16 Options on Stock Indexes, Foreign Currency, and Futures

Answers to Questions and Problems

1. Explain why interest payments on a foreign currency can be treated as analogous to a dividend on a common stock.

   For a stock, dividends represent a leakage of value from the asset. If dividends were not paid, the stock price would continue to grow at a higher rate, compounding the value of the dividends. The same is true for a currency. The interest rate paid by a currency represents a leakage of value from the currency. Therefore, dividends from common stock and interest payments from a currency can be treated in the same way for option pricing purposes.

2. Why do we assume that the cost-of-carry for a futures is the same as the risk-free rate?

   For pricing options on futures, the important consideration is that the futures price follow the cost-of-carry relationship very closely. This adherence to the cost-of-carry model is much more important than the exact amount of the cost-of-carry. The option pricing model for futures does not work well if there is not an adherence to the cost-of-carry model. Thus, it is mainly a matter of convenience that we assume the cost-of-carry to equal the risk-free rate. In the real world, this assumption performs very well for financial futures, but it performs less well for futures on agricultural goods.

3. Explain how to adjust a price lattice for an underlying good that makes discrete payments.

   The text considers three types of dividend payments: constant proportional payments, occasional payments equal to a percentage of the asset value, and occasional payments of a fixed dollar amount. In every case, the presence of the dividend requires an adjustment in the stock price lattice. In essence, the nodes in the stock price lattice must be decreased by the present value of the dividends that will occur from the time represented by that node until the expiration of the option. Dividends occurring after the expiration date of the option play no role and may be disregarded. Once the stock price lattice has been adjusted to reflect the dividends, the corresponding lattice for the put or call can be worked through in the normal way to price the option correctly.

4. If a European and an American call on the same underlying good have different prices when all of the terms of the two options are identical, what does this difference reveal about the two options? What does it mean if the two options have identical prices?

   If the two have an identical price, it means that the early exercise feature of the American option has no value. Any difference in the prices will stem from the value associated with the early exercise privilege.

5. Consider an option on a futures contract within the context of the binomial model. Assume that the futures price is 100.00, that the risk-free interest rate is 10 percent, that the standard deviation of the futures is .4, and that the futures expires in one year. Assuming that a call and a put on the futures also expire in one year, compute the binomial parameters U, D, and π_U. Now compute the expected futures price in one period. What does this reveal about the expected movement in futures prices?

$$U = e^{.4\sqrt{1}} = 1.4918; \quad D = \frac{1}{U} = .6703$$

$$\pi_U = \frac{e^{(.10 - .10)1} - .6703}{1.4918 - .6703} = .4013$$

   The expected price movement is:

$$.4013\,(1.4918) + .5987\,(.6703) = 1.00$$

   Thus, the futures price is not expected to change over the next year. In general, this will be true for futures. The futures price already impounds the expected price change in the asset between the current date and the expiration of the futures contract.

6. For a call and a put option on a foreign currency, compute the Merton model price, the binomial model price for a European option with three periods, the Barone-Adesi and Whaley model price, and the binomial model price with three periods for American options. Data are as follows: The foreign currency value is 2.5; the exercise price on all options is 2.0; the time until expiration is 90 days; the risk-free rate of interest is 7 percent; the foreign interest rate is 4 percent; the standard deviation of the foreign currency is .2.

   The prices in the following table show that the American call and put have no exercise potential. The difference between the Merton and Whaley model put prices, on the one hand, and the binomial model put prices, on the other, is due to the very few periods being employed.

	Merton Model	**European Binomial**	**Whaley Model**	**American Binomial**
Call	.5104	.5097	.5104	.5097
Put	.0008	0	.0008	0

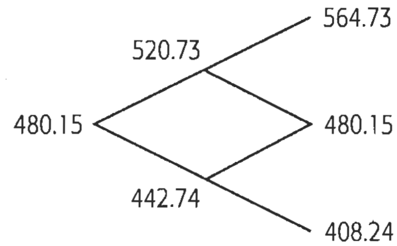
7. Consider a call and a put on a stock index. The index price is 500.00, and the two options expire in 120 days. The standard deviation of the index is .2, and the risk-free rate of interest is 7 percent. The two options have a common exercise price of 500.00. The stock index will pay a dividend of 20.00 index units in 40 days. Find the European and American option prices according to the binomial model, assuming two periods. Be sure to draw the lattices for the stock index and for all of the options that are being priced.

$$U = e^{.2\sqrt{\frac{60}{365}}} = 1.0845; \quad D = \frac{1}{1.0845} = .9221$$

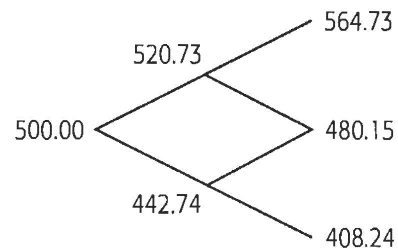
$$\pi_U = \frac{1.0116 - .9221}{1.0845 - .9221} = .5510$$

   As the following price lattices show, the European and American calls are worth \$19.20. The European put is worth \$27.67, and the American put is worth \$30.21.

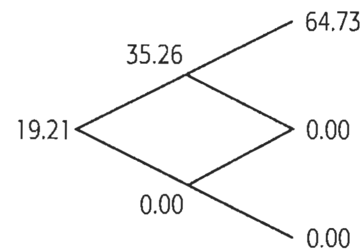
Stock Price Lattice



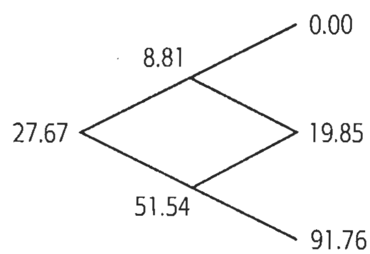
Adjusted Stock Price



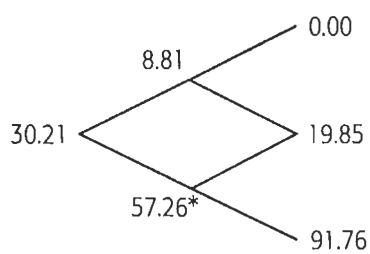
American and European Call Price



European Put Price



American Put Price



8. Consider two European calls and two European put options on a foreign currency. The exercise prices are \$.90 and \$1.00, giving a total of four options. All options expire in one year. The current risk-free rate is 8 percent, the foreign interest rate is 5 percent, and the standard deviation of the foreign currency is .3. The foreign currency is priced at \$.80. Find all four option prices according to the Merton model. Compare the ratio of the option prices to the ratio of the exercise prices. What does this show?

With $X = \$.90$, the call is worth \$.0640, and the put is worth \$.1338. With $X = \$1.00$, the call is worth \$.0392, and the put is worth \$.2014. Comparing the ratio of the option prices to the ratio of exercise prices shows that the call is more sensitive to a change in the exercise price than is the put. A change of about 10 percent in the exercise price causes about a 63 percent change in the call price, but only about a 51 percent change in the put price.

9. You work for the Treasury Department of a large midwestern soap company and your job is to evaluate proposals from OTC dealers. Your company currently has an equity-index swap that pays your company \$250 times the difference between the prevailing S&P 500 stock index price one month from now and the index level of 1000 (if the difference is positive) or zero if the difference is negative. You want to restructure the deal so as to extend the life of the option to a year past the original expiration date (13 months from today). You want to restructure the deal without paying any additional cash over and above what has been agreed to for the 30-day contract. If current volatility is 30 percent (annual), the current S&P 500 price is 1200, the current short-term interest rate is a constant six percent (annual), and the annual dividend yield is 3%, how much would you expect the dealer to alter the option's strike price in consideration for extending the option's life by one year? Assume 30 days per month, 360 days per year.

This problem illustrates a typical deal between corporate treasury departments and derivatives dealers. The first thing you need to recognize is that the payoff structure of this deal makes it an over-the-counter, privately-negotiated call option, held by the corporate treasury and sold by the dealer. In solving the problem it does not matter if you multiply out the \$250 per index point difference. The problem is completely scalable. It is easiest to figure out the option value first then multiply by \$250.

To start, calculate the current value of the option under the specified terms using Option! At 30 days the call is worth \$202.49 (or \$202.49 * \$250 total). This is the amount in cash the corporate treasury does not want to exceed. Extending the life of the call one year, to 390 days, under the specified terms yields a price of \$270.87 (or \$270.87 * \$250 total). The trick now is to determine the strike price at this maturity that will yield \$202.49 (\$202.49 * \$250 total). Solving iteratively for the strike price we find that \$1114 works (\$1114 * \$250 total). So, in summary, the dealer is willing to extend the life of the deal one year in exchange for raising the exercise price \$1114 (\$1114 * \$250 total). Of course, this problem arises because of the corporate treasury's desire not to payout additional cash.

10. In the American-Analytical Futures Option Pricing Model, futures volatility and spot volatility are assumed to be equal. Is this a reasonable assumption?

It is reasonable to assume that spot volatility equals futures volatility if the correlation between changes in spot prices is weakly correlated to changes in interest rates. This can be demonstrated most easily by assuming that the correlation is zero (i.e., nonstochastic interest rates).

(1) Denote $\sigma_S^2 = \text{Var}\,[\ln(S_{t-1}/S_t)]$

(2) Denote $\sigma_F^2 = \text{Var}\,[\ln(F_{t-1}/F_t)]$

Sustituting $S_t = F_t e^{-r(t-t)}$ from the cost-of-carry relationship between futures and spot prices into equation (1) yields:

$\sigma_S^2 = \text{Var}\,[\ln(F_{t-1}e^{-r(t-t-1)}/F_t e^{-r(t-t)})]$

rewriting:

$\sigma_S^2 = \text{Var}\,[\ln(F_{t-1}e^{r}/F_t)]$

$= \text{Var}[\ln(F_{t-1}/F_t) + r]$

$= \text{Var}[\ln(F_{t-1}/F_t) + \text{Var}(r) + 2\text{Cov}[r,\ln(F_{t-1}/F_t)]$

Imposing the Black model's assumption that interest rates are nonstochastic yields:

$\sigma_S^2 = \text{Var}[\ln(F_{t-1}/F_t) + 0 + 0$

Substituting equation 2 into the above expression completes the proof as $\sigma_S^2 = \sigma_F^2$.