Chapter 17: Vertical and Conglomerate Mergers

Learning Objectives:

Students should learn to:

1. Apply the complementary goods model to the analysis of vertical mergers.
2. Demonstrate the idea of double marginalization and its impact prices, profits, and consumer welfare.
3. Analyze a model where vertical integration allows or enhances price discrimination.
4. Analyze a model where vertical merger facilitates market foreclosure. Because the integrated firm has a lower acquisition cost from its own upstream supplier and because it has a higher margin on downstream sales to its own affiliate, the integrated firm will not buy from the other upstream suppliers and will not sell to other downstream firms.
5. Solve problems involving oligopolistic vertical mergers.
6. Relate scale and scope, transactions costs, proprietary information, and agency problems to incentives for conglomerate mergers.
7. Recognize that vertical integration can eliminate wasteful double-marginalization, intensify competition, and yield lower prices for consumers. Understand recent regression analysis showing this result in the ready-mixed concrete industry.

Suggested Lecture Outline:

Spend two fifty-minute long lectures on this chapter.

Lecture 1:

1. Models with complementary goods
2. Price discrimination
3. Market foreclosure
4. Gains from vertical integration
5. Evidence from ready-mixed concrete.

Lecture 2:

1. Oligopolistic vertical mergers
2. Conglomerate mergers
3. Mergers and the theory of firms

Suggestions for the Instructor:

1. Assign a problem set on complementary goods a week or so before these lectures would be useful.
2. Motivate vertical and horizontal mergers with examples.
3. “What if you were in this position” questions about vertical foreclosure are useful in motivating the section on this topic.
4. It is helpful to discuss the ideas of coordination, transactions costs, asset specificity, and hold-up problems in the context of incomplete contracts. Students easily relate to apartment rental contracts.

Solutions to End of the Chapter Problems:

Problem 1

1. Norman International has a monopoly in the manufacture of whatsits. Each whatsit requires exactly one richet as an input and incurs other variable costs of \$5 per unit. Richets are made by PepRich Inc., which is also a monopoly. The variable costs of manufacturing richets are \$5 per unit. Assume that the inverse demand for whatsits is

$$p_w = 50 - q_w$$

where p_w is the price of whatsits in dollars per unit and q_w is the quantity of whatsits offered for sale by Norman International.

(a) Write down the profit function for Norman International assuming that the two monopolists act as independent profit-maximizing companies, with Norman International setting a price p_w for whatsits and PepRich setting a price p_r for richets. Hence, derive the profit-maximizing price for whatsits as a function of the price of richets, and use this function to obtain the derived demand for richets.

Profits for Norman International are given by revenue minus the cost of richets and other variable

costs. If a richet costs p_r per unit we obtain

$$\begin{aligned}\pi_{NI} &= (50 - q_w)q_w - p_r q_w - 5q_w \\ &= 50q_w - q_w^2 - p_r q_w - 5q_w \\ &= (45 - p_r)q_w - q_w^2\end{aligned}$$

Taking the derivative with respect to q_w and solving for the optimal level of output (q_w) will yield

$$\frac{d\pi_{NI}}{dq_w} = \frac{d\left((45 - p_r)q_w - q_w^2\right)}{dq_w} = 0$$

$$\Rightarrow 45 - p_r - 2q_w = 0$$

$$\Rightarrow q_w = \frac{45 - p_r}{2}$$

The price of whatsits is then

$$\begin{aligned}p_w &= 50 - q_w \\ &= 50 - \frac{45 - p_r}{2} \\ &= \frac{55 + p_r}{2}\end{aligned}$$

We can then write the price of richets in inverse demand or price dependent form

$$\begin{aligned}p_r &= 45 - 2q_w \\ &= 45 - 2q_r\end{aligned}$$

(b) Use your answer in a to write down the profit function for PepRich. Hence, derive the profit-maximizing price of richets. Use this to derive the profit-maximizing price of whatsits. Calculate the sales of whatsits (and so of richets) and calculate the profits of the two firms.

The profits of PepRich are given by revenue from the sales of richets minus their cost.

Revenue is given by

$$R_{PR} = (45 - 2q_r)q_r$$

Profits are given by

$$\pi_{PR} = (45 - 2q_r)q_r - 5q_r$$
$$= 40q_r - 2q_r^2$$

If we take the derivative of profit with respect to q_r and set equal to zero we obtain

$$\frac{d\pi_{PR}}{dq_r} = \frac{d(40q_r - 2q_r^2)}{dq_r} = 0$$
$$\Rightarrow 40 - 4q_r = 0$$
$$\Rightarrow 4q_r = 40$$
$$\rightarrow q_r = 10$$

The price of richets is then given by

$$p_r = 45 - 2q_r = 45 - 20 = 25$$

The price of whatsits is given by

$$p_w = \frac{55 + p_r}{2}$$
$$= \frac{55 + 25}{2}$$
$$= \frac{80}{2} = 40$$

Sales of whatsits are

$$q_w = 50 - p_w = 50 - 40 = 10.$$

Profits of Norman International are given by revenue minus the cost of the richets minus the other variable costs or

$$\pi_{NI} = (p_w)(10) - (p_r)(10) - (5)(10)$$
$$= (40)(10) - (25)(10) - (5)(10)$$
$$= 400 - 250 - 50$$
$$= 100$$

Profits of PepRich are given by

$$\pi_{PR} = (p_r)(10) - (5)(10)$$
$$= (25)(10) - (5)(10)$$
$$= 250 - 50$$
$$= 200$$

Problem 2

(a) Now assume that these two firms merge to form NPR International. Write down the profit function for NPR given that it sets a price p_w for whatsits. Hence, calculate the postmerger profit-maximizing price for whatsits, sales of whatsits, and the profits of NPR.

Profits for NPR are given by revenue minus the cost of richets (5) and other variable costs. Profits are

$$\pi_{NPR} = (50 - q_w)q_w - 5q_w - 5q_w$$
$$= 40q_w - q_w^2$$

Taking the derivative with respect to q_w and solving for the optimal level of output (q_w) will yield the optimal sales of whatsits.

$$\frac{d\pi_{NPR}}{dq_w} = \frac{d\left(40q_w - q_w^2\right)}{dq_w} = 0$$
$$\Rightarrow 40 - 2q_w = 0$$
$$\rightarrow q_w = 20$$

The price of whatsits is then

$$p_w = 50 - q_w = 50 - 20 = 30$$

Profits for the combined firm are given by

$$\pi_{NPR} = p_w q_w - 5q_w - 5q_w$$
$$= 30q_w - 5q_w - 5q_w$$
$$= 20q_w$$
$$= (20)(20)$$
$$= 400$$

(b) Confirm that this merger has increased the joint profits of the two firms while reducing the price charged to consumers. By how much has consumer surplus been increased by the merger in the market for whatsits?

The joint profits are 400. The profits of NI alone were 100 and the profits of PepRich alone were 200 for a total of 300. Thus, the profits as a merged firm are larger. The price to consumers of 30 is lower than in the case of separate firms when the price of whatsits was 40.

We can compute consumer surplus most easily by finding the area of the rectangle bounded by the two prices and the original quantity and then adding the area of the triangle with height equal to the change in price and base equal to the change in quantity. If we let (p_1, q_1) be the initial price quantity pair for whatsits and (p_2, q_2) be the subsequent pair we obtain

$$\Delta CS = (p_1 - p_2)q_1 + \frac{1}{2}(p_1 - p_2)(q_2 - q_1)$$
$$= (40 - 30)10 + \frac{1}{2}(40 - 30)(20 - 10)$$
$$= (10)(10) + \frac{1}{2}(10)(10)$$
$$= 100 - 50$$
$$= 150$$

Thus consumers are better off with the merger.

(c) Assume that the two firms expect to last forever and that the discount factor R is 0.9. What is the largest sum that PepRich would be willing to pay the owners of Norman International to take over Norman International? What is the lowest sum that the owners of Norman International would be willing to accept? (Hint: Calculate the present value of the profit streams of the two firms before and after the merger, and notice that neither firm will want to be worse off with the takeover than without it.)

We can compute the net present value of the merged firm as a perpetuity where we divide the constant annual profit level by the interest rate. If the discount factor is given by $R = \frac{1}{1+r}$ then the interest rate r is given by $r = \frac{1-R}{R} = \frac{0.1}{0.9} = 0.111$. The net present values of the firms are as follows

$$NPV(PepRich + NI) = \frac{400}{0.11\overline{1}} = 3{,}600$$

$$NPV(NI) = \frac{100}{0.11\overline{1}} = 900$$

$$NPV(PepRich) = \frac{200}{0.11\overline{1}} = 1{,}800$$

For example, PepRich could offer NI 1,500 and still have a net present value of 2,100. NI in this case would have a net present value of 1,500 and both would be better off than as independent firms. Therefore, PepRich would pay up to 1,800 for Norman International while Norman International would be thrilled with an amount over 900.

Problem 3

Now assume that PepRich gets the opportunity to sell to an overseas market for whatsits, controlled by a monopolist FC Hu Inc., which has the same operating costs in making whatsits as Norman International. PepRich knows that it will have to pay transport costs of $2 per richet to supply the overseas market. Inverse demand for whatsits in this market is

$$p_w = 40 - q_w/2.$$

(a) Repeat your calculations for problem 1a.

Profits for FC Hu Inc. are given by revenue minus the cost of richets and other variable costs. If a richet costs p_r per unit we obtain

$$\begin{aligned}
\pi_S &= (40 - \frac{q_w}{2})q_w - p_r q_w - 5q_w \\
&= 40q_w - \frac{q_w^2}{2} - p_r q_w - 5q_w \\
&= (35 - p_r)q_w - \frac{q_w^2}{2}
\end{aligned}$$

Taking the derivative with respect to q_w and solving for the optimal level of output (q_w) will yield

$$\frac{d\pi_S}{dq_w} = \frac{d\left((35 - p_r)q_w - \frac{q_w^2}{2}\right)}{dq_w} = 0$$

$$\Rightarrow 35 - p_r - q_w = 0$$

$$\rightarrow q_w = 35 - p_r$$

The price of whatsits is then

$$
\begin{aligned}
p_w &= 40 - \frac{q_w}{2} \\
&= 40 - \frac{35 - p_r}{2} \\
&= \frac{45 + p_r}{2}
\end{aligned}
$$

We can also write the price of richets in inverse demand or price dependent form

$$p_r = 35 - q_w$$

While not asked for, it might be interesting to consider the optimal production level for PepRich if they only sell richets to FC Hu. First find the maximum profit for PepRich, assuming they sell to FC Hu.

$$
\begin{aligned}
\pi_{PR} &= (35 - q_r)q_r - 5q_r - 2q_r \\
&= 28q_r - q_r^2
\end{aligned}
$$

If we take the derivative of profit with respect to q_r and set equal to zero we obtain

$$
\begin{aligned}
\frac{d\pi_{PR}}{dq_r} = \frac{d(28q_r - q_r^2)}{dq_r} &= 0 \\
\Rightarrow 28 - 2q_r &= 0 \\
\Rightarrow q_r &= 14 \\
\Rightarrow p_r &= 35 \\
&= 21
\end{aligned}
$$

Profits of PepRich are given by

$$
\begin{aligned}
\pi_{PR} &= (21)(14) - (5)(14) - (2)(14) \\
&= 196
\end{aligned}
$$

as compared to 200 in the case of selling to Norman International. FC Hu would have profits of

$$
\begin{aligned}
\pi_S &= (33)(14) - (21)(14) - (5)(14) \\
&= 98
\end{aligned}
$$

The authorities in the overseas market are contemplating taking an antidumping action, accusing PepRich of dumping richets into its market. They calculate that by doing so, they will induce PepRich to offer to take over FC Hu. Assume that PepRich has limited access to funds, so that it can take over only one of Norman International and FC Hu.

(b) Are the overseas authorities correct in their calculations? (Hint: Compare the maximum amounts that PepRich would be willing to pay for Norman International and FC Hu).

Profits for the new merged firm (SPR) are given by revenue minus the cost of richets (5) minus the cost of transport (assuming they continue with overseas production) and other variable costs. Profits are

$$
\begin{aligned}
\pi_{SPR} &= \left(40 - \frac{q_w}{2}\right)q_w - 5q_w - 2q_w - 5q_w \\
&= 28q_w - \frac{q_w^2}{2}
\end{aligned}
$$

Taking the derivative with respect to q_w and solving for the optimal level of output (q_w) will yield the optimal sales of whatsits.

$$\frac{d\pi_{SPR}}{dq_w} = \frac{d\left(28q_w - \frac{q_w^2}{2}\right)}{dq_w} = 0$$

$$\Rightarrow 28 - q_w = 0$$

$$\Rightarrow q_w = 28$$

$$\rightarrow p_w = 40 - \frac{28}{2}$$

$$= 26$$

Profits for the combined firm are given by

$$\begin{aligned}\pi_{SPR} &= p_w q_w - 5q_w - 5q_w - 2q_w \\ &= 26q_w - 5q_w - 5q_w - 2q_w \\ &= 14q_w \\ &= (14)(28) \\ &= 392\end{aligned}$$

The net present value of buying FC Hu as compared to Norman International is found from computing the following net present values where + indicates merger and the / indicates a buyer/seller relationship.

$$NPV(PepRich + NI) = \frac{400}{0.11\bar{1}} = 3{,}600$$

$$NPV(PepRich + SI) = \frac{392}{0.11\bar{1}} = 3{,}528$$

$$NPV(PepRich/NI) = \frac{196}{0.11\bar{1}} = 1{,}800$$

$$NPV(PepRich/FC\ Hu) = \frac{196}{0.11\bar{1}} = 1{,}764$$

$$NPV(NI) = \frac{100}{0.11\bar{1}} = 900$$

$$NPV(FC\ Hu) = \frac{98}{0.11\bar{1}} = 882$$

Assuming the manufacturers of whatsits sell at their reservation value, PepRich is better off merging with Norman International.

Problem 4

Go back to the conditions of question 8, so that PepRich is supplying only Norman International. But now assume that the manufacture of each whatsit requires exactly one richet and one zabit. Zabits are made by ZabCorp., another monopolist, whose variable costs are $2.50 per zabit.

(a) Assume that the three firms act independently to maximize profit. Calculate the resulting prices of richets, zabits, and whatsits and the profits of the three firms. Remember that

$$p_w = 50 - q_w$$

Now let p_z denote the price of zabits and p_r the price of richets. Profit for Norman International is given by

$$\pi_{NI} = (50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w$$

Taking the derivative with respect to q_w and solving for the optimal output will yield

$$\frac{d\pi_{NI}}{dq_w} = \frac{d\big((50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w\big)}{dq_w} = 0$$
$$\Rightarrow 50 - 2q_w - p_r - p_z - 5 = 0$$
$$\rightarrow q_w = \frac{45 - p_r - p_z}{2}$$

The price of whatsits is then

$$p_w = 50 - q_w = 50 - \frac{45 - p_r - p_z}{2} = \frac{55 + p_r + p_z}{2}$$

We can also write the price of richets and zabits in inverse demand or price dependent form

$$p_r = 45 - p_z - 2q_w$$

$$p_z = 45 - p_r - 2q_w$$

Revenue for the two supply firms (PepRich and Zabcorp.) is given by

$$R_{PR} = (45 - p_z - 2q_r)q_r$$
$$R_Z = (45 - p_r - 2q_z)q_z$$

Setting marginal revenue equal to marginal cost will give

$$MR_{PR} = 45 - p_z - 4q_r = 5 = MC_{PR}$$
$$MR_Z = 45 - p_r - 4q_z = 2.5 = MC_Z$$
$$\Rightarrow q_r = \frac{40 - p_z}{4}$$
$$\Rightarrow q_z = \frac{42.5 - p_r}{4}$$

Since $q_w = q_r = q_z$ we can write

$$p_z = 40 - 4q_w$$
$$p_r = 42.5 - 4q_w$$

Plugging this into the equation for the optimal q_w will give

$$q_w = \frac{45 - p_r - p_z}{2} = \frac{45 - (40 - 4q_w) - (42.5 - 4q_w)}{2} = \frac{-37.5 + 8q_w}{2}$$

$$\Rightarrow q_w = 6.25 \Rightarrow p_w = 43.75$$

The market clearing quantities of p_r and p_z are then

$$p_z = 40 - 4q_w = 40 - 25 = 15$$
$$p_r = 42.5 - 4q_w = 42.5 - 25 = 17.5$$

Computing profits will give the following

$$\begin{aligned}\pi_{NI} &= (50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w \\ &= (50 - 6.25)(6.25) - (17.5)(6.25) - (15)(6.25) - (5)(6.26) \\ &= (6.25)(6.25) = 39.0625 \\ \pi_{PR} &= (45 - p_z - 2q_r)q_r - 5q_r \\ &= (45 - 15 - 12.5 - 5)(6.25) \\ &= (12.5)(6.25) = 78.125 \\ \pi_Z &= (45 - p_r - 2q_z)q_z - 2.5q_z \\ &= (45 - 17.5 - 12.5 - 2.5)(6.25) \\ &= (12.5)(6.25) = 78.125\end{aligned}$$

(b) Assume an infinite life for all three firms and a discount factor R = 0.9. PepRich and ZabCorp. are each contemplating a takeover of Norman International. Which of these two companies would win the bidding for Norman International? What will be the effect of the winning takeover on consumer surplus in the market for whatsits?

We now need to consider the profits of each of the merged firms versus the individual profits computed in a. First consider the PepRich and Norman International merger denoted NPR. The profits of the merged firm are given by

$$\begin{aligned}\pi_{NPR} &= (50 - q_w)q_w - 5q_w - p_z q_w - 5q_w \\ &= (40 - p_z - q_w)q_w\end{aligned}$$

Taking the derivative with respect to q_w and solving for the optimal output will yield

$$\frac{d\pi_{NPR}}{dq_w} = \frac{d\big((40 - p_z - q_w)q_w\big)}{dq_w} = 0$$

$$\Rightarrow 40 - p_z - 2q_w = 0$$

$$\rightarrow q_w = \frac{40 - p_z}{2}$$

The price of whatsits is then

$$\begin{aligned}p_w &= 50 - q_w \\ &= 50 - \frac{40 - p_z}{2} \\ &= \frac{60 + p_z}{2}\end{aligned}$$

We can write the price of zabits in inverse demand or price dependent form

$$p_z = 40 - 2q_w$$

We can now compute revenue for Zabcorp. and set marginal revenue equal to marginal cost as follows

$$\begin{aligned}R_z &= (40 - 2q_z)q_z \\ MR_z &= 40 - 4q_z = 2.5 \\ \Rightarrow q_z &= \frac{37.5}{4} = 9.375\end{aligned}$$

Since $q_w = q_z$ we can write

$$\begin{aligned}p_z &= 40 - 2q_z \\ &= 40 - 18.75 \\ &= 21.25\end{aligned}$$

We can then find p_w from

$$p_w = \frac{60 + p_z}{2}$$
$$= \frac{60 + 21.25}{2}$$
$$= 40.625$$

Profits for the combined firm (NPR) are

$$\pi_{NPR} = (40 - p_z - q_w)q_w$$
$$= (40 - 21.25 - 9.375)(9.375)$$
$$= (9.375)(9.375) = 87.890625$$

The net present value of this is

$$NPV(PepRich + NI) = \frac{87.890625}{0.11\overline{1}} = 791.015625$$

The net present value of PepRich from a was

$$NPV(PepRich) = \frac{78.125}{0.11\overline{1}} = 703.125$$

Therefore PepRich could afford to pay (878.90625 - 781.25) = $87.890625 for Norman International. Profits in this new market for Zabcorp. are

$$\pi_z = p_z q_z - 2.5q_z$$
$$= 21.25q_z - 2.5q_z$$
$$= (18.75)(9.375) = 175.78125$$

Now consider the merger of Zabcorp. and Norman International denoted ZN. Proceeding as before we write the profit for the merged firm, choose the optimal quantity of q_w, and then find the price of richets. The profits of the merged firm are given by

$$\pi_{ZN} = (50 - q_w)q_w - 2.5q_w - p_r q_w - 5q_w$$
$$= (42.5 - p_r - q_w)q_w$$

Taking the derivative with respect to q_w and solving for the optimal output will yield

$$\frac{d\pi_{ZN}}{dq_w} = \frac{d\big((42.5 - p_r - q_w)q_w\big)}{dq_w} = 0$$
$$\Rightarrow 42.5 - p_r - 2q_w = 0$$
$$\Rightarrow q_w = \frac{42.5 - p_r}{2}$$

The price of whatsits is then

$$p_w = 50 - q_w$$
$$= 50 - \frac{42.5 - p_r}{2}$$
$$= \frac{57.5 + p_r}{2}$$

We can write the price of richets in inverse demand or price dependent form

$$p_r = 42.5 - 2q_w$$

We can now compute revenue for PepRich and set marginal revenue equal to marginal cost as follows

$$R_r = (42.5 - 2q_r)q_r$$
$$MR_r = 42.5 - 4q_r = 5$$
$$\Rightarrow q_r = \frac{37.5}{4} = 9.375$$

Since $q_w = q_z$ we can write

$$\begin{aligned} p_r &= 42.5 - 2q_r \\ &= 42.5 - 18.75 \\ &= 23.75 \end{aligned}$$

We can then find p_w from

$$\begin{aligned} p_w &= \frac{57.5 + p_r}{2} \\ &= \frac{57.5 + 23.75}{2} \\ &= 40.625 \end{aligned}$$

Profits for the combined firm (ZN) are

$$\begin{aligned} \pi_{ZN} &= (42.5 - p_r - q_w)q_w \\ &= (42.5 - 23.75 - 9.375)(9.375) \\ &= (9.375)(9.375) = 87.890625 \end{aligned}$$

The net present value of this is

$$NPV(Zabcorp. + NI) = \frac{87.890625}{0.11\overline{1}} = 791.015625$$

The net present value of Zabcorp. from (a) was

$$NPV(Zabcorp.) = \frac{78.125}{0.11\overline{1}} = 703.125$$

Hence Zabcorp can afford to pay (878.90625 - 781.25) = \$87.890625 for Norman International.

Both firms can afford the same amount and so it is not clear who will win the bidding. However, since the price of whatsits is \$40.625, which is less than the price of \$43.75 in the original monopoly problem a, consumers are better off. Quantity demanded also increases from 6.125 to 9.375. Consumers' surplus is most easily computed finding the area of the rectangle bounded by the two prices and the original quantity and then adding the area of the triangle with height equal to the change in price and base equal to the change in quantity. If we let (p_1, q_1) be the initial price quantity pair for whatsits and (p_2, q_2) be the subsequent pair we obtain

$$\begin{aligned} \Delta CS &= (p_1 - p_2)q_1 + \frac{1}{2}(p_1 - p_2)(q_2 - q_1) \\ &= (43.75 - 40.625)6.25 + \frac{1}{2}(43.75 - 40.625)(9.375 - 6.25) \\ &= (3.125)(6.25) + \frac{1}{2}(3.125)(3.125) \\ &= 14.6484375 \end{aligned}$$

Thus consumers are better off with either merger. What is obvious is that Norman International will not accept either offer of \$87.890625 since the present value of its profit stream before merger is

$$NPV(Norman) = \frac{39.0625}{0.1111} = 351.5625$$

Problem 5

As an alternative to buying Norman International, the owners of PepRich and ZabCorp. contemplate merging to form PRZ, which will control the manufacture of both richets and zabits.

(a) Calculate the impact of this merger on (1) the prices of richets, zabits, and whatsits, (2) the profits of these firms, and (3) consumer surplus in the whatsit market.

We now have one firm controlling the production of Richets and Zabits. The profits for Norman International are

$$\pi_{NI} = (50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w$$

Taking the derivative with respect to q_w and solving for the optimal output will yield

$$\frac{d\pi_{NI}}{dq_w} = \frac{d\left((50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w\right)}{dq_w} = 0$$

$$\Rightarrow 50 - 2q_w - p_r - p_z - 5 = 0$$

$$\Rightarrow q_w = \frac{45 - p_r - p_z}{2}$$

as before. The price of Whatsits is then

$$p_w = 50 - q_w = 50 - \frac{45 - p_r - p_z}{2} = \frac{55 + p_r + p_z}{2}$$

Since the combined firm of PepRich and Zabcorp. knows that their product is demanded in fixed proportions, they realize that only the total price is relevant to Norman International. Therefore they face an inverse demand curve of

$$p_r + p_z = 45 - 2q_w$$

Revenue for the merged supply firm is given by

$$R_{RZ} = (45 - 2q_{rz})q_{rz}$$

where the subscript rz denotes the quantity of Richets or Zabits and RZ denotes the merged firm. The marginal cost of producing a Richet and a Zabit is 7.5. Setting marginal revenue equal to marginal cost will give

$$MR_{RZ} = 45 - 4q_r = 7.5$$

$$\Rightarrow q_{rz} = \frac{37.5}{4}$$

$$= 9.375$$

Since $q_w = q_{rz}$ we can write

$$p_z + p_r = 45 - 2q_w = 26.25$$

$$p_w = \frac{55 + p_r + p_z}{2}$$

$$= \frac{55 + 26.25}{2} = 40.625$$

Plugging this into the equation for the optimal q_w will give

$$q_w = \frac{45 - (p_r + p_z)}{2}$$

$$= \frac{45 - 26.25}{2}$$

$$= 9.375$$

Computing profits will give the following Comparing consumer surplus in this situation versus the situation in a we obtain

$$\pi_{NI} = (50 - q_w)q_w - (p_r + p_z)q_w - 5q_w$$
$$= (50 - 9.735)(9.735) - (26.25)(9.375) - (5)(9.375)$$
$$= (9.375)(9.375) = 87.890625$$
$$\pi_{RZ} = (45 - 2q_{rz})q_{rz} - 7.5q_{rz}$$
$$= (45 - 18.75 - 7.5)(9.375)$$
$$= (18.75)(9.375) = 175.78125$$
$$\Delta CS = (p_1 - p_2)q_1 + \frac{1}{2}(p_1 - p_2)(q_2 - q_1)$$
$$= (43.75 - 40.625)6.25 + \frac{1}{2}(43.75 - 40.625)(9.375 - 6.25)$$
$$= (3.125)(6.25) + \frac{1}{2}(3.125)(3.125)$$
$$= 14.6484375$$

just as before since the prices and quantities in the Whatsit market are the same with any of the mergers.

(b) Which merger will be preferred

(1) by consumers of whatsits?

Any of the mergers give the same surplus to consumers so they would like a merger.

(2) by the owners of PepRich and ZabCorp.?

These firms are better off if the other one merges with Norman International.

(3) by the owners of Norman International?

If the owners of Norman International are involved in a merger they must share profits, which are less in total than the total of what they, and the merging firm got before merger so they would not like to merge. The best for them is for the supply firms to merge.

Problem 6

(Harder) Ginvir and Sipep are Bertrand competitors in the market for carbonated drinks. Consumers consider their products to be differentiated with the demands for the products of the two firms given by the inverse demand functions

$P_G = 25 - q_G - q_S/2$ for Ginvir and
$P_S = 25 - q_S - q_G/2$ for Sipep

Both companies need syrup to make their drinks that is supplied by two competing companies, NorSyr and BenRup. These companies incur costs of $5 per unit in making the syrup. Both Ginvir and Sipep can use the syrup of either supplier.

(a) Confirm that competition between NorSyr and BenRup leads to the syrup being priced at $5 per unit.

NorSyr and BenRup produce identical goods which are "nondifferentiated" in that they are perfect substitutes for Ginvir and Sipep. Consequently, Ginvir and Sipep buy from the producer who charges the lowest price. If NorSyr and BenRup charge the same price, we assume that each firm faces a demand schedule equal to half of the market demand at the common price. The market demand for syrup is Q = *D(w)* where w is the price of syrup. Since producing syrup costs $5 per unit, the profit of each firm is

$$\pi^{NorSyr}(w_N, w_B) = (w_N - 5)D_N(w_N, w_B)$$
$$\pi^{BenRup}(w_N, w_B) = (w_B - 5)D_B(w_N, w_B)$$

where the demand for the output of firm *i*, denoted D_i, is given by

$$D_N(w_N, w_B) = \begin{cases} D(w_N) & \text{if } w_N < w_B \\ \frac{1}{2}D(w_N) & \text{if } w_N = w_B \\ 0 & \text{if } w_N > w_B \end{cases}$$

for NorSyr and

$$D_B(w_N, w_B) = \begin{cases} D(w_B) & \text{if } w_B < w_N \\ \frac{1}{2}D(w_B) & \text{if } w_N = w_B \\ 0 & \text{if } w_B > w_N \end{cases}$$

for BenRup Combining the profit and demand expressions we obtain

$$\pi^{NorSyr}(w_N, w_B) = \begin{cases} (w_N - 5)D(w_N) & \text{if } w_N < w_B \\ \left(\dfrac{w_N - 5}{2}\right)D(w_N) & \text{if } w_N = w_B \\ 0 & \text{if } w_N > w_B \end{cases}$$

$$\pi^{BenRup}(w_N, w_B) = \begin{cases} (w_B - 5)D(w_B) & \text{if } w_B < w_N \\ \left(\dfrac{w_B - 5}{2}\right)D(w_B) & \text{if } w_N = w_B \\ 0 & \text{if } w_B > w_N \end{cases}$$

The aggregate profit, as usual, cannot exceed the monopoly profit. Each firm can guarantee itself a nonnegative profit by charging a price above marginal cost so we are looking for prices between marginal cost ($5) and the monopoly price. A Bertrand equilibrium is a pair of prices (w_N, w_B) such that each firm's price maximizes that firm's profit given the other firm's price, that is

$$\pi^{NorSyr}(w_N^*, w_B^*) \geq \pi^{NorSyr}(w_N, w_B^*)$$

$$\pi^{BenRup}(w_N^*, w_B^*) \geq \pi^{BenRup}(w_N^*, w_B).$$

We can show that the firms will each charge the same price and that it will be equal to marginal cost ($5) as follows. Consider, for example, the case where NorSyr charges a price

$$w_N^* > w_B^* > c.$$

Then NorSyr has no demand, and its profit is zero. On the other hand, if NorSyr charges

$$w_N = w_B^* - \varepsilon$$

(where ε is positive and "small"), it obtains the entire market demand, $D(w_B^* - \varepsilon)$, and has a positive profit margin of

$$w_B^* - \varepsilon - 5.$$

Therefore, NorSyr cannot be acting in its own best interest if it charges w_N^*. Now suppose that

$$w_N^* = w_B^* > c.$$

The profit of firm NorSyr is

$$\pi^{NorSyr}(w_N^*, w_B^*) = \left(\frac{w_N^* - 5}{2}\right) D(w_N^*).$$

If NorSyr reduces its price slightly to $w_N^* - \varepsilon$, its profit becomes

$$\pi^{NorSyr}(w_N^* - \varepsilon, w_B^*) = \left(w_N^* - \varepsilon - 5\right) D(w_N^* - \varepsilon).$$

which is greater for small ε. The market share of the firm increases in a discontinuous manner. Because no firm will charge less than the unit cost c (doing so would make negative profit), we are left with one or two firms charging exactly c. To show that both firms charge c, suppose that

$$w_N^* > w_B^* = c.$$

Then BenRup, which makes no profit, could raise its price slightly, still supply all the demand, and make a positive profit—a contradiction.

(b) What are the resulting equilibrium prices for Ginrip and Sipep and what are their profits? Profit for Ginrip is given by price minus marginal cost multiplied by the quantity sold or

$$\pi^{Ginrip}(P_G, P_S) = (P_G - 5)\, q_G(P_G, P_S).$$

Similarly for Sipep we obtain

$$\pi^{Sipep}(P_G, P_S) = (P_S - 5)\, q_S(P_G, P_S).$$

We are given the inverse demand system for the two firms. Because the firms are Bertrand competitors, the optimal prices are determined by taking the derivatives of the two profit expressions with respect to price and then solving the system for P_G and P_S. To write the profit system in terms of prices, we need to solve the inverse demand system for quantity as a function of price. First rewrite each inverse demand function as follows.

$$P_G = 25 - q_G - \frac{q_S}{2}$$

$$P_S = 25 - q_S - \frac{q_G}{2}$$

$$\Rightarrow q_S = 25 - P_S - \frac{q_G}{2}$$

$$\rightarrow q_G = 25 - P_G - \frac{q_S}{2}$$

Then substitute for q_S in the last expression as follows

$$q_G = 25 - P_G - \frac{q_S}{2}$$

$$= 25 - P_G - \frac{\left(25 - P_S - \frac{q_G}{2}\right)}{2}$$

$$= 25 - P_G - 12.5 + \frac{P_S}{2} + \frac{q_G}{4}$$

$$\Rightarrow \frac{3}{4} q_G = 12.5 - P_G + \frac{P_S}{2}$$

$$\Rightarrow q_G = 16\frac{2}{3} - \frac{4}{3} P_G + \frac{2}{3} P_S$$

$$= 16.6\bar{6} - 1.3\bar{3} P_G + 0.6\bar{6} P_S.$$

We find q_S by substitution as follows

$$\begin{aligned} q_S &= 25 - P_S - \frac{q_G}{2} \\ &= 25 - P_S - \frac{\left(16.6\bar{6} - 1.3\bar{3}P_G + 0.6\bar{6}P_S\right)}{2} \\ &= 25 - P_S - 8.3\bar{3} + 0.6\bar{6}P_G - 0.3\bar{3}P_S \\ &= 16.6\bar{6} - 1.3\bar{3}P_S + 0.6\bar{6}P_G \end{aligned}$$

We can now write the two profit equations in the following form

$$\begin{aligned} \pi^{Ginrip}(P_G, P_S) &= (P_G - 5)\left(16.6\bar{6} - 1.3\bar{3}P_G + 0.6\bar{6}P_S\right) \\ &= 16.6\bar{6}P_G - 1.3\bar{3}P_G^2 + 0.6\bar{6}P_GP_S + 6.6\bar{6}P_G - 3.3\bar{3}P_S - 83.3\bar{3} \\ &= 23.3\bar{3}P_G - 1.3\bar{3}P_G^2 + 0.6\bar{6}P_GP_S - 3.3\bar{3}P_S - 83.3\bar{3} \\ \pi^{Sipep}(P_G, P_S) &= (P_S - 5)\left(16.6\bar{6} - 1.3\bar{3}P_S + 0.6\bar{6}P_G\right) \\ &= 16.6\bar{6}P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_GP_S + 6.6\bar{6}P_S - 3.3\bar{3}P_G - 83.3\bar{3} \\ &= 23.3\bar{3}P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_GP_S - 3.3\bar{3}P_G - 83.3\bar{3}. \end{aligned}$$

Differentiating with respect to P_G and P_S we obtain the following system of equations

$$\begin{aligned} \pi^{Ginrip} &= 23.3\bar{3}P_G - 1.3\bar{3}P_G^2 + 0.6\bar{6}P_GP_S - 3.3\bar{3}P_S - 83.3\bar{3} \\ \frac{\partial \pi^{Ginrip}}{P_G} &= 23.3\bar{3} - 2.6\bar{6}P_G + 0.6\bar{6}P_S = 0 \\ \pi^{Sipep}(P_G, P_S) &= 23.3\bar{3}P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_GP_S - 3.3\bar{3}P_G - 83.3\bar{3} \\ \frac{\partial \pi^{Sipep}}{P_S} &= 23.3\bar{3} - 2.6\bar{6}P_S + 0.6\bar{6}P_G = 0 \\ &= 23.3\bar{3} + 0.6\bar{6}P_G - 2.6\bar{6}P_S = 0. \end{aligned}$$

Now multiply the first order condition for Sipep by four and subtract from the first order equation for Ginrip and solve for P_S.

$$\begin{aligned} 23.3\bar{3} - 2.6\bar{6}P_G + 0.6\bar{6}P_S &= 0 \\ 93.3\bar{3} + 2.6\bar{6}P_G - 10.6\bar{6}P_S &= 0 \\ \Rightarrow 116.6\bar{6} - 10P_S &= 0 \\ \rightarrow P_S &= 11.6\bar{6}. \end{aligned}$$

We obtain P_G by substitution:

$$\mathbf{23.33 - 2.66P_G + 0.66)11.66) = 0}$$

$\Rightarrow$ **$2.66P_G = 31.11$**

$\Rightarrow$ **$P_G = 11.66$**

Quantities may also be obtained by substitution.

$$q_G = 16.66 - 1.33P_G + 0.66P_S$$
$$= 16.6\overline{6} - 1.3\overline{3}(11.6\overline{6}) + 0.6\overline{6}(11.6\overline{6})$$
$$= 16.66 - 15.55 + 7.77$$
$$= 8.8\overline{8}$$
$$q_S = 16.6\overline{6} - 1.3\overline{3}P_S + 0.6\overline{6}P_G$$
$$= 16.66 - 1.33(11.66) + 0.66(11.66)$$
$$= 8.8\overline{8}.$$

Profits are straightforward to compute.

$$\pi^{Ginrip}(P_G, P_S) = (P_G - 5)\,q_G(P_G, P_S)$$
$$= (11.6\overline{6} - 5)(8.8\overline{8})$$
$$= (6.66)(8.88)$$
$$= 59.259259$$
$$\pi^{Sipepp}(P_G, P_S) = (P_S - 5)\,q_S(P_G, P_S)$$
$$= (11.66 - 5)(8.88)$$
$$= 59.259\overline{259}.$$

Total profits for the two firms, who are able to buy the syrup at marginal cost of $5 per unit, are 118.518518

Problem 7

Now suppose that Ginvir and NorSyr merge and that in doing so NorSyr no longer competes for Sipep's business.

(a) What price will BenRup now charge Sipep for the syrup?

Benrup is no longer in Bertrand competition with a homogeneous product as before, but is a monopolist in the supply of the input to Sipep. The profit maximization problem for Benrup is a follows.

$$\pi^{BenRup}(w_B) = (w_B - 5)D_B(w_N, w_B)$$
$$= (w_B - 5)q_S(P_g, P_S, w_B)$$

It takes one unit of syrup to make one unit of carbonated drink and the quantity of syrup supplied is now variable depending on the price of syrup (which is no longer fixed at $5.00 by competition). Differentiating profits with respect to w_B, we obtain

$$\pi^{BenRup}(w_B) = (w_B - 5)q_S(P_g, P_S, w_B)$$

$$\frac{\partial \pi^{BenRup}(w_B)}{w_B} = (w_B - 5)\left(\frac{\partial q_S(P_G, P_S, w_B)}{w_B}\right) + q_S(P_g, P_S, w_B) = 0$$

Now rewrite profit for Sipep taking account of the fact that the cost of syrup is not \$5 but w_B. This will give

$$\begin{aligned}\pi^{Sipep}(P_G, P_S, w_B) &= (P_S - w_B)\left(16.6\bar{6} - 1.3\bar{3}P_S + 0.6\bar{6}P_G\right)\\ &= 16.6\bar{6}P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_G P_S + 1.3\bar{3}w_B P_S - 0.6\bar{6}w_B P_G - 16.6\bar{6}w_B\\ &= (16.6\bar{6} + 1.3\bar{3}w_B)P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_G P_S - 0.6\bar{6}w_B P_G - 16.6\bar{6}w_B\end{aligned}$$

Differentiating the profit equations for Sipep and Ginvir with respect to P_G and P_S we obtain the following system of equations

$$\pi^{Ginrip} = 23.3\bar{3}P_G - 1.3\bar{3}P_G^2 + 0.6\bar{6}P_G P_S - 3.3\bar{3}P_S - 83.3\bar{3}$$

$$\frac{\partial \pi^{Ginrip}}{P_G} = 23.3\bar{3} - 2.6\bar{6}P_G + 0.6\bar{6}P_S = 0$$

$$\pi^{Sipep}(P_G, P_S) = (16.6\bar{6} + 1.3\bar{3}w_B)P_S - 1.3\bar{3}P_S^2 + 0.6\bar{6}P_G P_S - 0.6\bar{6}w_B P_G - 16.6\bar{6}w_B$$

$$\frac{\partial \pi^{Sipep}}{P_S} = 16.6\bar{6} + 1.3\bar{3}w_B - 2.6\bar{6}P_S + 0.6\bar{6}P_G = 0$$

Now multiply the first order condition for Sipep by four and subtract from the first order equation for Ginrip and solve for P_S.

$$\begin{aligned}23.3\bar{3} - 2.6\bar{6}P_G + 0.6\bar{6}P_S &= 0\\ 66.6\bar{6} + 5.3\bar{3}w_B + 2.6\bar{6}P_G - 10.6\bar{6}P_S &= 0\\ \Rightarrow 90 + 5.3\bar{3}w_B - 10P_S &= 0\\ \rightarrow 10P_S &= 90 + 5.3\bar{3}w_B\\ \Rightarrow P_S &= 9 + 0.53\bar{3}w_B.\end{aligned}$$

We obtain P_G by substitution as follows

$$23.33 - 2.66P_G + 0.66(9 + 0.533w_B) = 0$$

$$\rightarrow 23.3\bar{3} - 2.6\bar{6}P_G + 6 + 0.35\bar{5}w_B = 0$$

$$\Rightarrow 2.6\bar{6}P_G = 29.3\bar{3} + 0.35\bar{5}w_B$$

$$\Rightarrow P_G = 11 + 0.13\bar{3}w_B.$$

Quantities are also obtained by substitution. First for Ginvir.

$$q_G = 16.66 - 1.33P_G + 0.66P_S$$

$$= 16.6\bar{6} - 1.3\bar{3}(11 + 0.13\bar{3}w_B) + 0.6\bar{6}(9 + 0.53\bar{3}w_B)$$

$$= 16.6\bar{6} - 14.6\bar{6} - 0.17\bar{7}w_B + 6 + 0.35\bar{5}w_B$$

$$= 8 + 0.17\bar{7}w_B$$

Then for Sipep.

$$q_S = 16.66 - 1.33P_S + 0.66P_G$$

$$= 16.6\bar{6} - 1.3\bar{3}(9 + 0.53\bar{3}w_B) + 0.6\bar{6}(11 + 0.13\bar{3}w_B)$$

$$= 16.6\bar{6} - 12 - 0.71\bar{1}w_B + 7.3\bar{3} + 0.08\bar{8}w_B$$

$$= 12 - 0.62\bar{2}w_B$$

Notice at this point that the prices and quantities for both firms will be the same if $w_B = 5$ as in part b. As w_B rises the price charged by Sipep will rise and the quantity sold will fall. We solve for w_B by using the profit maximization condition for Benrup and substituting as appropriate. Before substituting in this expression note that

$$\frac{\partial q_S(P_G, P_S, w_B)}{w_B} = -0.62\bar{2}$$

Now substitute in the first order condition

$$\frac{\partial \pi^{BenRup}(w_B)}{w_B} = (w_B - 5)\left(\frac{\partial q_S(P_G, P_S, w_B)}{w_B}\right) + q_S(P_g, P_S, w_B) = 0$$

$$\rightarrow (w_B - 5)(-0.62\bar{2}) + 12 - 0.62\bar{2}w_B = 0$$

$$\Rightarrow -1.24\bar{4}w_B + 15.1\bar{1} = 0$$

$$\rightarrow 1.244w_B = 15.11$$

$$\rightarrow w_B = 12.142857$$

(b) What are the resulting profits to the three post-merger companies?
First we need to compute the equilibrium prices and quantities for each firm. First the prices.

$$\begin{aligned}
w_B &= 12.142857 \\
P_S &= 9 + 0.53\bar{3}w_B \\
&= 9 + 0.53\bar{3}(12.142857) \\
&= 15.4762 \\
P_G &= 11 + 0.13\bar{3}w_B \\
&= 11 + 0.13\bar{3}(12.142857) \\
&= 12.619.
\end{aligned}$$

Then the quantities are follows

$$\begin{aligned}
q_G &= 8 + 0.177w_B \\
&= 8 + 0.17\bar{7}(12.142857) \\
&= 10.1587 \\
q_S &= 12 - 0.62\bar{2}w_B \\
&= 12 - 0.62\bar{2}(12.142857) \\
&= 4.4\bar{4}
\end{aligned}$$

Profits are given by substitution.

$$\begin{aligned}
\pi^{BenRup}(w_B) &= (w_B - 5)q_S(P_g, P_S, w_B) \\
&= (12.142857 - 5)(4.4\bar{4}) \\
&= 31.746 \\
\pi^{Ginrip}(P_G, P_S) &= (P_G - 5)\,q_G(P_G, P_S) \\
&= (12.619 - 5)(10.1587) \\
&= 77.399 \\
\pi^{Sipepp}(P_G, P_S) &= (P_S - w_B)\,q_S(P_G, P_S) \\
&= (15.4762 - 12.142857)(4.4\bar{4}) \\
&= 14.8148
\end{aligned}$$

Total profits for the three firms are \$123.9598. Profits for the combined Benrup/Sipep firm are \$46.5608.

(c). Do BenRup and Sipep have an incentive also to merge?

If BenRup and Sipep merge, they will then be able to compete in a duopoly with Ginvir as in part b, with a cost of syrup of \$5. This will lead to profits of \$59.2593, which are higher than if they do not merge. So they will also merge. Given that Ginvir can anticipate this move, they will also not merge and so nothing will take place in the industry.

Problem 8

Profits with no vertical integration are:

$\pi_1^U = \pi_2^U = \dfrac{2\left(A - c^U - c^D\right)^2}{27B}$ for the upstream firms and $\pi_1^D = \pi_2^D = \dfrac{4\left(A - c^U - c^D\right)^2}{81B}$ for the downstream firms. Total profit for each upstream-downstream pair is: $\dfrac{10\left(A - c^U - c^D\right)^2}{81}$.

If both firms vertically integrate, they compete as Cournot duopolists, each with a constant marginal cost of $c^U + c^D$. Hence, each merged firm's output is:

$$q_1^D = q_2^D = \frac{A - c^U - c^D}{3B}$$

Total output is $Q = \frac{2\left(A - c^U - c^D\right)}{3B}$ and the retail price is $P = \frac{A + 2\left(c^U + c^D\right)}{3}$. Profit to each firm is $\pi_1 = \pi_2 = \frac{\left(A - c^U - c^D\right)^2}{9B}$. This is clearly less than the combined profit earned by an unintegrated pair of upstream and downstream firms.

Problem 9

(a) Assume that marginal cost is zero. Since demand is: $P = 100 - Q$, monopolization of the industry would mean a firm facing a marginal revenue described by: $MR = 100 - 2Q$. Equating marginal revenue with marginal cost ($c = 0$) implies $Q = 50 = P$, so that profit is $2500.

(b) If both firms reject the offer, each earns zero profit. If both accept the offer, the $1250 becomes a sunk cost. They will compete as Cournot duopolists with a marginal cost of zero. Normally, this would yield an output of 33.33 for each. However, since they are constrained by their allotment of 25 units each, they will simply sell out their entire stock. All fifty units will be sold and the market price will be $50. Each will earn 25 x $50 = $1250 in operating profit. This will just cover the charge by the monopoly supplier so that the net profit for each will again be zero. If only one downstream firms accepts the offer, she becomes a retail monopolist with a capacity of 25 units. She will sell all 25 (she would like to sell more) at a market-clearing price of $75 each for a total revenue of $1875—more than enough to cover the charge of $1250 from the upstream supplier. So, the payoff when only one firms accepts the offer is $1875 - $1250 = $625. The payoff matrix is then:

		Downstream Firm 2	
		Reject	Accept
Downstream Firm 1	Reject	(0,0)	(0,$625)
	Accept	($625,0)	(0,0)

Accept is a weakly dominant strategy (a slight reduction in the fee from $1250 to $1249 can make it strictly dominant). Therefore, the Nash Equilibrium is for both to accept the offer.

Problem 10

(a) In a sequential setting, if one firm accepts the offer the monopolist can subsequently make the offer to the latter firm and drive all downstream profit to zero. Whichever downstream firm goes first, can prevent this from happening and raise its profit by holding out for a better deal. The fact that a particular firm goes first means that the upstream supplier cannot sell to anyone else until it settles with the first retailer. This fact gives the firs downstream firm some bargaining power.

(b) Vertical integration eliminates all double marginalization. By foreclosing the alternative retailer, the integrated firm is in exactly the same position as the monopolist in 9a. Therefore, it can exactly duplicate that outcome of P = $50; Q = 50; and Profit = $2500.