Chapter 16: Horizontal Mergers

Learning Objectives

Students will learn to:

1. Explain the reasons behind mergers. Some of these have to do with increased efficiency and reduced costs of production and marketing. Others relate to the ability to enhance product quality or services.
2. Differentiate horizontal, vertical, complementary, and conglomerate mergers. Horizontal mergers take place between firms that produce or consume *substitute* goods at the same point in the vertical chain. Vertical mergers typically take place between firms at different stages in the chain or between firms at the same stage that produce or consume *complementary* goods. Outputs at one stage that are inputs at another stage are obviously complements in production. Conglomerate mergers involve the combination of firms without a clear substitute or a clear complementary relationship.
3. Analyze horizontal merger using a simple Cournot model.
4. Explain why there is a merger paradox in the Cournot merger model, why it makes some intuitive sense, and provide a reasoned discussion of the weaknesses of the model.
5. Set up and solve a simple three-firm Cournot model and then resolve the model assuming two of the firms merge leaving a two-firm duopoly. The student will be able to explain the merger paradox using this simple merger model.
6. Show how the results of the Cournot merger model differ if the firms involved have differing fixed or marginal costs either before or after the mergers, i.e. show that potential cost savings matter.
7. Explain why the Stackelberg merger model may be more realistic than the Cournot merger model. The student will be able to give reasons why a newly merged firm may be able to act as a Stackelberg leader.
8. Differentiate the conclusions of the alternative horizontal merger models (Cournot and multiple leader/follower Stackelberg) and relate them to the model assumptions.
9. Understand the importance of sequential play in which commitment is possible and the reason that mergers that might not happen in a simultaneous setting can happen in a sequential one.
10. Explain why a horizontal merger may make sense when the products are differentiated substitutes as compared to homogeneous goods. The student will be able to solve a simple circle location model involving the merger of two of the firms.
11. Solve a simple circle location model involving price discrimination. The student will be able to show how the market equilibrium in the price discriminating circle location model changes with the merger of two of the firms. The student will understand why mergers are a particularly valuable way to avoid price competition in a circle location model where price discrimination is possible and prevalent.
12. Explain the public policies towards horizontal merger.

Suggested Lecture Outline:

Spend two fifty-minute long lectures on this chapter.

Lecture 1:

1. Introduction
2. Cournot models of merger
3. Introduction to Stackelberg model of mergers

Lecture 2:

1. Analyze Sequential Merger Models
2. Location models of horizontal mergers
   i) No price discrimination
   ii) Price discrimination
3. Public Policies towards horizontal mergers

Suggestions for the Instructor:

1. The introduction to these lectures is important in motivating the topic.
2. Examples of horizontal mergers are very helpful in motivating the material.
3. Common examples of horizontal mergers include:

   Banking firms in neighboring communities or states

   Competing department stores in a given urban area or region

   Concrete ready-mix firms

   Trucking firms or railroads

   Airplane manufacturing firms (Boeing and everyone else)

   Retail hardware and garden supply stores
4. The results from on the N firm Cournot model should be reviewed briefly before presenting the merger results. A numerical homework problem might be given a week or so before beginning this set of lectures so that students have the ideas fresh in their minds.
5. The easiest way to motivate mergers is to consider a three-firm Cournot model and compare the total profits and profits per firm with a two-firm Cournot model. This is simpler than the general N firm case initially presented in the text. Relate this to the material on collusion and cartels and the incentives for firms to control total market supply. The merger paradox becomes more transparent as we move from three firms to two firms to a single firm.
6. It is important to discuss the limitations of the Cournot merger model. The model has often been criticized in the literature and the students will think of some of these criticisms themselves. Have the students try to think of how they would respond to the criticisms, as well.
7. The material on the multiple leader multiple follower Stackelberg model is quite complicated as far as algebraic manipulation. One approach is to take some time and go over this in detail as a review of much of the course to this point. An alternative is simply to state the results in general terms (show the final inequalities) and spend more time on the interpretation of the result. The use of a spreadsheet table here may be very useful, even if the instructor just presents the results rather than have the student create them.
8. Spend time motivating mergers in location models.

Solutions to End of the Chapter Problems:

Problem 1

(a) First consider the 3 symmetric firms. For the first firm profit is given by

$$\begin{aligned}\pi_1 &= Pq_1 - 20q_1 \\ &= (100 - q_1 - q_2 - q_3 - q_4)q_1 - 20q_1 \\ &= 100q_1 - q_1^2 - q_2q_1 - q_3q_1 - q_4q_1 - 20q_1\end{aligned}$$

If we maximize profit we obtain q_1 as a function of the other firms' outputs.

$$\begin{aligned}\frac{d\pi_1}{dq_1} &= 100 - 2q_1 - q_2 - q_3 - q_4 - 20 = 0 \\ \Rightarrow 2q_1 &= 80 - q_2 - q_3 - q_4 \\ \Rightarrow q_1 &= \frac{80 - q_2 - q_3 - q_4}{2}\end{aligned}$$

Since the first three firms are symmetric we know that $q_1 = q_2 = q_3$. This will then give

$$q_1 = \frac{80 - q_2 - q_3 - q_4}{2} \Rightarrow q_1 = \frac{80 - q_1 - q_1 - q_4}{2} \Rightarrow q_1 = 20 - \frac{q_4}{4}$$

This also implies that $q_1 + q_2 + q_3 = 60 - \frac{3}{4}q_4$

Now consider the profit maximization problem of firm 4.

$$\begin{aligned}\pi_4 &= Pq_4 - (20 + \gamma)q_4 \\ &= (100 - q_1 - q_2 - q_3 - q_4)q_4 - 20q_4 - \gamma q_4 \\ &= 100q_4 - q_1q_4 - q_2q_4 - q_3q_4 - q_4^2 - 20q_4 - \gamma q_4\end{aligned}$$

If we maximize profit we obtain q_4 as a function of the other firms' outputs.

$$\begin{aligned}\frac{d\pi_4}{dq_4} &= 100 - q_1 - q_2 - q_3 - q_4 - 20 - \gamma = 0 \\ \rightarrow 2q_4 &= 80 - q_1 - q_2 - q_3 - \gamma \\ \rightarrow q_4 &= \frac{80 - q_1 - q_2 - q_3 - \gamma}{2}\end{aligned}$$

If we substitute in the expression for $q_1 + q_2 + q_3$, we obtain

$$\begin{aligned}q_4 &= \frac{80 - q_1 - q_2 - q_3 - \gamma}{2} \\ &= \frac{80 - \gamma - \left(60 - \frac{3}{4}q_4\right)}{2} \\ &= \frac{20 - \gamma + \frac{3}{4}q_4}{2} \\ \Rightarrow \frac{5}{4}q_4 &= 20 - \gamma \\ \Rightarrow q_4 &= 16 - \frac{4}{5}\gamma\end{aligned}$$

We can then find the optimal levels of the first three firms by substituting in as follows

$$\begin{aligned} q_i &= 20 - \frac{1}{4} q_4 \\ &= 20 - \frac{1}{4} (16 - \frac{4}{5} \gamma) \\ &= 16 + \frac{1}{5} \gamma \end{aligned}$$

The price is given by

$$\begin{aligned} P &= 100 - q_1 - q_2 - q_3 - q_4 \\ &= 100 - 64 - \frac{3}{5}\gamma + \frac{4}{5}\gamma \\ &= 36 + \frac{1}{5} \gamma \end{aligned}$$

The profit for each symmetric firm is given by

$$\begin{aligned} \pi_i &= Pq_i - 20q_i \\ &= (36 + \frac{1}{5} \gamma)(16 + \frac{1}{5} \gamma) - (20)(16 + \frac{1}{5} \gamma) \\ &= (16 + \frac{1}{5} \gamma)(16 + \frac{1}{5} \gamma) \\ &= 256 + \frac{32}{5} \gamma + \frac{1}{25} \gamma^2 \end{aligned}$$

Profit for firm 4 is given by

$$\begin{aligned} \pi_4 &= Pq_4 - 20q_4 - \gamma q_4 \\ &= (36 + \frac{1}{5} \gamma)(16 - \frac{4}{5} \gamma) - (20+\gamma)(16 - \frac{4}{5} \gamma) \\ &= (16 - \frac{4}{5} \gamma)(16 - \frac{4}{5} \gamma) \\ &= 256 - \frac{128}{5} \gamma + \frac{16}{25} \gamma^2 \end{aligned}$$

The restrictions on the model are that price be greater than marginal cost and that quantities are nonnegative. Since marginal cost for the first three firms is equal to 20, this implies that

$$\begin{aligned} P = 36 + \frac{1}{5} \gamma \geq 20 \\ \rightarrow \gamma > -80 \end{aligned}$$

Since quantities must be positive we also have that

$$\begin{aligned} 16 + \frac{1}{5} \gamma \geq 0 \\ \Rightarrow \gamma > -80 \\ 16 - \frac{4}{5} \gamma \geq 0 \\ \rightarrow \gamma \leq 20 \end{aligned}$$

But we also require that marginal cost for the fourth firm is positive then, i.e., $\gamma > 0$. This then implies

$$\begin{aligned} 20 + \gamma \geq 0 \\ \rightarrow \gamma \geq -20 \end{aligned}$$

Combining the constraints and eliminating the redundant ones will yield

$$20 \leq \gamma \leq 20.$$

(b) There are now three firms, two identical and one different. Solution is as in part a. Denote the firms *m*, 3 and 4, where m is the combined firm. First consider the 2 symmetric firms. For the first firm profit is given by

$$\pi_m = Pq_m - 20q_m$$
$$= (100 - q_m - q_3 - q_4)q_m - 20q_m$$
$$= 100q_m - q_m^2 - q_3q_m - q_4q_m - 20q_m$$

If we maximize profit we obtain q_m as a function of the other firms' outputs.

$$\frac{d\pi_m}{dq_m} = 100 - 2q_m - q_3 - q_4 - 20 = 0$$
$$\Rightarrow 2q_m = 80 - q_3 - q_4$$
$$\rightarrow q_m = \frac{80 - q_3 - q_4}{2}$$

Since the merged firm and firm 3 are symmetric we know that $q_m = q_3$. This will then give

$$q_m = \frac{80 - q_3 - q_4}{2}$$
$$\Rightarrow q_m = \frac{80 - q_m - q_4}{2}$$
$$\Rightarrow \frac{3}{2}q_m = 40 - \frac{q_4}{2}$$
$$\Rightarrow q_m = 26.6\overline{6} - \frac{q_4}{3}$$

Now consider the profit maximization problem of firm 4.

$$\pi_4 = Pq_4 - (20 + \gamma)q_4$$
$$= (100 - q_m - q_3 - q_4)q_4 - 20q_4 - \gamma q_4$$
$$= 100q_4 - q_mq_4 - q_3q_4 - q_4^2 - 20q_4 - \gamma q_4$$

If we maximize profit we obtain q_4 as a function of the other firms' outputs.

$$\frac{d\pi_4}{dq_4} = 100 - q_m - q_3 - 2q_4 - 20 - \gamma = 0$$
$$\Rightarrow 2q_4 = 80 - q_m - q_3 - \gamma$$
$$\Rightarrow q_4 = \frac{80 - q_m - q_3 - \gamma}{2}$$

If we substitute in for the other firms we obtain

$$q_4 = \frac{80 - q_m - q_3 - \gamma}{2}$$

$$= \frac{80 - \gamma - \left(53.33\overline{3} - \frac{2}{3}q_4\right)}{2}$$

$$= \frac{26.66\overline{6} - \gamma + \frac{2}{3}q_4}{2}$$

$$\Rightarrow \frac{4}{3}q_4 = 26.66\overline{6} - \gamma$$

$$\Rightarrow q_4 = 20 - \frac{3}{4}\gamma$$

We can then find the optimal levels of the other two firms by substituting in as follows

$$q_i = 26.66\overline{6} - \frac{1}{3}q_4$$

$$= 26.66\overline{6} - \frac{1}{3}(20 - \frac{3}{4}\gamma)$$

$$= 20 + \frac{1}{4}\gamma$$

The price is given by

$$P = 100 - q_m - q_3 - q_4$$

$$= 100 - (40 + \frac{1}{2}\gamma) - (20 - \frac{3}{4}\gamma)$$

$$= 40 + \frac{1}{4}\gamma$$

The profit for each symmetric firm is given by

$$\pi_i = Pq_i - 20q_i$$

$$= (40 + \frac{1}{4}\gamma)(20 + \frac{1}{4}\gamma) - (20)(20 + \frac{1}{4}\gamma)$$

$$= (20 + \frac{1}{4}\gamma)(20 + \frac{1}{4}\gamma)$$

$$= 400 + 10\gamma + \frac{1}{16}\gamma^2$$

Profit for firm 4 is given by

$$\pi_4 = Pq_4 - 20q_4 - \gamma q_4$$

$$= (40 + \frac{1}{4}\gamma)(20 - \frac{3}{4}\gamma) - (20 + \gamma)(20 - \frac{3}{4}\gamma)$$

$$= (20 - \frac{3}{4}\gamma)(20 - \frac{3}{4}\gamma)$$

$$= 400 - 30\gamma + \frac{9}{16}\gamma^2$$

The combined profits from part a are

$$2\pi_i = (2)(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2)$$
$$= 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2$$
$$= 512 + 12.8\gamma + \frac{2}{25}\gamma^2$$

If γ is positive then the sum of the independent profits are higher since they are higher in every term. If γ is negative then the difference between (512 + 12.8(+ 2/25 (γ^2) and (400 - 30(+ 1/16 (γ^2) could shrink. Denote this difference as Δ = 112 - 2.8 - 0.0175γ^2. Consider this difference at the lower bound for γ (-20). The profits in the two cases are

$$\pi(sum) = 512 + 12.8\ (-20) + \frac{2}{25}\ (400) = 288$$
$$\pi(merge) = 400 + 10\ (-20) + \frac{1}{16}\ (400) = 225$$

The difference is 63. At γ = 0, the difference is 112. At γ = 20, the difference is 175.

(c) We assume that firms 2 and 3 are still independent. Since firm 4 has higher costs than firm 1, all production at the merged firm will take place using the facilities of firm 1. Thus we have a three-firm Cournot game where the firms are symmetric. We can proceed as in a, essentially ignoring firm 4.

For the merged firm profit is given by

$$\pi_m = Pq_m - 20q_m$$
$$= (100 - q_m - q_2 - q_3)q_m - 20q_m$$
$$= 100q_m - q_m^2 - q_2q_m - q_3q_m - 20q_m$$

If we maximize profit we obtain q_m as a function of the other firms' outputs.

$$\frac{d\pi_m}{dq_m} = 100 - 2q_m - q_2 - q_3 - 20 = 0$$
$$\Rightarrow 2q_m = 80 - q_2 - q_3$$
$$\Rightarrow q_m = \frac{80 - q_2 - q_3}{2}$$

Since the three firms are symmetric we know that $q_m = q_2 = q_3$. This will then give

$$q_m = \frac{80 - q_2 - q_3}{2} \Rightarrow q_m = \frac{80 - q_m - q_m}{2} \Rightarrow q_m = 20$$

Price is given by

$$P = 100 - q_m - q_2 - q_3 = 100 - 60 = 40$$

The profits for each firm are given by

$$\pi_i = Pq_i - 20q_i$$
$$= (40)(20) - (20)(20)$$
$$= 800 - 400$$
$$= 400$$

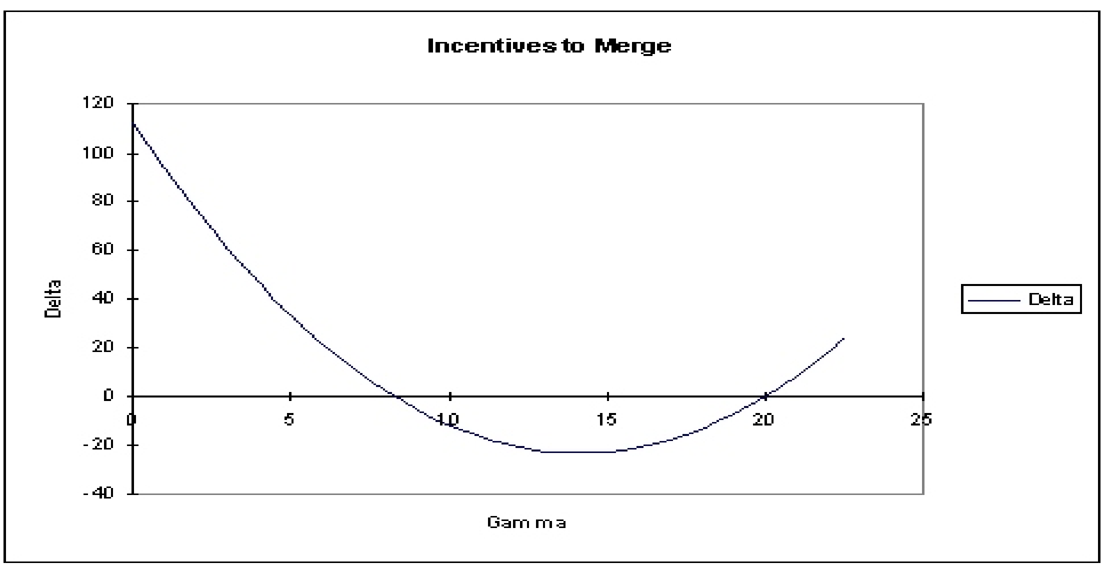
So firms 2 and 3 each have profits of 400. In a, the profits of firm 2 were $\pi_2 = \left(16 + \frac{1}{5}\gamma\right)^2$. The largest possible value of γ is 20 so the maximum profit for firm 2 in the initial problem is 400. Thus, firm 2 cannot lose from this merger. We now need to compare this to the profits that occurred in a to see how firms 1 and 4 do. The combined profits of firm 1 and firm 4 in part a are

$$\pi_1 = (16 + \frac{1}{5}\gamma)^2$$
$$= 256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2$$
$$\pi_4 = (16 - \frac{4}{5}\gamma)^2$$
$$= 256 - \frac{128}{5}\gamma + \frac{16}{25}\gamma^2$$
$$\Rightarrow \pi_1 + \pi_4 = 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2$$

The question is whether this is smaller than 400 for positive γ. If it is, there is an incentive to merge. For example, if $\gamma = 10$ then the profits in (i) are given by

$$\pi_1 = (16 + \frac{1}{5}10)^2 = 324$$
$$\pi_4 = (16 - \frac{4}{5}10)^2 = 64$$
$$\Rightarrow \pi_1 + \pi_4 = 512 - \frac{96}{5}10 + \frac{17}{25}100 = 388$$

This is clearly less than 400 so the firms would want to merge. We can solve this in more general form by writing the difference in profits as $\Delta = 112 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2$. If we can find all values of γ such that $\Delta < 0$, we know all values of γ where there is an incentive to merge. Since this is a quadratic equation we can graph it as follows.



Problem 2

(a) From Problem 1b, we have the following optimal quantities and market price, which are not affected by fixed costs.

$$q_m = q_3 = 20 + \frac{1}{4}\gamma$$
$$q_4 = 20 - \frac{3}{4}\gamma$$
$$P = 40 + \frac{1}{4}\gamma$$

Profits are now given by

$$\pi_m = 400 + 10\gamma + \frac{1}{16}\gamma^2 - bF$$
$$\pi_3 = 400 + 10\gamma + \frac{1}{16}\gamma^2 - F$$
$$\pi_4 = 400 - 30\gamma + \frac{9}{16}\gamma^2 - F$$

Combined profits for firms 1 and 2 with no merger are

$$2\pi_i = \pi(sum) = (2)(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2 - F)$$
$$= 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2 - 2F$$
$$= 512 + 12.8\gamma + \frac{2}{25}\gamma^2 - 2F$$

The merger is profitable if

$$\pi(merge) = 400 + 10\gamma + \frac{1}{16}\gamma^2 - bF \geq 512 + 12.8\gamma + \frac{2}{25}\gamma^2 - 2F = \pi(sum)$$

We can rearrange this expression to obtain

$$2F - bF \geq 112 + 2.8\gamma + 0.0175\gamma^2$$
$$\Rightarrow F(2 - b) \geq 112 + 2.8\gamma + 0.0175\gamma^2$$
$$0.0175\gamma^2 + 2.8\gamma + 112 \leq F(2 - b)$$
$$0.0175\gamma^2 + 2.8\gamma + 112 + F(b - 2) \leq 0$$
$$0.0175(\gamma^2 + 160\gamma + 6{,}400) + F(b - 2) \leq 0$$
$$0.0175(\gamma + 80)^2 + F(b - 2) \leq 0$$
$$0.0175(\gamma + 80)^2 \leq F(2 - b)$$
$$(\gamma + 80)^2 \leq \frac{F(2 - b)}{0.0175}$$
$$\gamma + 80 \leq \frac{\sqrt{F}(2 - b)^{\frac{1}{2}}}{0.13228756}$$
$$\gamma \leq \frac{\sqrt{F}(2 - b)^{\frac{1}{2}}}{0.13228756} - 80$$

Now to interpret the condition. If b is relatively close to 1, then the right-hand side of the expression will be large and the left-hand side is more likely to be less than it. If F is large in general and b is not close to 2, then the left-hand side is more likely to be less than it. Suppose that b is equal to 1 so that the merged firm has the same fixed costs as one of the merging firms.

Then we can write the above expression as

$$\gamma \le \frac{\sqrt{F}}{0.13228756} - 80$$

Large values of F (with b = 1) will make it more likely a merger is profitable. Low values of γ will also encourage the merger.

(b) From Problem 1c, we have the following optimal quantities and market price, which are not affected by fixed costs.

$$q_m = q_2 = q_3 = 20$$
$$P = 40$$

Profits are now given by

$$\pi_m = 400 - bF$$
$$\pi_2 = \pi_3 = 400 - F$$

Combined profits for firms 1 and 4 with no merger are

$$\pi_1 + \pi_4 = 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2 - 2F$$

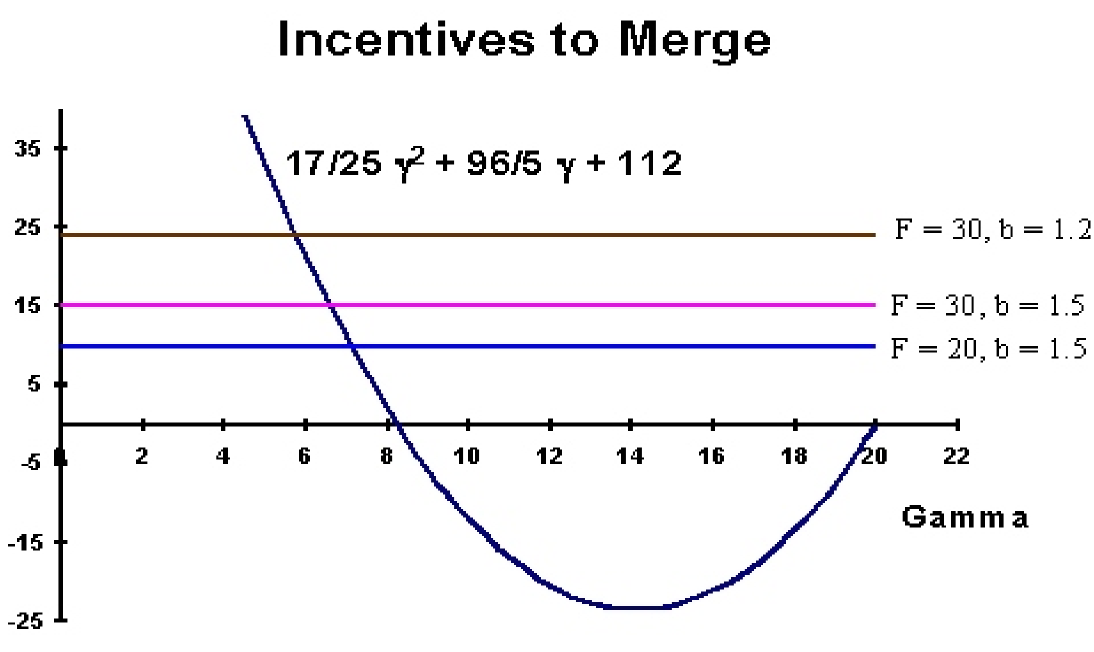
The merger is profitable if

$$\pi(merge) = 400 - bF \ge 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2 - 2F = \pi(sum)$$

We can rearrange this expression to obtain

$$\frac{17}{25}\gamma^2 - \frac{96}{5}\gamma + 112 \le F(2-b)$$

Now to interpret the condition. If b is relatively close to 1, then the right-hand side of the expression will be large and the left-hand side is more likely to be less than it. If F is large in general and b is not close to 2, then the left-hand side is more likely to be less than it. We can repeat the diagram from Chapter Problem 1 part c.



(c) Mergers that create cost savings by economizing on fixed costs or by eliminating high cost firms are more likely to be profitable.

Problem 3

(a) We now have a market with three firms, two identical and one (the leader) different. Denote the firms *l*, 3 and 4, where *l* denotes the combined leader firm. First consider the response of the 2 follower firms. For the third firm profit is given by

$$\begin{aligned}\pi_3 &= Pq_3 - 20q_3\\ &= (100 - q_\ell - q_3 - q_4)q_3 - 20q_3\\ &= 100q_3 - q_\ell q_3 - q_3^2 - q_4 q_3 - 20q_3\end{aligned}$$

If we maximize profit taking the other outputs as given, we obtain q_3 as a function of the other firms' outputs.

$$\begin{aligned}\frac{d\pi_3}{dq_3} &= 100 - q_\ell - 2q_3 - q_4 - 20 = 0\\ \Rightarrow 2q_3 &= 80 - q_\ell - q_4\\ \Rightarrow q_3 &= \frac{80 - q_\ell - q_4}{2}\end{aligned}$$

Since firm 4 and firm 3 are symmetric we know that $q_4 = q_3$. This will then give

$$\begin{aligned}q_3 &= \frac{80 - q_\ell - q_4}{2}\\ \Rightarrow q_3 &= \frac{80 - q_\ell - q_3}{2}\\ \Rightarrow \frac{3}{2}q_3 &= 40 - \frac{q_\ell}{2}\\ \rightarrow q_3 = q_4 &= 26.6\overline{6} - \frac{q_\ell}{3}\end{aligned}$$

The total output is

$$q_3 + q_4 = 53.33\overline{3} - \frac{2q_\ell}{3}$$

Now consider the profit maximization problem of the leader firm. This firm will take into account the best response function of the follower firms.

$$\begin{aligned}\pi_\ell &= Pq_\ell - 20q_\ell\\ &= (100 - q_\ell - q_3 - q_4)q_\ell - 20q_\ell\\ &= 100q_\ell - q_\ell^2 - q_3 q_\ell - q_4 q_\ell - 20q_\ell\\ &= 100q_\ell - q_\ell^2 - (26.66\overline{6} - \frac{q_\ell}{3})q_\ell - (26.66\overline{6} - \frac{q_\ell}{3})q_\ell - 20q_\ell\\ &= 100q_\ell - q_\ell^2 - (53.33\overline{3} - \frac{2q_\ell}{3})q_\ell - 20q_\ell\\ &= 80q_\ell - q_\ell^2 - 53.33\overline{3}q_\ell + \frac{2}{3}q_\ell^2\\ &= 26.66\overline{6} - \frac{1}{3}q_\ell^2\end{aligned}$$

If we maximize profit we obtain q_R as a function of the other firms' outputs.

$$\pi_l = 26.66\bar{6}q_l - \frac{1}{3}q_l^2$$

$$\frac{d\pi_l}{dq_l} = 26.66\bar{6} - \frac{2}{3}q_l$$

$$\rightarrow \frac{2}{3}q_l = 26.66\bar{6}$$

$$\Rightarrow q_l = 40$$

If we substitute in for the follower firms, we obtain

$$q_3 = q_4 = 26.66\bar{6} - \frac{1}{3}q_l$$

$$= 26.66\bar{6} - \frac{1}{3}(40)$$

$$= 26.666 - 13.333$$

$$= 13.33\bar{3}$$

The price is given by

$$P = 100 - q_l - q_3 - q_4 = 33\frac{1}{3}$$

The profits for each symmetric follower firm are given by

$$\pi_i = Pq_i - 20q_i \quad i = 3,4$$

$$= (33\frac{1}{3})(13\frac{1}{3}) - (20)(13\frac{1}{3})$$

$$= (13\frac{1}{3})^2$$

$$= 177.7\bar{7}$$

Profit for the leader firm is

$$\pi_l = Pq_l - 20q_l$$

$$= (33\frac{1}{3})(40) - (20)(40)$$

$$= (13\frac{1}{3})(40)$$

$$= 533\frac{1}{3}$$

This is larger than the sum of the profits from problem 1b, which were

$$2\pi_i = (2)(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2)$$

$$= 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2$$

$$= 512 + 12.8\gamma + \frac{2}{25}\gamma^2$$

since if $\gamma = 0$ then 533 1/3 > 512. The product price of 33 1/3 is lower than the price of 36 in 1a or 40 in 1b. The profits of the non-merged of follower firms (177.77) are smaller than in either 1a (256) or 1b (400) when $\gamma = 0$.

(b) In this situation we will have two Cournot competitors in the leader group. Thus, we can solve the problem as a symmetric Cournot duopoly. Denote them as R and m for the initial leaders and followers who merge. Profits for the newly merged firm are

$$\begin{aligned}\pi_m &= Pq_m - 20q_m \\ &= (100 - q_\ell - q_m)q_m - 20q_m \\ &= 100q_m - q_\ell q_m - q_m^2 - 20q_m\end{aligned}$$

If we maximize profit we obtain q_m as a function of the other firm's output.

$$\begin{aligned}\frac{d\pi_m}{dq_m} &= 100 - q_\ell - 2q_m - 20 = 0 \\ \rightarrow 2q_m &= 80 - q_\ell \\ \rightarrow q_m &= \frac{80 - q_\ell}{2} \\ &= 40 - \frac{q_\ell}{2}\end{aligned}$$

Since the two firms are symmetric we know that $q_R = q_m$. This will then give

$$\begin{aligned}q_m &= \frac{80 - q_\ell}{2} \\ q_m &= \frac{80 - q_m}{2} \\ \Rightarrow \frac{3}{2}q_m &= 40 \\ \Rightarrow q_m &= 26.66\overline{6}\end{aligned}$$

The price is given by

$$\begin{aligned}P &= 100 - 26.666 - 26.666 \\ &= 46.66\overline{6}\end{aligned}$$

The profit for each firm is given by

$$\begin{aligned}\pi_i &= Pq_i - 20q_i \\ &= (46\frac{2}{3})(26\frac{2}{3}) - (20)(26\frac{2}{3}) \\ &= (26\frac{2}{3})^2 \\ &= 711.1\overline{1}\end{aligned}$$

This is much larger than the sum of follower firm profits each of which was 177.77.

Problem 4

(a) The total profit of the firms 1 and 2 after merger as a Stackelberg leader is 533 1/3. The sum of the profits from being part of a Cournot market is 512. Thus a total fixed cost for both firms of more than 533 1/3 - 512 = 21 1/3 would make the merger not desirable.

(b) In the Stackelberg equilibrium, the follower firms each make 177.77 for a total profit of 355.55. In the two firm Cournot model the profit of each member of the duopoly has a profit of 711.11. If the cost of merging is more than 711.11 - 355.55 = 355.55, then these two firms will not merge.

The following table summarizes the results.

Model	**Q**	**P**	**q1 & q2**	**q3 & q4**	**q1**	**q2**	**q3**	**q4**
Monopoly	40	60						
Cournot (4 firms)	64	36			16	16	16	16
Stackelberg (2 followers)	66.66	33.33	40				13.33	13.33
Cournot (2 firms)	53.33	46.66	26.66	26.66				

Problem 5

(a) Assume that the number of firms in the market is ten, that is, N = 10, and that, as in question 4, a two-firm merger requires that each of the merging firms incurs a fixed cost of f prior to the merger. Derive a relationship, f(L), between f and the size of the leader group, L such that if f > f(L), the two-firm merger will be unprofitable. Calculate f(L) for L = 1, 2, 3, 4, and 5 to confirm that f(L) is decreasing in L. Interpret this result.

First write the expression that specifies when a two-firm merger is profitable and then subtract f from the lhs so that the merger is still profitable after paying the fixed cost f.

$$\left(\frac{(A-c)^2}{B(L+2)^2(N-L-1)}\right) \geq 2\left(\frac{(A-c)^2}{B(L+1)^2(N-L+1)^2}\right)$$

and if still profitable

$$\left(\frac{(A-c)^2}{B(L+2)^2(N-L-1)}\right) - f \geq 2\left(\frac{(A-c)^2}{B(L+1)^2(N-L+1)^2}\right)$$

Rearranging we obtain the desired expression for when a merger is still profitable. If f is larger than the expression then the merger will not be profitable.

$$f < \left(\frac{(A-c)^2}{B(L+2)^2(N-L-1)}\right) - 2\left(\frac{(A-c)^2}{B(L+1)^2(N-L+1)^2}\right) \Rightarrow \textbf{merger profitable}$$

$$f > \left(\frac{(A-c)^2}{B(L+2)^2(N-L-1)}\right) - 2\left(\frac{(A-c)^2}{B(L+1)^2(N-L+1)^2}\right) \Rightarrow \textbf{merger not profitable}$$

Note that once L = 8, a merger of the final two firms leads to a model with no followers. If N = 10, we obtain the following table.

L	Profit of Leader Firm	2*Profit of Follower	Profit L - 2*Profit F
1	88.88889	32.00000	56.88889
2	57.14286	17.55830	39.58456
3	42.66667	12.50000	30.16667
4	35.55556	10.44898	25.10658
5	32.65306	9.87654	22.77652
6	33.33333	10.44898	22.88435
7	39.50617	12.50000	27.00617
8	64.00000	17.55830	46.44170

If f is larger than the rightmost column, then the merger will not be profitable. Thus, the function is decreasing in L until L reaches 6, after which it is increasing in L. The intuition is that the advantage of being a leader is greater for the first few firms but falls as more join the group. Eventually the advantage of being the leader stops falling as the remaining follower firms gain some market advantage.

(b) In this case, once L = 6, a merger of the final two firms leads to a model with no followers. If N = 8, we obtain the following table.

L	Profit of Leader Firm	2*Profit of Follower	Profit L - 2*Profit F
1	118.51852	50.00000	68.51852
2	80.00000	29.02494	50.97506
3	64.00000	22.22222	41.77778
4	59.25926	20.48000	38.77926
5	65.30612	22.22222	43.08390
6	100.00000	29.02494	70.97506

If f is larger than the rightmost column, then the merger will not be profitable. Thus, the function is decreasing in L until L reaches 4, after which it is increasing in L. The intuition is that the advantage of being a leader is greater for the first few firms but falls as more join the group. Eventually the advantage of being the leader stops falling as the remaining follower firms gain some market advantage.

Problem 6

(a) We have three firms in this problem as compared to the five firms in the Appendix to this chapter. The notation is as follows.

L Length of the city or circumference of the circle N Number of consumers who are uniformly distributed along the circle n Number of firms or products in the market, in this case n = 3 m_j Mill price of the product produced by the jth firm t Unit transportation cost r_{ik} Distance the marginal consumer is from the location of firm i in the direction of firm k

We can take any one of the three firms as typical of the others. So consider firm 2. Demand for this firm from consumers to its left is Nr_{12}, where r_{12} is the marginal consumer given by

$$m_2 + tr_{12} = m_1 + t\left(\frac{L}{n} - r_{12}\right)$$
$$\Rightarrow tr_{12} = m_1 + \frac{tL}{n} - tr_{12} - m_2$$
$$\rightarrow 2tr_{12} = m_1 - m_2 + \frac{tL}{n}$$
$$\Rightarrow r_{12} = \frac{m_1 - m_2}{2t} + \frac{L}{2n}$$
$$= \frac{m_1 - m_2}{2t} + \frac{L}{6}$$

So demand to the left of firm 2 is given by $D_2(\text{left}) = Nr_{12}$, i.e.,

$$D_2(left) = Nr_{12}$$
$$= N\left[\frac{m_1 - m_2}{2t} + \frac{L}{2n}\right]$$
$$= \frac{Nm_1}{2t} - \frac{Nm_2}{2t} + \frac{NL}{2n}$$
$$= \frac{Nm_1}{2t} - \frac{Nm_2}{2t} + \frac{NL}{6}$$

Similarly, demand from consumers to the right of firm 2 is Nr_{23} , where r_{23} is

$$r_{23} = \frac{m_3 - m_2}{2t} + \frac{L}{6}$$

So demand to the right of firm 2 is given by $D_2(right) = Nr_{23}$, i.e.,

$$D_2(right) = Nr_{23}$$
$$= N\left[\frac{m_3 - m_2}{2t} + \frac{L}{2n}\right]$$
$$= \frac{Nm_3}{2t} - \frac{Nm_2}{2t} + \frac{NL}{2n}$$
$$= \frac{Nm_3}{2t} - \frac{Nm_2}{2t} + \frac{NL}{6}$$

And total demand is given by

$$D = D_2(left) + D_2(right)$$
$$= \frac{Nm_1}{2t} - \frac{Nm_2}{2t} + \frac{NL}{2n} + \frac{Nm_3}{2t} - \frac{Nm_2}{2t} + \frac{NL}{2n}$$
$$= \frac{Nm_1}{2t} - \frac{2Nm_2}{2t} + \frac{Nm_3}{2t} + \frac{NL}{n}$$
$$= N\left[\frac{m_1 - 2m_2 + m_3}{2t} + \frac{L}{n}\right]$$
$$= N\left|\frac{m_1 - 2m_2 + m_3}{2t} + \frac{L}{3}\right|$$

Firm 2's profit is, therefore,

$$\pi_2 = m_2N\left|\frac{m_1 - 2m_2 + m_3}{2t} + \frac{L}{n}\right|$$
$$= N\left(\frac{m_1m_2 - 2m_2^2 + m_3m_2}{2t} + \frac{m_2L}{n}\right)$$
$$= N\left(\frac{m_1m_2 - 2m_2^2 + m_3m_2}{2t} + \frac{m_2L}{3}\right)$$

Differentiate this with respect to m_2 to give the first-order condition for firm 2:

$$\frac{\partial \pi}{\partial m_2} = N\left(\frac{m_1 - 4m_2 + m_3}{2t} + \frac{L}{n}\right) = 0$$

Since the three firms are identical, in equilibrium we must have $m_1 = m_2 = m_3$. Let this common value be denoted by m* and substitute in the first order condition to obtain

$$N\left(\frac{m_1 - 4m_2 + m_3}{2t} + \frac{L}{n}\right) = 0$$

$$\Rightarrow N\left(\frac{m* - 4m* + m*}{2t} + \frac{L}{n}\right) = 0$$

$$\Rightarrow \left(\frac{-2m*}{2t} + \frac{L}{n}\right) = 0$$

$$\Rightarrow \frac{m*}{t} = \frac{L}{n}$$

$$\Rightarrow m* = \frac{tL}{n}$$

$$* = \frac{tL}{3}$$

Now substitute in the values for this problem where L = 1 and t = 0.5. This will give

$$m* = \frac{tL}{3}$$

$$= \frac{0.5}{3}$$

$$= \frac{1}{6}$$

Given the equal prices, each store will sell to 1/6 of the total customers on each side of it, or 1/3 of total customers. For example for firm 2, demand is given by

$$D_2 = N(r_{12} + r_{23})$$

$$= N\left(\frac{m_1 - 2m_2 + m_3}{2t} + \frac{L}{3}\right)$$

$$= N\left(\frac{m* - 2m* + m*}{2t} + \frac{L}{3}\right)$$

$$= \frac{NL}{3}$$

$$= \frac{100}{3} = 33\frac{1}{3}$$

(b) Profits will be given by the product of price and quantity or

$$\pi_i = \left(33\frac{1}{3}\right)\left(\frac{1}{6}\right)$$

$$= 5.5\overline{5}$$

Problem 7

(a) We can write the profits for each of the three firms as follows.

$$\pi_1 = Nm_1\left(\frac{m_3 - m_1}{2t} + \frac{m_2 - m_1}{2t} + \frac{L}{n}\right)$$
$$= N\left(\frac{m_1m_3 + m_1m_2 - 2m_1^2}{2t} + \frac{m_1L}{n}\right)$$
$$\pi_2 = Nm_2\left(\frac{m_1 - m_2}{2t} + \frac{m_3 - m_2}{2t} + \frac{L}{n}\right)$$
$$= N\left(\frac{m_2m_1 + m_2m_3 - 2m_2^2}{2t} + \frac{m_2L}{n}\right)$$
$$\pi_3 = Nm_3\left(\frac{m_1 - m_3}{2t} + \frac{m_2 - m_3}{2t} + \frac{L}{n}\right)$$
$$= N\left(\frac{m_3m_1 + m_3m_2 - 2m_3^2}{2t} + \frac{m_3L}{n}\right)$$

After the merger, the merged firm chooses m_1 and m_2 to maximize aggregate profit $\pi_1 + \pi_2$, while firm 3 chooses its price to maximize its individual profits. This means that we have three first-order conditions to solve:

$$\frac{\partial(\pi_1+\pi_2)}{\partial m_1} = \frac{\partial \pi_1}{\partial m_1} + \frac{\partial \pi_2}{\partial m_1}$$
$$= N\left(\frac{m_3 + m_2 - 4m_1}{2t} + \frac{L}{n}\right) + \frac{Nm_2}{2t} = 0$$
$$\frac{\partial(\pi_1+\pi_2)}{\partial m_2} = \frac{\partial \pi_1}{\partial m_2} + \frac{\partial \pi_2}{\partial m_2}$$
$$= \frac{Nm_1}{2t} + N\left(\frac{m_3 + m_1 - 4m_2}{2t} + \frac{L}{n}\right) = 0$$
$$\frac{\partial \pi_3}{\partial m_3} = N\left(\frac{m_1 + m_2 - 4m_3}{2t} + \frac{L}{n}\right) = 0$$

We need to solve these three equations in three unknowns (m_1, m_2, m_3). Making the substitution that n = 3 and rewriting the first equation we obtain

$$N\left[\frac{m_3 + m_2 - 4m_1}{2t} + \frac{L}{3}\right] = -\frac{Nm_2}{2t}$$
$$\Rightarrow \left[\frac{m_3 + m_2 - 4m_1}{2t} + \frac{L}{3}\right] = -\frac{m_2}{2t}$$

Now rearrange and simplify

$$\left(\frac{m_3 + m_2 - 4m_1}{2t} + \frac{L}{3}\right) = -\frac{m_2}{2t}$$

$$\Rightarrow \frac{m_3 + 2m_2 - 4m_1}{2t} = -\frac{L}{3}$$

$$\Rightarrow m_3 + 2m_2 - 4m_1 = -\frac{2tL}{3}$$

$$\Rightarrow 4m_1 - 2m_2 - m_3 = \frac{2tL}{3}$$

Now do the same with the second equation

$$\frac{Nm_1}{2t} + N\left(\frac{m_3 + m_1 - 4m_2}{2t}\right) = -\frac{NL}{3}$$

$$\Rightarrow \frac{m_1}{2t} + \left(\frac{m_3 + m_1 - 4m_2}{2t}\right) = -\frac{L}{3}$$

$$\Rightarrow \frac{m_3 + 2m_1 - 4m_2}{2t} = -\frac{L}{3}$$

$$\Rightarrow m_3 + 2m_1 - 4m_2 = -\frac{2tL}{3}$$

$$\Rightarrow -2m_1 + 4m_2 - m_3 = \frac{2tL}{3}$$

And the third equation

$$N\left(\frac{m_1 + m_2 - 4m_3}{2t}\right) = -\frac{NL}{3}$$

$$\rightarrow \left(\frac{m_1 + m_2 - 4m_3}{2t}\right) = -\frac{L}{3}$$

$$\rightarrow m_1 + m_2 - 4m_3 = -\frac{2tL}{3}$$

$$\Rightarrow -m_1 - m_2 + 4m_3 = \frac{2tL}{3}$$

Now subtract the second equation from the first

$$4m_1 - 2m_2 - m_3 = \frac{2tL}{3}$$

$$2m_1 + 4m_2 - m_3 = \frac{2tL}{3}$$

$$\rightarrow 6m_1 - 6m_2 = 0$$

$$\rightarrow m_1 = m_2$$

This means that the merged firm will charge the same price at both of its outlets. Denote this common output level by m* and substitute back in the first equation.

$$4m_1 - 2m_2 - m_3 = \frac{2tL}{3}$$

$$\Rightarrow 4m^* - 2m^* - m_3 = \frac{2tL}{3}$$

$$\Rightarrow 2m^* - m_3 = \frac{2tL}{3}$$

$$\rightarrow 2m^* = m_3 + \frac{2tL}{3}$$

$$\Rightarrow m^* = \frac{m_3}{2} + \frac{2tL}{6}$$

Now substitute m* for m_1 and m_2 in the third equation

$$-m_1 - m_2 + 4m_3 = \frac{2tL}{3}$$

$$\Rightarrow -2m^* + 4m_3 = \frac{2tL}{3}$$

$$\Rightarrow (-2)\left(\frac{m_3}{2} + \frac{2tL}{6}\right) + 4m_3 = \frac{2tL}{3}$$

$$\rightarrow \frac{-2m_3}{2} - \frac{4tL}{6} + 4m_3 = \frac{2tL}{3}$$

$$\rightarrow 3m_3 = \frac{2tL}{3} + \frac{4tL}{6}$$

$$\Rightarrow m_3 = \frac{2tL}{9} + \frac{4tL}{18}$$

$$= \frac{8tL}{18} = \frac{4tL}{9}$$

We get m_1 and m_2 by substituting m_3 in the equation for m*.

$$m^* = \frac{m_3}{2} + \frac{2tL}{6}$$

$$= \frac{9}{2} + \frac{2tL}{6}$$

$$= \frac{4tL}{18} + \frac{6tL}{18}$$

$$= \frac{10tL}{18} = \frac{5tL}{9}$$

Now plug in the values for this problem where L = 1 and t = 0.5. We then have

$$m* = m_1 = \frac{5tL}{9} = \frac{2.5}{9} = \frac{5}{18}$$

$$m* = m_2 = \frac{5tL}{9} = \frac{2.5}{9} = \frac{5}{18}$$

$$m_3 = \frac{4tL}{9} = \frac{2}{9} = \frac{4}{18}$$

The merger has led to higher prices. Profits are given substituting the optimal prices in the profit functions. First for outlet one

$$\pi_1 = N\left(\frac{m_1 m_3 + m_1 m_2 - 2m_1^2}{2t} + \frac{m_1 L}{3}\right)$$

$$= 100\left(\left(\frac{5}{18}\right)\left(\frac{4}{18}\right) + \left(\frac{5}{18}\right)\left(\frac{5}{18}\right) - 2\left(\frac{5}{18}\right)^2 + \frac{\frac{5}{18}}{3}\right)$$

$$= 100\left(\frac{20}{324} + \frac{25}{324} - \frac{50}{324} + \frac{30}{324}\right)$$

$$= 100\left(\frac{25}{324}\right)$$

$$= \frac{2{,}500}{324} = \frac{625}{81} = 7.716$$

Now for the other outlet owned by the merged firm

$$\pi_2 = N\left(\frac{m_2 m_3 + m_2 m_1 - 2m_2^2}{2t} + \frac{m_2 L}{3}\right)$$

$$= 100\left(\left(\frac{5}{18}\right)\left(\frac{4}{18}\right) + \left(\frac{5}{18}\right)\left(\frac{5}{18}\right) - 2\left(\frac{5}{18}\right)^2 + \frac{\frac{5}{18}}{3}\right)$$

$$= 100\left(\frac{25}{324}\right)$$

$$= \frac{2{,}500}{324} = \frac{625}{81} = 7.716$$

So profits for the merged firm are

$$\pi(merged) = \pi_1 + \pi_2 = \frac{625}{81} + \frac{625}{81} = \frac{1{,}250}{81}$$

$$= 15.432$$

Profits for firm 3 are

$$\pi_3 = N\left(\frac{m_3 m_1 + m_3 m_2 - 2m_3^2}{2t} + \frac{m_3 L}{3}\right)$$

$$= 100\left(\left(\frac{4}{18}\right)\left(\frac{5}{18}\right) + \left(\frac{4}{18}\right)\left(\frac{5}{18}\right) - 2\left(\frac{4}{18}\right)^2 + \frac{\frac{4}{18}}{3}\right)$$

$$= 100\left(\frac{20}{324} + \frac{20}{324} - \frac{32}{324} + \frac{24}{324}\right)$$

$$= 100\left(\frac{32}{324}\right)$$

$$= \frac{3{,}200}{324} = \frac{800}{81} = 9.8765$$

All firms are much better off, the merged ones more so. The merger is profitable

(b) Let Firms 1, 2 and 3 locate at 1/6, ½ and 5/6, respectively. In equilibrium, Firm 1 serves consumers who are located in [0, 1/3], Firm 2 serves consumers who are located on [1/3, 2/3] and Firm 3 serves consumers who located on [2/3, 1]. Consider the competition for consumers that lie between two firms. For a consumer who lies midway between two store locations, e.g., at 1/3 or 2/3, the products of the two nearest firms are perfect substitutes. Hence, price competition to serve this consumer will lead each firm to offer a price equal to marginal cost, which we again assume is zero for simplicity. Prices will then rise linearly to consumers as their location moves closer to one store or the other. For a consumer located precisely at the same location as a store, the nearest competitor is 1/3 miles away. Hence, the store can charge this consumer marginal cost plus $s/3$ = \$0.50/3 = \$0.167. The average price charged by a store on a particular side that faces competition from rival on that same side is then (\$0 + \$0.167) = \$0.08333 or about 8 cents. Because the town is a line and not a circle, matters for stores 1 and 3 are a little different on their far sides where they face no direct competition. A consumer at 5/6 plus epsilon, for example, would have to travel 1/3 plus epsilon to by-pass store 3 and purchase from store 2. So, store 3 can charge this consumer a price of $\$0.50/(3 + \varepsilon)$. The price then rises linearly to the most remotely located consumer (in this case, the consumer at location 1) who can be charged a price of \$0.50/2, since his nearest option is ½ miles away. So, on the end segments, the average price is [(\$0.50/3) + (\$0.50/2)]/2 = \$0.208 or about 21 cents.

Each firm serves 33.33 customers. Profits to firm 2 are 33.33x(\$0.0833) = \$2.78. Profits to firms 1 and 3 are: 16.67 x (\$0.0833) + 16.67 x (\$0.208) = \$1.39 + \$3.47 = \$4.86. Price discrimination intensifies competition where there are rivals. However, without uniform pricing that imposes the same price to all consumers, price discrimination leads to higher prices where consumers have inelastic demands because few substitutes are available.

A merger in this price discrimination case between say firms 1 and 2, will have the following effects. The price to all consumers to the right (east) of firm 2 will remain unchanged. Nothing has happened to alter the strategic interaction between the store at firm 2's location and the remaining, non-merged firm, firm 3.

For consumers west of store 1 (address = 0 to 1/6), and consumers between stores 1 and 2 that are now commonly owned, the only constraints on pricing are: 1) the maximum willingness to pay less transport cost; and 2) the price at firm 3 plus the cost of getting to that store. The binding constraint will be whichever is lower and, in this case, it's the latter. Consider the consumer midway between stores 1 and 2. She can buy at either store at total cost of either $p_1 + 0.5/6$ or $p_2 + 0.5/6$. Alternatively she can buy from store 3 at price $p_3 + 0.5/2$. Since firm 3 can profitably serve this consumer at a price of $c = 0$, the maximum price that can be charged this consumer is \$0.5/2 = \$0.25. For consumers to the east of this point but still to the west of store 2, the maximum price falls linearly as the distance to store 2 (and firm 3) diminishes. For the consumer located just at store 2, the maximum price that can be charged this consumer is 0.5/3 or \$0.167. Thus, the 16.67 consumers located between x = 1/3 and x = 1/2 pay an average price of \$0.208, or about 21 cents. Profit from this group is therefore \$3.47.

For consumers located to the west of x = 1/3, the total expense of buying from firm 3 rises as the distance to firm three grows. The maximum price that store 1 can charge to a consumer just west of x = 1/3 is $p_3 + 0.5/2$. As before, the fact that firm 3 can profitably serve this customer at a price just above marginal cost $c = 0$, means that this maximum is \$0.25. From that point, the price rises linearly to consumers as their location moves further west. The maximum price that can be charged to consumers at the west end of town (x = 0) is p_3 + 5(\$0.5)/6 = \$2.5/6 = \$0.417. Store 1 would of course like to charge a higher price to its consumers of $V - td$ where V = \$5, t = \$0.5 and d is the consumer's distance to store 1. However, since no consumer is more than 1/6 of a mile from store 1, such a high price is not feasible given the (distant) competition from firm 3. The average price paid by the 33.33 consumers located from $x = 0$ to $x = 1/6$ is therefore [$0.416 + $0.25]/2 = $0.33 and the profit from this group is therefore 33.33 x $0.33 = $11.

Of course, the average price charged to the 16.67 consumers who live just east of store 2 but west of the midpoint between that store and firm 3, i.e., those between $x = 1/2$ and $x = 2/3$, remains as before $0.0833. So, profit from this group is still 16.67 x $0.0833 = $1.39. Total post-merger profits to firms 1 and 2 then are: $3.47 + $11 + $1.39 = $15.86. The merger is highly profitable.

Problem 8

Note: There is typographical errors in this problem. First, the setting inverted the value for *h*. That is, $h = \frac{\varepsilon}{\varepsilon - 1}$ ***not*** $\frac{\varepsilon - 1}{\varepsilon}$. We then have $p = \frac{\varepsilon}{\varepsilon - 1} c = hc$. Taking logs then yields: $\ln p = \ln h + \ln c$. Differentiation then yields the first desired result, $\frac{dp}{p} = \frac{dh}{h} + \frac{dc}{c}$. Denote the post-merger price cost margin as *h*'. We then have that $h' = h + \Delta h = \frac{(1-\delta)\varepsilon}{(1-\delta)\varepsilon - 1}$. We may then solve for

$$\frac{\Delta h}{h} = \frac{h' - h}{h} = \frac{h'}{h} - 1 = \frac{(1-\delta)\varepsilon}{(1-\delta)\varepsilon - 1} x \frac{(\varepsilon - 1)}{\varepsilon} - 1 = \frac{(1-\delta)(\varepsilon - 1)}{(1-\delta)\varepsilon - 1} - \frac{(1-\delta)\varepsilon - 1}{(1-\delta)\varepsilon - 1} = \frac{\delta}{(1-\delta)\varepsilon - 1}$$ as required.

Problem 9

From above we have that with $\varepsilon = 2$ and $\delta = 0.1$, $\frac{\Delta h}{h} = \frac{0.1}{(1-0.1)2 - 1} = \frac{0.1}{0.8} = 0.125$. That is, the price-cost margin will rise by 12.5% as a result of the merger. To keep prices constant, this means that the merger must lead to cost efficiencies of 12.5 percent ($\frac{\Delta c}{c} = -0.125$) .