swap can be replicated by a fixed-for-fixed currency swap, combined with a plain vanilla interest rate swap. If a party enters the dollar pay-fixed swap that we have analyzed and also enters a plain vanilla interest rate swap, the cash flows would be as shown in the following table:

	Synthetic Plain Vanilla Currency Swap–Dollar Payer Perspective					
Date (Semiannual Period)	Dollar-Pay Fixed-for-Fixed Currency Swap		Receive-Fixed Interest Rate Swap		Resulting Synthetic Plain Vanilla Currency Swap	
	Receipts	Payments	Receipts	Payments	Receipts	Payments
0	$100,000,000	¥13,350,000,000	$0	$0	$100,000,000	¥13,350,000,000
1	¥356,151,300	$3,614,650	3,614,650	– 3,385,000	¥356,151,300	– 3,385,000
2	¥356,151,300	$3,614,650	3,614,650	– 3,466,750	¥356,151,300	– 3,466,750
3	¥356,151,300	$3,614,650	3,614,650	– 3,559,346	¥356,151,300	– 3,559,346
4	¥356,151,300	$3,614,650	3,614,650	– 3,632,720	¥356,151,300	– 3,632,720
5	¥356,151,300	$3,614,650	3,614,650	– 3,547,017	¥356,151,300	– 3,547,017
6	¥356,151,300	$3,614,650	3,614,650	– 3,822,745	¥356,151,300	– 3,822,745
7	¥356,151,300	$3,614,650	3,614,650	– 3,820,546	¥356,151,300	– 3,820,546
8	¥356,151,300	$3,614,650	3,614,650	– 3,637,964	¥356,151,300	– 3,637,964
9	¥356,151,300	$3,614,650	3,614,650	– 3,660,495	¥356,151,300	– 3,660,495
10	¥356,151,300	$3,614,650	3,614,650	– 3,716,481	¥356,151,300	– 3,716,481
10	¥13,350,000,000	$100,000,000	$0	$0	¥13,350,000,000	$100,000,000

20 The Swaps Market: Introduction

Answers to Questions and Problems

1. Explain the differences between a plain vanilla interest rate swap and a plain vanilla currency swap.

   In a plain vanilla interest rate swap, one party pays a fixed rate of interest based on a given nominal amount, while the second party pays a floating rate of interest based on the same nominal amount. No principal is exchanged in the agreement. In a plain vanilla foreign currency swap, there are three different sets of cash flows. First, at the initiation of the swap, the two parties actually do exchange cash. Second, the parties make periodic interest payments to each other during the life of the swap agreement. In the plain vanilla currency swap, one party typically pays dollars at a floating rate, and the payer of the nondollar currency pays a fixed rate. Third, at the termination of the swap, the parties again exchange the principal.

2. Explain the role that the notional principal plays in understanding interest rate swap transactions. Why is this principal amount regarded as only notional? How does it compare with a deliverable instrument in the interest rate futures market?

   The deliverable instrument in the interest rate futures market is like the notional principal in determining the scale of the daily settlement cash flows on the futures contracts. For many futures, however, actual delivery is possible but also avoidable. With an interest rate swap, the notional principal is not ever delivered. If we think of the Eurodollar futures contract, which is only cash-settled, the analogy between the notional principal and the underlying $1,000,000 Eurodollar deposit on a futures is quite close.

   In interest rate swaps, all of the cash flows are based on a notional amount—notional, because the notional principal is not actually paid. This is essentially a matter of convenience in helping to conceptualize the transaction. The entire contract could be stated without regard to the principal amount. One definition of *notional* is "existing in idea only."

3. Consider a plain vanilla interest rate swap. Explain how the practice of net payments works.

   In a typical interest rate swap, each party is scheduled to make payments to the other at certain dates. For the fixed payer, these amounts are certain, but the payments that the floating payer will have to make are unknown at the outset of the transaction. In each period, each party will owe a payment to the other. Rather than make two payments, the party owing the greater amount simply pays the difference between the two obligations.

4. Assume that you are a money manager seeking to increase the yield on your portfolio and that you expect short-term interest rates to rise more than the yield curve would suggest. Would you rather pay a fixed long-term rate and receive a floating short rate, or the other way around? Explain your reasoning.

You would prefer to pay a fixed long-term rate and receive a floating short-term rate. The initial short-term rate that you receive will merely be the spot rate that prevails today. However, if your hunch is correct, the short-term rate will rise more than the market expects, and you will then receive that higher rate. Because your payments are fixed, you will reap a profit from your insight.

5. Assume that the yield curve is flat, that the swaps market is efficient, and that two equally creditworthy counterparties engage in an interest rate swap. Who should pay the higher rate, the party that pays a floating short-term rate or the party that pays a fixed long-term rate? Explain.

   They should pay the same. If the yield curve is flat, short-term rates equal long-term rates. Barring a change in rates, the two parties should pay the same amounts to each other in each period. If interest rates change, however, the payments will no longer be the same. If interest rates rise, the party paying the floating payment will lose; if rates fall, the floating payer will benefit.

6. In a currency swap, counterparties exchange the same sums at the beginning and the end of the swap period. Explain how this practice relates to the custom of making interest payments during the life of the swap agreement.

   At the outset, the two parties exchange cash denominated in two currencies. Each party pays interest on the currency it receives from the other. Thus, the exchange of currencies is the basis for computing all of the interest payments that will be made over the life of the agreement. The interest payments can be either fixed or floating on both sides of the swap.

7. Explain why a currency swap is also called an "exchange of borrowings."

   In a currency swap, both parties pay and both parties receive actual cash at the outset of the transaction. In effect, each has borrowed from the other, so they have exchanged borrowings.

8. What are the two major kinds of swap facilitators? What is the key difference between the roles they play?

   They are swap brokers and swap bankers (swap dealers). The swap broker helps complete a swap by bringing counterparties together and perhaps by providing consulting services. The swap broker does not take a financial position in the transaction. By contrast, a swap banker or swap dealer will take a financial position to help the two parties complete their transaction. As the swap market has developed, the swap dealer has come to predominate. Most swap facilitators today are swap dealers who willingly act as counterparties to swaps.

9. In the context of interest rate swaps, "basis risk" is the risk arising from an unanticipated change in the yield relationship between the two instruments involved in the swap. Explain how basis risk affects a swap dealer. Does it affect a swap broker the same way? Explain.

   Basis risk affects a swap dealer because it changes the gross profit margin that the dealer will receive. For example, assume that a swap dealer agrees to pay LIBOR and receive the two-year T-note rate plus 60 basis points. This agreement is based upon the yield spread between LIBOR and the two-year T-note when the swap is initiated. This spread can change due to shifts in the term structure, but it can also change due to political disturbances or other causes. Basis risk arises from changes of the second kind. For example, political unrest in Europe might cause LIBOR to rise relative to US Treasury rates. In our example, the dealer would have to pay a higher rate without receiving any correlatively higher rate. This problem does not affect the swap broker, because the swap broker does not take a risk position in the transaction.

10. Assume a swap dealer attempts to function as a pure financial intermediary avoiding all interest rate risk. Explain how such a dealer may yet come to bear interest rate risk.

    Ideally, a pure financial intermediary would take no risk position in the transactions it helps to consummate. In the real world, however, there are few things that are pure. A swap dealer might wish to avoid all risk positions, but swap dealers enter many transactions that are likely to leave the dealer with an unbalanced portfolio and an exposure to interest rate risk. Thus, even risk-averse swap dealers often find themselves with undesired risk positions that must be hedged away.

11. Two parties enter an interest rate swap paid in arrears on the following terms: a seven-year tenor, annual payments, $100 million notional principal, a fixed rate of 6.75 percent, with LIBOR as the floating rate. Assume that the following LIBOR spot rates are observed at each of the following dates. From the perspective of the receive-fixed side of the deal, what is the cash flow at each payment date of the swap? What role does the swap rate observed at the termination of the swap (year 7) play in the analysis?

Year (Date of Observation)	One-Year LIBOR (Rate Actually Observed)	Receive Fixed Cash Flow
0	0.0680	$0
1	0.0575	$6,750,000 − $6,800,000 = −$50,000
2	0.0875	$6,750,000 − $5,750,000=$1,000,000
3	0.0674	$6,750,000 − $8,750,000= − $2,000,000
4	0.0600	$6,750,000 − $6,740,000= $10,000
5	0.0700	$6,750,000 − $6,000,000= $750,000
6	0.0655	$6,750,000 − $7,000,000= − $250,000
7	0.0685	$6,750,000 − $6,550,000= $200,000

The observation of LIBOR at year 7 when the swap terminates plays no role in the analysis of the swap, as it affects no payment. This is the case, because each floating payment on the swap is "determined in advance and paid in arrears."

12. A plain vanilla foreign currency swap has just been arranged between parties ABC and XYZ. ABC has agreed to pay dollars based on LIBOR, while XYZ will pay British pounds at a fixed rate of 7 percent. The current exchange rate is £1 = $1.65. The notional principal is £100 million = $165 million. The tenor of the swap is seven years, and the swap has annual payments paid in arrears. Complete the following table showing the **periodic** cash outflows only for each party at each relevant period of the swap. (Ignore the exchange of principal.)

Year of Observation	LIBOR Rate Observed (%)	XYZ Sterling Pay Outflows	ABC Dollar Pay Outflows
0	6.5800	0	0
1	5.870	£7,000,000	0.0658 × $165,000,000 = $10,857,000
2	6.745	£7,000,000	0.0587 × $165,000,000 = $9,685,500
3	6.550	£7,000,000	0.06745 × $165,000,000 = $11,129,250
4	6.100	£7,000,000	0.0655 × $165,000,000 = $10,807,500
5	6.800	£7,000,000	0.0610 × $165,000,000 = $10,065,000
6	6.350	£7,000,000	0.0680 × $165,000,000 = $11,220,000
7	6.450	£7,000,000	0.0635 × $165,000,000 = $10,477,500

13. A swap dealer holds the following portfolio of interest rate swaps, all with annual payments, all with floating payments equal to LIBOR.

Swap	Notional Principal ($ million)	Tenor (years)	Fixed Rate (%)	Dealer's Position
A	20	3	7.000	Receive-Fixed
B	30	5	6.500	Pay-Fixed
C	25	4	7.250	Pay-Fixed
D	50	7	7.300	Receive-Fixed
E	10	2	6.750	Receive-Fixed

A. Complete the following table showing the dealer's position for each payment in each year. For example, the entry for a given year t and a given swap with a fixed rate of 8 percent, and a notional principal of $15 million, would be of the form: $(LIBOR_{t-1} - 8.00) \times \$15,000,000$.

	The Swap Dealer's Anticipated Cash Flows					
Year	**Swap A**	**Swap B**	**Swap C**	**Swap D**	**Swap E**	**Dealer's Net Position**
1	\$1,400,000 $- LIBOR_0 \times$ \$20,000,000	−\$1,950,000 $+ LIBOR_0 \times$ \$30,000,000	−\$1,812,500 $+ LIBOR_0 \times$ \$25,000,000	\$3,650,000 $- LIBOR_0 \times$ \$50,000,000	\$675,000 $- LIBOR_0 \times$ \$10,000,000	\$1,962,500 $- LIBOR_0 \times$ \$25,000,000
2	\$1,400,000 $- LIBOR_1 \times$ \$20,000,000	−\$1,950,000 $+ LIBOR_1 \times$ \$30,000,000	−\$1,812,500 $+ LIBOR_1 \times$ \$25,000,000	\$3,650,000 $- LIBOR_1 \times$ \$50,000,000	\$675,000 $- LIBOR_1 \times$ \$10,000,000	\$1,962,500 $- LIBOR_1 \times$ \$25,000,000
3	\$1,400,000 $- LIBOR_2 \times$ \$20,000,000	−\$1,950,000 $+ LIBOR_2 \times$ \$30,000,000	−\$1,812,500 $+ LIBOR_2 \times$ \$25,000,000	\$3,650,000 $- LIBOR_2 \times$ \$50,000,000	0	\$1,287,500 $- LIBOR_2 \times$ \$15,000,000
4	0	−\$1,950,000 $+ LIBOR_3 \times$ \$30,000,000	−\$1,812,500 $+ LIBOR_3 \times$ \$25,000,000	\$3,650,000 $- LIBOR_3 \times$ \$50,000,000	0	−\$112,500 $+ LIBOR_3 \times$ \$5,000,000
5	0	−\$1,950,000 $+ LIBOR_4 \times$ \$30,000,000	0	\$3,650,000 $- LIBOR_4 \times$ \$50,000,000	0	\$1,700,000 $- LIBOR_4 \times$ \$20,000,000
6	0	0	0	\$3,650,000 $- LIBOR_5 \times$ \$50,000,000	0	\$3,650,000 $- LIBOR_5 \times$ \$50,000,000
7	0	0	0	\$3,650,000 $- LIBOR_6 \times$ \$50,000,000	0	\$3,650,000 $- LIBOR_6 \times$ \$50,000,000

B. Appraise the dealer's net risk position.

Across the years, the dealer's current position appears to be weighted toward the receive-fixed side of the swap market. But the dealer's ultimate exposure depends on the LIBOR rates that materialize. For example, in year 1, a $LIBOR_0$ higher than (about) 8 percent gives a net outflow for the year. If LIBOR rates are in the range of the dealer's fixed rate commitments (6.5 to 7.3 percent), then the dealer's position is weighted toward the receive-fixed side of the deal. The major risk facing the dealer is rising LIBOR rates.

C. Recommend transactions that the dealer might use to reduce the net risk.

The dealer needs to protect against rising short-term rates, particularly in the more distant horizon. In the ordinary conduct of swap dealing, the dealer could reduce exposure by acting as the pay-fixed counterparty. This would be particularly desirable for swaps with a tenor of 6–7 years. The dealer might also consider using Eurodollar futures to hedge some of the risk. For example, by selling Eurodollar futures with expirations that match the existing cash flows, the dealer could reduce interest rate risk. Note that any such effort would only protect against rate changes that arise in the future. To fully appraise the risk exposure of this dealer, it would be helpful to know the current term structure.

14. Consider the swap indication schedule shown in the table below. Two parties, A and B, arrange a plain vanilla interest rate swap with the Bank as intermediary. In effect, A and B are counterparties to each other as described below, but their individual swaps are actually negotiated with the Bank. Party A enters a receive-fixed plain vanilla swap, while Party B enters a pay-fixed plain vanilla swap. Both swaps have a notional principal of $50 million and a five-year tenor. Both swaps have annual payments made in arrears.

	Sample Swap Indication Pricing Bank's Fixed Rates: (T-Note Rate Plus Indicated Basis Points)		
Maturity (years)	**Bank Pays**	**Bank Receives**	**T-Note Yields (%)**
1	23	27	5.74
2	29	33	5.67
3	33	37	5.60
4	37	40	5.55
5	40	44	5.49

A. For each of the parties, state exactly the commitment that they undertake in their swap agreements.

Party A will receive a fixed rate of the 5-year T-Note rate plus 40 basis points, which equals 5.89 percent of a $50 million notional principal. Thus, the fixed inflow will be $2,945,000 each year. In return, Party A will pay (in arrears) LIBOR × $50,000,000. Party B will pay a fixed rate equal to the T-Note yield of 5.49 percent plus 44 basis points, which equals 5.93 percent of a $50 million notional principal. This fixed payment will be $2,965,000 each year. In return, Party B will receive (in arrears) LIBOR × $50,000,000.

B. What net cash flows will the Bank anticipate at each relevant date? What interest rate risk does the Bank face?

For each year t, the net cash flow for the bank will be:

$$\$2,965,000 - \$2,945,000 + LIBOR_{t-1} \times \$50,000,000 - LIBOR_{t-1} \times \$50,000,000 = \$20,000$$

Based on these two deals, the bank does not face interest rate risk.

C. In the event of default by either party, analyze the interest rate risk position of the Bank.

Default by either party exposes the bank to interest rate risk. If Party A, the receive-fixed counterparty to the bank, defaults, the bank is left to honor its commitments to Party B. This means that the bank must receive-fixed/pay-floating, so the bank is exposed to rising interest rates. If Party B defaults, the bank is left only with its pay-fixed/receive-floating obligations to Party A. If Party B defaults, the bank is exposed to interest rate risk from falling rates.

15. What is the difference between a seasonal and a roller coaster swap?

    In a seasonal swap, the notional principal varies according to a fixed plan, typically rising and falling according to a regular plan. In a roller coaster swap, the notional principal of the swap first increases and then amortizes to zero over the remaining life of the swap.

16. Compare and contrast an accreting and an amortizing swap.

    In an amortizing swap, the notional principal is reduced over time, while the notional principal in an accreting swap increases over time. Generally, these changes in notional principal occur on a schedule established when the swap agreement is negotiated.

17. "An equity swap is nothing but a commodity swap." Do you agree or disagree with this statement? Explain.

    In essence, the statement is correct. A commodity swap involves one party making payments that are fixed relative to the price of a commodity and the other party making payments that float with the value of the underlying commodity. In an equity swap, the agreement has the same structure, except the underlying commodity may be thought of as a stock index or a stock portfolio.

18. Consider two plain vanilla interest rate swaps that have the same notional principal, the same fixed rate, and the same initial floating rate. One swap has a tenor of five years, while the second has a tenor of ten years. Assume that you take a pay-fixed position in the ten-year swap and a receive-fixed position in the five-year swap. What kind of instrument do these transactions create? Explain, assuming that the term structure is flat. What difference would it make if the term structure were not flat?

    The net payment from these swaps during the first five years is zero, because the two exactly cancel each other. In each period, the pay-fixed swap requires a fixed payment and a floating receipt. By contrast, the receive-fixed swap generates a fixed receipt and a floating payment. The fixed payments are the same, and the floating payments are the same during the first five years. The pair of swaps, therefore, generates no cash flows in the first five years. After the tenor of the five-year swap, a five-year pay-fixed swap remains. Therefore, from the point of view of the initial contracting, the pair of swaps is equivalent to a forward pay-fixed swap with a tenor of five years that is to be initiated in five years. Notice that this outcome results from the fact that the yield curve is flat, as evidenced by the same fixed rate on two swaps with different tenors. If the term structure were not flat, at least one of the swaps would be an off-market swap, since they both have the same fixed rate. Therefore, you would have to make or receive an up-front payment on each swap that was an off-market swap.

19. "A swaption is essentially a portfolio of options on futures or options on forwards." Is this statement correct? Explain.

    This is false. A swaption is a single option on a swap agreement, not a portfolio of options. The swaption may also be regarded as a single option on a portfolio of forward contracts. However, the swap agreement may be viewed as a portfolio of forward contracts.

20. Generally, political unrest in Europe is accompanied by an increase in the yield differential between Eurocurrency deposit rates and US T-bill rates. Explain how to construct a basis swap to profit from such a development. Explain how this might be related to a TED spread in futures.

    In a basis swap, both parties make floating rate payments, but the payments are tied to different indexes. Thus, to exploit generally unexpected political turmoil in Europe, one might agree to pay based on US T-bill rates and to receive based on Eurocurrency deposit rates. In a TED spread in the futures market, one trades a T-bill futures contract against a Eurodollar futures contract. In this specific case, a trader would sell the Eurodollar futures and buy the T-bill futures to exploit a widening yield differential. This kind of swap is also known as a cross-currency basis swap when the Eurocurrency is not a Eurodollar.

21. Using interest rate swaps based on US Treasury instruments, explain how to create a yield curve swap that will profit if the yield curve has an upward slope and the curve steepens. Explain how this might be related to the NOB spread in futures.

To profit from steepening yield curve, a trader could agree to receive floating payments based on the yield of a long-term instrument (such as a T-bond) and to make floating payments based on the yield of a short-term instrument (such as a T-bill). If the yield curve steepens, the rate received, the long-term rate, will rise more than the rate being paid, the short-term rate. In the NOB trade in the futures market, a trader trades a T-note futures contract against a T-bond futures contract. Thus, to profit from a steepening yield curve, the trader would sell T-bond futures and buy T-note futures. (The NOB trade is discussed in Chapter 5.)

22. Explain how two fixed-for-floating foreign currency swaps might be combined to create a fixed-for-fixed foreign currency swap.

Two fixed-for-floating currency swaps can be combined to created a fixed-for-fixed swap. Assume that in one swap a firm pays floating payments in Currency A and receives fixed payments in Currency B. The firm also has a second swap in which it makes fixed payments in Currency C and receives floating payments in Currency A. Assuming that these two swaps have the same notional amount, the payments in Currency A offset each other. This leaves the firm receiving fixed payments in Currency B and making fixed payments in Currency A.

23. Why are only positive swap values used to measure counterparty credit risk?

Only positive swap values are of interest in determining swap credit exposure. This is because with negative- or zero-value swaps (i.e., out-of-the-money or at-the-money swaps), the counterparty owes nothing in the event of a default.

24. What is Acherry picking@? Who does cherry picking complicate scenario analysis of swap credit risk?

It is possible that in the event of bankruptcy, a bankruptcy court may allow each swap to run until its settlement, maturity, or expiration date, and then close out only those swaps that have positive replacement cost. Selectively closing out only those swaps with positive value is called **cherry picking**. The possibility of cherry picking is a scenario that must be accounted for in measuring potential counterparty credit exposure.

25. What is replacement cost? Why is potential replacement cost an important consideration for measuring swap counterparty credit risk?

Current replacement cost is the amount required to replace the swap in the event of counterparty default today. Current replacement cost alone does not accurately portray the potential credit risk over the life of the swap. A counterparty might default at some future date with swap values significantly different than current swap values. The potential loss is larger because the replacement cost can potentially become larger over the life of the swap.

26. What is the key difference between a credit default swap and a total return swap?

The key difference between a credit default swap and a total return swap is the fact that the credit default swap provides protection against specific credit events. The total return swap provides protection against loss of value irrespective of cause. Finally, either credit default swaps or total return swaps entail two sources of credit exposure: one from the underlying reference asset and another from possible default by the counterparty to the transaction.

27. Explain how an interest rate swap can be viewed as a portfolio of forward rate agreements.

We have already noted that a swap may be regarded as a portfolio of forward contracts. For example, a swap agreement with quarterly payments based on Eurodollar deposit rates is essentially similar to a strip of Eurodollar futures contracts in which the futures maturities match the payment dates on the swap.