8 Stock Index Futures: Refinements

Answers to Questions and Problems

1. Explain the market conditions that cause deviations from a computed fair value price and that give rise to no-arbitrage bounds.

   The villains are market imperfections, principally transaction costs. When trading is sufficiently costly, the futures price can deviate somewhat from fair value, and no market forces will arise to drive the futures price back to its fair value. The greater the costs of trading, the farther the futures price can stray from its theoretical fair value without arbitrage coming into play to restore the relationship. These trading costs include: the bid-asked spread and direct transaction costs such as brokerage commissions and taxes. Also, restrictions on the use of the proceeds from short sales can be important.

2. The No-Dividend Index consists only of stocks that pay no dividends. Assume that the two stocks in the index are priced at $100 and $48, and assume that the corresponding cash index value is 74.00. The cost of carrying stocks is 1 percent per month. What is the fair value of a futures contract on the index that expires in one year?

$$\text{Fair Value} = 74.00(1.01)^{12} = 83.3851$$

3. Using the same facts as in Problem 2, assume that the round-trip transaction cost on a futures is $30. The contract size, we now assume, is for 1,000 shares of each stock. Trading stocks costs $.05 per share to buy and the same amount to sell. Based on this additional information, compute the no-arbitrage bounds for the futures price.

   From the cash-and-carry transactions we would buy the stocks, carry them to expiration, and sell the futures. This strategy would cost:

Purchase and carry stock:	$-\$148{,}000(1.01)^{12} = -\$166{,}770$
Stock transaction cost:	$+1{,}000(2)(\$.05) = -\$100$
Futures transaction cost:	$-\$30$
Total Outlay:	$-\$166{,}900$

   For this strategy to generate a profit, the futures must exceed 83.450 per contract. For the reverse cash-and-carry, we would sell the stocks, invest the proceeds, and buy the futures:

Sell stock; invest proceeds:	$-\$148{,}000(1.01)^{12} = -\$166{,}770$
Stock transaction cost:	$+1{,}000(2)(\$.05) = -\$100$
Futures transaction cost:	$-\$30$
Total Inflow:	$-\$166{,}640$

For this strategy to generate a profit, the futures must be less than 83.320 per contract. The no-arbitrage bounds on the futures range from 83.320 to 83.450.

4. Using the facts in Problems 2 and 3, we now consider differential borrowing and lending costs. Assume that the 1 percent per month is the lending rate and assume that the borrowing rate is 1.5 percent per month. What are the no-arbitrage bounds on the futures price now?

   From the cash-and-carry transactions we would buy the stocks, carry them to expiration, and sell the futures. Now the financing cost is 1.5 percent per month. This strategy would cost:

Purchase and carry stock:	$-\$148{,}000(1.015)^{12} = -\$176{,}951$
Stock transaction cost:	$+1{,}000(2)(\$.05) = -\$100$
Futures transaction cost:	$-\$30$
Total Outlay:	$-\$176{,}821$

   For this strategy to generate a profit, the futures must exceed 88.411 per contract. The reverse cash-and-carry strategy is unaffected because the lending rate is still 1 percent. Therefore, the no-arbitrage bounds on the futures range from 83.320 to 88.411.

5. Using the facts in Problems 2–4, assume now that the short seller receives the use of only half of the funds in the short sale. Find the no-arbitrage bounds.

   The cash-and-carry transactions are the same as in Problem 4 so they give an upper no-arbitrage bound of 88.411. For the reverse cash-and-carry, we would sell the stocks, invest the proceeds, and buy the futures:

Sell stock; invest 50% of proceeds:	$+\$74{,}000(1.01)^{12} = \$83{,}385$
Stock transaction cost:	$-1{,}000(2)(\$.05) = -\ \$100$
Futures transaction cost:	$-\$30$
Recoup 50% of unused funds:	$+\$74{,}000$
Total Inflow:	$+\$157{,}255$

   For this strategy to generate a profit, the futures must be less than 78.628 per contract. The no-arbitrage bounds on the futures range from 78.628 to 88.411.

6. Consider the trading of stocks in an index and trading futures based on the index. Explain how different transaction costs in the two markets might cause one market to reflect information more rapidly than the other.

   Let us assume that it is more costly to trade the individual stocks represented in the index than it is to trade the futures based on the index. (Once in a while we assume something consistent with reality.) Traders with information about the future direction of stock prices will want to exploit that information as cheaply as possible. Therefore, they will be likely to trade futures rather than the stocks in the index. Trading futures causes the futures price to adjust, and through arbitrage links, the stock price adjusts to the new futures price. In this scenario, the futures market reflects the new information before the stock market does.

7. For index arbitrage, explain how implementing the arbitrage through program trading helps to reduce execution risk.

   Execution risk is the risk that the actual trade price will not equal the anticipated trade price. The discrepancy arises largely from the delay between order entry and order execution. By using program trading, orders are conveyed to the floor more quickly and receive more rapid execution. (At least this is true in the absence of exchange-imposed delays on program trades.) Therefore, the use of program trading techniques should help to reduce execution risk.

8. Index arbitrageurs must consider the dividends that will be paid between the present and the futures expiration. Explain how overestimating the dividends that will be received could affect a cash-and-carry arbitrage strategy.

Assume a trader estimates a dividend rate that is higher than the actual dividend rate that will be achieved. Further assume that the market as a whole correctly forecasts the dividend rate. For this investor, a strategy of cash-and-carry arbitrage will appear to be more attractive than it really is. This trader will be expecting to receive more dividends than will actually be forthcoming, so the trader will underestimate the net cost of carrying stocks forward. This overestimate of the dividend rate could lead the trader to expect a profit from the trade that will evaporate when adjusted for the actual dividends that will be received.

9. Explain the difference between the beta in the Capital Asset Pricing Model and the beta one finds by regressing stock returns against returns on a stock index.

   The beta of the CAPM is a theoretical entity. The CAPM beta is a measure based on the relationship between a particular security and an unobserved and probably unobservable market portfolio. The beta estimated by regressing stock returns against the returns on an index is an estimate of that ideal CAPM beta. Because the index fails to capture the true market portfolio, the actually estimated beta must fail to capture the true CAPM beta. Nonetheless, the estimated beta may be a useful approximation of the true CAPM beta.

10. Explain the difference between an ex-ante and an ex-post minimum risk hedge ratio.

    The ex-ante minimum risk hedge ratio is estimated using historical data. In hedging practice, this estimated hedge ratio is applied to a future time period. Almost certainly the hedge ratio that would have minimized risk in the future period (the ex-post hedge ratio) will not equal the estimated ex-ante hedge ratio. However, the ex-post minimum risk hedge ratio can only be known after the fact. Therefore, we must expect some inaccuracy in estimating a hedge ratio ex-ante and comparing it with the ideal ex-post hedge ratio.

11. Assume you hold a well-diversified portfolio with a beta of 0.85. How would you trade futures to raise the beta of the portfolio?

    Buy a stock index futures. In effect, this action levers up the initial investment in stocks, effectively raising the beta of the stock investment. In principle, this levering up can continue to give any level of beta a trader desires.

12. An index fund is a mutual fund that attempts to replicate the returns on a stock index, such as the S&P 500. Assume you are the manager of such a fund and that you are fully invested in stocks. Measured against the S&P 500 index, your portfolio has a beta of 1.0. How could you transform this portfolio into one with a zero beta without trading stocks?

    Sell S&P 500 Index futures in an amount equal to the value of your stock portfolio. After this transaction you are effectively long the index (your stock holdings) and short the index by the same amount (your short position in the futures). As a result, you are effectively out of the stock market, and the beta of such a position must be zero.

13. You hold a portfolio consisting of only T-bills. Explain how to trade futures to create a portfolio that behaves like the S&P 500 stock index.

    Buy S&P 500 Index futures. You should buy an amount of futures that equals the value of funds invested in T-bills. The resulting portfolio will replicate a portfolio that is fully invested in the S&P 500.

14. In portfolio insurance using stock index futures, we noted that a trader sells additional futures as the value of the stocks falls. Explain why traders follow this practice.

    The goal of portfolio insurance is to keep the value of a portfolio from falling below a certain level or, alternatively expressed, to ensure that the return achieved on a portfolio over a given horizon achieves a certain minimum level. At the same time, portfolio insurance seeks to retain as much potential for beating that minimum return as is possible. The difference between the portfolio's current value and the value it must have to meet the minimum target we will call the *cushion*. If the portfolio has no cushion, the only way to ensure that the portfolio will achieve the target return, or the target value, is for the portfolio to be fully hedged.

We now consider the trader's response if the portfolio value is above the minimum level, that is, if there is some cushion and stock prices fall. The drop in stock prices reduces the cushion, so the trader must move to a somewhat more conservative position. This requires hedging a greater portion of the portfolio, which the trader does by selling futures. Therefore, an initial drop in prices requires the selling of futures, and each subsequent drop in prices requires the sale of more futures.

15. Casey Mathers, manager of the Zeta Corporation's equity portfolio, hires a new assistant, Alec. Alec is pretty sharp and immediately questions Casey's decision to hedge an anticipated $10 million withdrawal. Casey had hedged the portfolio using the S&P 500 index futures contract. In calculating the hedge, Casey used the portfolio beta of 1.2 which was computed using the S&P 500 Index.

A. Explain to Casey why his hedge may not be a risk minimization hedge.

Casey's hedge may not be a risk minimization hedge because the beta used in calculating the hedge ratio was computed using the S&P 500 index. It is the index futures contract, though, that is used for hedging. So theoretically the portfolio's beta computed using the futures contract prices is what should be used in calculating the hedge. Additionally, the S&P 500 index futures contract is not the only possible hedging vehicle available. For example, there are the Dow Jones Industrial Average index futures and the NYSE Composite index futures. The portfolio's returns may be more highly correlated with one of these other contracts than it is with the S&P 500 index futures contract.

Alec calculates possible risk-minimizing betas for the Zeta portfolio using the S&P 500 index futures, the Dow Jones Industrial Average futures, and the NYSE Composite index futures, with the following results:

	S&P 500 Index Futures	DJIA Futures	NYSE Composite Index Futures
Contract size	$250 × index	$10 × index	$500 × index
Current quote (MAR) contract	1110.59	8715.00	559.30
$Beta_{RM}$	1.30	1.35	1.10
R^2	0.83	0.75	0.90

B. Given Alec's results, is the S&P 500 futures index the most appropriate hedging vehicle? Be sure to justify your answer.

In risk-minimization hedging the best vehicle to use is the instrument with the highest R^2. The R^2 tells the percentage of portfolio returns that is explained by the hedging instrument's price returns. Examining the R^2 figures from Alec's results, it can be seen that the NYSE Composite index contract has the highest R^2 of 90%. This is greater than the S&P 500 contract's R^2 of 83%. The NYSE Composite contract would be the better hedging instrument according to the risk-minimization technique.

C. Design a risk minimization hedge using Alec's results.

Using Alec's results the risk minimizing hedge would be accomplished by selling NYSE Composite index futures. The number of contracts to sell is computed as:

$$N = -1.10\frac{\$10{,}000{,}000}{559.30\,(\$500)} = -39.3 \text{ contracts}$$

Alec would recommend selling 39 contracts.

D. Will Alec's hedging strategy turn out to be superior to Casey's hedging strategy? Justify your answer.

While Alec's hedging strategy will probably be superior to Casey's, this is not guaranteed, because the hedge calculation is based on historic relationships that are measured with error. This relationship will not hold exactly in the future.

16. Raymond J. Johnson, Jr. manages a \$20 million equity portfolio. It has been designed to mimic the S&P 500 index. Ray has a hunch that the market is going south during the coming month. He has decided that he wants to eliminate his exposure for the next month and take off for Montana to go fishing. Ray has the following information at hand:

S&P 500 index futures with 1 month to delivery:	1084.50
Dividend yield on Ray's portfolio:	2.1%
S&P 500 index today:	1081.40

A. Design a hedge to eliminate Ray's market risk for the next month.

Ray, in effect, would like to sell his portfolio for a month and put the money into T-bills. The transactions costs make this strategy cost prohibitive though. Alternatively, Ray could sell futures contracts. Then over the next month any losses in the cash market will be offset by gains in the futures market. To eliminate his exposure to the market, Ray would calculate the number of S&P 500 futures contracts as:

$$N = -1.0\frac{\$20,000,000}{(1084.50)(\$250)} = -73.8 \text{ contracts}$$

Ray should sell 74 S&P 500 futures contracts.

B. Compute the return he can expect to receive over the next month.

The futures index pricing relationship is:

$$F_{0,t} = S_0(1 + C - \text{DIVYLD})$$

where:

$F_{0,t}$ = index futures value
S_0 = spot price
C = cost-of-carry
DIVYLD = dividend yield

The cost-of-carry can also be viewed as an implied repo rate. This is the return Ray will receive over the next month:

$$C = \left(\frac{F_{0,t}}{S_0} + \text{DIV} - 1\right)12$$

$$C = \left(\frac{74(1084.50)(250)}{20,000,000} + \frac{.021}{12} - 1\right)12$$

$$C = 5.9\%$$

Ray can expect to earn a 5.9% annual return over the next month.

17. Remember Ray? He is the guy running the S&P 500 index fund who wanted to go fishing. Ray has changed his mind. The fishing reports from Montana were not favorable so he has decided not to go. Since he is not leaving, he had decided to devise a portfolio insurance strategy for his \$20 million portfolio. His objective is to not let his portfolio value fall below \$18 million.

A. Design a portfolio insurance strategy that applies no hedges for portfolio values at or above $20 million and is fully hedged at or below $18 million.

Portfolio insurance is a dynamic hedging strategy that applies hedges as the hedged asset falls in value and removes hedges as the hedged asset increases in value. When the asset value is falling, hedges are applied until the assets are fully hedged. Theoretically a money manager could prevent the value of his positions from falling below some pre-specified level because further decline in the assets' value is offset by futures gains. Ray wishes to apply hedges as the value of this portfolio falls from $20 million (no hedges) down to $18 million (fully hedged). Assume that Ray will apply the hedges in a linear fashion. That is, the percentage of assets hedged will be given by:

$$\% \text{ hedged} = \max\left(\left[\frac{20 - \max(V_P, 18)}{20 - 18}\right], 0\right)$$

When $V_P < 18$, the percentage of the portfolio hedged is 100%. When $V_P > 20$, the percentage of the portfolio hedged is 0%.

B. On day one, the stock portfolio value falls from $20 million to $19.4 million, and the S&P 500 futures price falls to 1052. What action should Ray take?

If the portfolio value falls from $20 million to 19.4 million on day 1, the percentage of the assets that should be hedged is:

$$\% \text{ hedged} = \left(\left[\frac{20 - \max(19.4, 18)}{20 - 18}\right], 0\right)$$

$$\% \text{ hedged} = 30\%$$

Thirty percent of the assets should be hedged. This can be accomplished by selling contracts calculated as follows:

$$N = -\ 1.0\,(0.30)\frac{\$19{,}400{,}000}{(1{,}052)\,(\$250)} = -\ 22.13$$

At the end of day 1 Ray should sell 22 S&P 500 futures contracts.

C. On day two, the value of Ray's portfolio increases by 2%. The S&P 500 futures contract increases to 1073. What would be the change in value of Ray's portfolio (including any hedges that may be in place)? What action should Ray take?

Cash Market Gain	Assets increase by 2% to $19,788,000
Future Market Loss	22(1052 − 1073) ($250) = − $115,500
End of Day 2 Assets	$19,788,000 − $115,500 = $19,672,500

The change in value of Ray's portfolio on day 2 is $272,500. Part of the portfolio must be liquidated to mark the futures contract to market. At the end of day 2 Ray needs to adjust his hedge. The new percentage to hedge is:

$$\% \text{ hedged} = \frac{20 - 19.67}{20 - 18} = 16.5\%$$

The number of contracts to achieve this hedge is:

$$N = -\ 1.0\,(0.164)\frac{\$19{,}672{,}500}{(1{,}073)\,(\$250)} = -\ 12.02$$

Ray is already short 22 contracts so he is over-hedged. He should buy 10 S&P 500 futures contracts to bring his futures position to 12 short.

D. Is Ray really protected against his portfolio value falling below $18 million in value? Explain.

The protection Ray has depends upon several factors. First, the amount of protection depends on how closely Ray monitors his position. If Ray does not closely monitor his position, the portfolio value could fall below $18 million before Ray places a hedge. The more closely Ray monitors, and the more frequently he adjusts the hedge, the better the protection. Second, if market frictions prevent Ray from applying hedges in a timely fashion, the portfolio value could fall below $18 million. For many investors, this occurred during the stock market crash in 1987. The order flow overwhelmed the order handling and reporting systems of the stock market. Also, the price information coming from the stock markets was stale. This kind of event could mislead Ray in his hedging decision-making process. In this kind of extreme situation, Ray's assets could fall below $18 million before Ray knew it

18. What real-world complications can hinder the effectiveness of a stock index arbitrage strategy?

Dividend payments may be suspended or altered; 2) the composition of underlying index continually changes through time; 3) transaction costs; 4) stock exchange restrictions on short sales of stock; 5) the prices observed on trading screens may not reflect the actual state of the market at the time arbitrageurs make their decisions; 6) trade execution risk, i.e., the arbitrageur=s order may move the price; and 7) stock exchange "collars" on index arbitrage on days of large price moves. This is not an exhaustive list.

19. It is 7:00 a.m and the New York Stock Exchange does not open until 9:30 a.m. The current S&P 500 e-mini nearby futures contract is trading on the CME's Globex system at 1150.00 index points. There are 21 days remaining until contract expiration. You expect the underlying index to pay out dividend equivalent to 1.1 index points over the contract's remaining life. You expect your financing cost to be 1.9 percent per year. If the closing price for the index from the previous day was 1135.00, use the cost of carry relationship to determine whether you expect the stock market to open higher or lower based on current information.

The annualized financing cost of an arbitrage position is 1.90 percent, or .0190 in decimal form. The financing cost for 21 days is therefore $(21/365)*.0190 = .001093$. The expected dividend return is 1.1 index points. Therefore, the fair value stock index price is:

$(1150 + 1.1)/(1 + .001093) = 1149.843$ index points.

Since the index closed at 1135, based on information at 7:00 it appears that the market will open strongly higher.