3 Futures Prices

Answers to Questions and Problems

1. Explain the function of the settlement committee. Why is the settlement price important in futures markets in a way that the day's final price in the stock market is not so important?

   In futures markets, the settlement committee determines the settlement price for each contract each day. The settlement price estimates the true value of the contract at the end of the day's trading. In active markets, the settlement price will typically equal the last trade price. In inactive markets, the settlement price is the committee's estimate of the price at which the contract would have traded at the close, if it had traded. The settlement price is important, because it is used to calculate margin requirements and the cash flows associated with daily settlement. In the stock market, there is no practice comparable to daily settlement, so the closing price in the stock market lacks the special significance of the futures settlement price.

2. Open interest tends to be low when a new contract expiration is first listed for trading, and it tends to be small after the contract has traded for a long time. Explain.

   When the contract is first listed for trading, open interest is necessarily zero. As traders take positions, the open interest builds. At expiration, open interest must again be zero. Every contract will have been fulfilled by offset, delivery, or an EFP. Therefore, as the contract approaches the expiration month, many traders will offset their positions to avoid delivery. This reduces open interest. In the expiration month, deliveries that occur further reduce open interest. Also, EFPs typically reduce open interest. This creates a pattern of very low open interest in the contract's early days of trading, followed by increases, followed by diminution, followed by the contract's extinction.

3. Explain the distinction between a normal and an inverted market.

   In a normal market, prices for more distant expirations are higher than prices for earlier expirations. In an inverted market, prices for more distant expirations are lower than prices for earlier expirations.

4. Explain why the futures price converges to the spot price and discuss what would happen if this convergence failed.

   The explanation for convergence at expiration depends on whether the market features delivery or cash settlement, but in each case, convergence depends on similar arbitrage arguments. We consider each type of contract in turn. For a contract with actual delivery, failure of convergence gives rise to an arbitrage opportunity at delivery. The cash price can be either above or below the futures price, if the two are not equal. If the cash price exceeds the futures price, the trader buys the future, accepts delivery, and sells the good in the cash market for the higher price. If the futures price exceeds the cash price, the trader buys the good on the cash market, sells a futures, and delivers the cash good in fulfillment of the futures. To exclude both types of arbitrage simultaneously, the futures price must equal the cash price at expiration. Minor discrepancies can

exist, however. These are due to transaction costs and the fact that the short trader owns the options associated with initiating the delivery sequence.

For a contract with cash settlement, failure of convergence also implies arbitrage. Just before delivery, if the futures price exceeds the cash price, a trader can sell the futures, wait for expiration, and the futures price will be set equal to the cash price. This gives a profit equal to the difference between the cash and futures. Alternatively, if the cash price is above the futures price, and expiration is imminent, the trader can buy the futures and wait for its price to be marked up to equal the cash price. Thus, no matter whether the futures price is above or below the cash price, a profit opportunity will be available immediately.

In short, the futures and cash price converge at expiration to exclude arbitrage, and failure of convergence implies the existence of arbitrage opportunities.

5. Is delivery, or the prospect of delivery, necessary to guarantee that the futures price will converge to the spot price? Explain.

   No, delivery is not necessary. As explained in the answer to question 4, cash settlement will also lead to convergence of the cash and the futures at expiration.

6. As we have defined the term, what are the two key elements of *academic arbitrage*?

   The two elements are riskless profit and zero investment. Each condition is necessary for academic arbitrage, and the two conditions are jointly sufficient.

7. Assume that markets are perfect in the sense of being free from transaction costs and restrictions on short selling. The spot price of gold is \$370. Current interest rates are 10 percent per year, compounded monthly. According to the cost-of-carry model, what should the price of a gold futures contract be if expiration is six months away?

   In perfect markets, the cost-of-carry model gives the futures price as:

$$F_{0,t} = S_0(1 + C)$$

   The cost of carrying gold for six months is $(1 + .10/12)^6 - 1 = .051053$. Therefore, the futures price should be:

$$F_{0,t} = \$370(1.051053) = \$388.89$$

8. Consider the information in question 7. Round-trip futures trading costs are \$25 per 100-ounce gold contract, and buying or selling an ounce of gold incurs transaction costs of \$1.25. Gold can be stored for \$.15 per month per ounce. (Ignore interest on the storage fee and the transaction costs.) What futures prices are consistent with the cost-of-carry model?

   Answering this question requires finding the bounds imposed by the cash-and-carry and reverse cash-and-carry strategies. For convenience, we assume a transaction size of one 100-ounce contract. For the cash-and-carry, the trader buys gold and sells the futures. This strategy requires the following cash outflows:

Buy gold	−\$370(100)
Pay transaction costs on the spot	−\$1.25(100)
Pay the storage cost	−\$.15(100)(6)
Sell futures	0
Borrow to finance these outlays	+\$37,215

   Six months later, the trader must:

Pay the transaction cost on one futures	−\$25
Repay the borrowing	−\$39,114.95
Deliver on futures	?

Net outlays at the outset were zero, and they were $39,139.95 at the horizon. Therefore, the futures price must exceed $391.40 an ounce for the cash-and-carry strategy to yield a profit.

The reverse cash-and-carry incurs the following cash flows. At the outset, the trader must:

Sell gold	+$370(100)
Pay transaction costs on the spot	−$1.25(100)
Invest funds	−$36,875
Buy futures	0

These transactions provide a net zero initial cash flow. In six months, the trader has the following cash flows:

Collect on investment	$+\$36{,}875(1 + .10/12)^6 = \$38{,}757.59$
Pay futures transaction costs	−$25
Receive delivery on futures	?

The breakeven futures price is therefore $387.33 per ounce. Any lower price will generate a profit. From the cash-and-carry strategy, the futures price must be less than $391.40 to prevent arbitrage. From the reverse cash-and-carry strategy, the price must be at least $387.33. (Note that we assume there are no expenses associated with making or taking delivery.)

9. Consider the information in questions 7 and 8. Restrictions on short selling effectively mean that the reverse cash-and-carry trader in the gold market receives the use of only 90 percent of the value of the gold that is sold short. Based on this new information, what is the permissible range of futures prices?

   This new assumption does not affect the cash-and-carry strategy, but it does limit the profitability of the reverse cash-and-carry trade. Specifically, the trader sells 100 ounces short but realizes only $.9(\$370)(100) = \$33{,}300$ of usable funds. After paying the $125 spot transaction cost, the trader has $33,175 to invest. Therefore, the investment proceeds at the horizon are: $\$33{,}175(1 + .10/12)^6 = \$34{,}868.69$. Thus, all of the cash flows are:

Sell gold	+$370(100)
Pay transaction costs on the spot	−$1.25(100)
Broker retains 10 percent	−$3,700
Invest funds	−$33,175
Buy futures	0

   These transactions provide a net zero initial cash flow. In six months, the trader has the following cash flows:

Collect on investment	$34,868.69
Receive return of deposit from broker	$3,700
Pay futures transaction costs	- $25
Receive delivery on futures	?

   The breakeven futures price is therefore $385.44 per ounce. Any lower price will generate a profit. Thus, the no-arbitrage condition will be fulfilled if the futures price equals or exceeds $385.44 and equals or is less than $391.40.

10. Consider all of the information about gold in questions 7–9. The interest rate in question 7 is 10 percent per annum, with monthly compounding. This is the borrowing rate. Lending brings only 8 percent, compounded monthly. What is the permissible range of futures prices when we consider this imperfection as well?

The lower lending rate reduces the proceeds from the reverse cash-and-carry strategy. Now the trader has the following cash flows:

Sell gold	+$370(100)
Pay transaction costs on the spot	−$1.25(100)
Broker retains 10 percent	−$3,700
Invest funds	−$33,175
Buy futures	0

These transactions provide a net zero initial cash flow. Now the investment will yield only $33,175 $(1 + .08/12)^6 = \$34{,}524.31$. In six months, the trader has the following cash flows:

Collect on investment	$34,524.31
Pay futures transaction costs	− $25
Receive delivery on futures	?
Return gold to close short sale	0
Receive return of deposit from broker	$ 3,700

Total proceeds on the 100 ounces are $38,199.31. Therefore, the futures price per ounce must be less than $381.99 for the reverse cash-and-carry strategy to profit. Because the borrowing rate has not changed, the bound from the cash-and-carry strategy remains at $391.40. Therefore, the futures price must remain within the inclusive bounds of $381.99 to $391.40 to exclude arbitrage.

11. Consider all of the information about gold in questions 7–10. The gold futures expiring in six months trades for $375 per ounce. Explain how you would respond to this price, given all of the market imperfections we have considered. Show your transactions in a table similar to Table 3.8 or 3.9. Answer the same question, assuming that gold trades for $395.

If the futures price is $395, it exceeds the bound imposed by the cash-and-carry strategy, and it should be possible to trade as follows:

	Cash-and-Carry Arbitrage	
$t = 0$	Borrow $37,215 for 6 months at 10%.	+$37,215.00
	Buy 100 ounces of spot gold.	−37,000.00
	Pay storage costs for 6 months.	−90.00
	Pay transaction costs on gold purchase.	−125.00
	Sell futures for $395.	0.00
	Total Cash Flow	$0
$t = 6$	Remove gold from storage.	$0
	Deliver gold on futures.	+39,500.00
	Pay futures transaction cost.	−25.00
	Repay debt.	−39,114.95
	Total Cash Flow	+$360.05

If the futures price is $375, the reverse cash-and-carry strategy should generate a profit as follows:

	Reverse Cash-and-Carry Arbitrage	
$t = 0$	Sell 100 ounces of gold short.	+$37,000.00
	Pay transaction costs.	−125.00
	Broker retains 10%.	−3,700.00
	Buy futures.	0
	Invest remaining funds for 6 months at 8%.	−33,175.00
	Total Cash Flow	$0
$t = 6$	Collect on investment.	−$34,524.31
	Receive delivery on futures.	−37,500.00
	Return gold to close short sale.	0
	Receive return of deposit from broker.	+3,700.00
	Pay futures transaction cost.	−25.00
	Total Cash Flow	+$699.31

12. Explain the difference between pure and quasi-arbitrage.

In a pure arbitrage transaction, the arbitrageur faces full transaction costs on each transaction comprising the arbitrage. For example, a retail customer with no initial position in the market, who attempts arbitrage, would be attempting pure arbitrage. By contrast, a quasi-arbitrage transaction occurs when a trader faces less than full transaction costs. The most common example arises in reverse cash-and-carry arbitrage, which requires short selling. For example, in stock index arbitrage, holding a large portfolio allows a trader to simulate a short sale by selling part of the portfolio from inventory. Therefore, this trader faces less than the full transaction costs due to the preexisting position in the market. By contrast, the pure arbitrage trade would require the actual short sale of the stocks, and short selling does not provide the full proceeds to earn interest in the reverse cash-and-carry transactions.

13. Assume that you are a gold merchant with an ample supply of deliverable gold. Explain how you can simulate short selling and compute the price of gold that will bring you into the market for reverse cash-and-carry arbitrage.

The breakeven price for reverse cash-and-carry arbitrage depends principally on the transaction costs the trader faces. With an existing inventory of gold, the trader can simulate short selling by selling a portion of the inventory. Further, because the trader already actually owns the gold, she can have full use of the proceeds of the sale. Therefore, the gold owner's reverse cash-and-carry transactions are similar to those in problem 10:

Reverse Cash-and-Carry Arbitrage

$t = 0$	Sell 100 ounces of gold short.	+$37,000.00
	Pay transaction costs.	−125.00
	Buy futures.	0
	Invest funds for 6 months at 8%.	−36,875.00
	Total Cash Flow	$0
$t = 6$	Collect on investment.	−$38,374.80
	Return gold to close short sale.	0
	Pay futures transaction cost.	−25.00
	Receive delivery on futures. (Note: This is the futures price to give zero cash flow.)	+38,349.80
	Total Cash Flow	+$0

Therefore, if the futures price is $383.498 per ounce, the reverse cash-and-carry transactions give a zero cash flow. This is the breakeven price for reverse cash-and-carry. If the futures price is less that $383.498 per ounce, reverse cash-and-carry arbitrage will be possible for the trader who holds an initial inventory of gold. In problem 10, the price of gold has to be less than $381.99 for reverse cash-and-carry arbitrage to work. The trader there faced full transaction costs, due to the lack of a preexisting inventory.

14. Assume that silver trades in a full carry market. If the spot price is $5.90 per ounce and the futures that expires in one year trades for $6.55, what is the implied cost-of-carry? Under what conditions would it be appropriate to regard this implied cost-of-carry as an implied repo rate?

If the market is at full carry, then $F_{0,t} = S_0(1 + C)$ and $C = F_{0,t}/S_0 - 1$. With our values, $C = \$6.55/\$5.90 - 1 = .110169$. It would be appropriate to regard this implied cost-of-carry as an implied repo rate if the only carrying cost were the financing cost. This is approximately true for silver.

15. What is *normal backwardation?* What might give rise to normal backwardation?

Normal backwardation is the view that futures prices normally rise over their life. Thus, prices are expected to rise as expiration approaches. The classic argument for normal backwardation stems from Keynes. According to Keynes, hedgers are short in the aggregate, so speculators must be net long. Speculators provide their risk-bearing services for an expected profit. To have an expected profit, the futures price must be less than the expected future spot price at the time the speculators assume their long positions. Therefore,

given unbiased expectations regarding future spot prices, we expect futures prices to rise over time to give the speculators their compensation. This leads directly to normal backwardation.

16. Assume that the CAPM beta of a futures contract is zero, but that the price of this commodity tends to rise over time very consistently. Interpret the implications of this evidence for normal backwardation and for the CAPM.

Because futures trading requires no investment, positive returns on long futures positions can be consistent with the CAPM only if futures have positive betas. With a zero beta (by our assumption) and a zero investment to acquire a long futures position (by the structure of the market), the CAPM implies zero expected returns. Therefore, a zero beta and positive returns is inconsistent with the CAPM. Even with zero beta, positive returns are consistent with normal backwardation resulting from speculators assuming long positions and being rewarded for their risk-bearing services.

17. Explain why futures and forward prices might differ. Assume that platinum prices are positively correlated with interest rates. What should be the relationship between platinum forward and futures prices? Explain.

Futures are subject to daily settlement cash flows, while forwards are not. If the price of the underlying good is not correlated with interest rates, futures and forward prices will be equal. If the price of the underlying good is positively correlated with interest rates, a long trader in futures will receive daily settlement cash inflows when interest rates are high and the trader can invest that cash flow at the higher rate from the time of receipt to the expiration of the futures. Because forwards have no daily settlement cash flows, they are unable to reap this benefit. Therefore, if a commodity's price is positively correlated with interest rates, there will be an advantage to a futures over a forward. Thus, for platinum in the question, the futures price of platinum should exceed the forward price. The opposite price relationship can occur if there is negative correlation. Generally, this price relationship is not sufficiently strong to be observed in the market.

18. Consider the life of a futures contract from inception to delivery. Explain two fundamental theories on why the futures prices might exhibit different volatility at different times over the life of the contract.

According to the Samuelson hypothesis, price volatility will be greater when more information about the price of the good is being revealed. According to this view, this tends to happen as the futures comes to expiration, particularly for agricultural goods. Therefore, the Samuelson hypothesis suggests that the volatility of futures prices should increase over the life of the contract.

There are several other theories that attempt to relate contract maturity and volatility. First, there seems to be some evidence for believing that volatility is higher for some commodities in certain seasons, particularly at times when information about the harvest of some good is reaching the market. With this view, volatility depends on the time of the year and not so much on the contract's expiration. Second, volatility also differs depending on the day of the week. Third, volatility is autocorrelated. High volatility in one month begets high volatility in the next month.

19. What is a limit order? How does placing a limit order provide an option to other traders?

    A limit order is a conditioned order that requires that a trade not be executed unless the price reaches a predetermined level. For example, a corn producer may give instructions to their broker to sell March corn only if the price reaches $2.70 per bushel. Or a buyer may give instructions not to buy March corn unless the price falls below $2.50 per bushel. Limit orders are used by traders as a means of communicating their instructions to their broker.

    Traders who submit limit orders give other traders the choice of whether to trade with the order or not. This choice can be characterized as an option to trade. For example, a limit order to sell gives other traders the option to buy at the limit price. The option to buy a set quantity at a set price is by definition a call option. A limit order to buy gives other traders the option to sell at the limit price. The option to sell a set quantity at a set price is by definition a put option.

20. What is market microstructure?

    Market microstructure is a branch of financial economics that analyzes how trading technology influences the trading characteristics of a financial market. For example, market microstructure techniques can be applied to the futures market to determine whether the information content of prices formed in an open outcry trading environment differs from the information content of prices formed in an electronic trading environment. Trading technology defines what traders can do and what they can know. Broadly defined, trading technology includes such things as the physical layout of a market, trading protocols and rules, market governance, and information systems available to traders. Trading characteristics include the determinants of the market price, market liquidity, transaction costs, volatility, and trading profits.

21. What is the difference between fundamental volatility and transitory volatility?

    Fundamental volatility is caused by the actions of informed traders who base their trades on the arrival of new fundamental information. As informed traders conduct their trades, market prices change to reflect the new information. Fundamental volatility can be good for the economy because it helps investors allocate scarce capital to its highest valued use. Transitory volatility is caused by the trading decisions of speculators who are uninformed about market fundamentals and trade instead information they glean from the trading process itself.

22. What is the implied repo rate? What information does the implied repo provide about the relationship between cash and futures prices?

Most participants in the futures markets face a financing charge on a short-term basis that is equivalent to the repo rate, that is, the interest rate on repurchase agreements. In a repurchase agreement, a person sells securities at one time, with the understanding that they will be repurchased at a certain price at a later time. Most repurchase agreements are for one day only and are known, accordingly, as overnight repos. The repo rate is relatively low, exceeding the rate on Treasury bills by only a small amount.

In trading vernacular, the theoretical rate of return on a cost-of-carry strategy is the *implied repo rate*. An arbitrageur calculates the implied repo rate and compare it to his own financing cost (proxied by the actual repo rate) to determine whether or not an arbitrage opportunity exists. In a wellBfunctioning market without arbitrage opportunities, the implied repo rate is equivalent to the actual repo rate. Deviations from this relationship lead to arbitrage opportunities in a perfect market.