Chapter 11: Dynamic Games and First and Second Movers

Learning Objectives

Students should learn to:

1. Extend the reaction function ideas developed in the Cournot duopoly model to a model of sequential behavior due to Stackelberg.
2. Explain the difference between the Nash equilibrium of a simultaneous form game (Cournot) and an extensive form game (Stackelberg).
3. Apply sequential games in the context of price competition.
4. Explain the importance of order in sequential games.
5. Gain an intuitive understanding of credibility, credible commitments, and credible threats.

Suggested Lecture Outlines:

Spend two fifty-minute long lectures on this chapter.

Lecture 1:

1. Simultaneous vs. Sequential Games
2. Stackelberg model
3. Numerical examples / problems

Lecture 2:

1. Sequential games for price competition
2. Numerical problems
3. Credibility, Credible commitments

Suggestions for the Instructor:

1. It is very useful to view the Stackelberg model as an extension of Cournot based on the previously derived best response function for the follower firm. Then ask the students what they would do if they knew the structure of the game and knew they always had to move first and could not change their output once they moved. Stress the idea that the leader will use the other firm's best response function (not a fixed quantity) in making the decision.
2. Spend time to graphically solve the Stackelberg model.
3. Point out it doesn't always pay to be the leader as in the sequential version of paper-rock-scissors.
4. Then argue why, it may pay to be a follower, in a price competition.

Solutions to the End of the Chapter Problems:

Problem 1

(a) Firm 2 chooses its quantity to maximize

$$\pi_2 = Q_2(1000 - 4Q_1 - 4Q_2) - 20Q_2$$

$$\frac{\partial \pi_2}{\partial Q_2} = 1000 - 4Q_1 - 8Q_2 - 20 = 0 \Rightarrow Q_2 = \frac{1}{8}(980 - 4Q_1)$$

Now, Firm 1 chooses its quantity to maximize

$$\pi_1 = Q_1(1000 - 4Q_1 - 4Q_2) - 20Q_1 = Q_1\left(980 - 4Q_1 - \frac{1}{2}(980 - 4Q_1)\right) = \frac{1}{2}Q_1(980 - 4Q_1)$$

$$\frac{\partial \pi_1}{\partial Q_1} = 980 - 8Q_1 = 0 \Rightarrow Q_1 = \frac{980}{8} = 122.5 \Rightarrow Q_2 = 61.25$$

(b) There is no non-negative c such that the leader and the follower have the same market share. To see, consider $c = 0$. Then the leader's quantity is 120, whereas the follower's quantity is less than 120. As c increases, the market share of the leader goes up and the market share of the follower goes down.

Problem 2

Let p_1 be the price charged by Ben and p_2 be the price charged by Will. Let x be the location of a consumer who is indifferent between buying from Ben and Will.
Therefore,

$$p_1 + x = p_2 + (10 - x) \Rightarrow x = \frac{1}{2}(p_2 - p_1 + 10)$$

Consequently, the demand faced by Ben is

$$D_1(p_1, p_2) = \left(\frac{1000}{10}\right)\left(\frac{1}{2}\right)(p_2 - p_1 + 10) = 500 + 50(p_2 - p_1)$$

The demand faced by Will is

$$D_2(p_1, p_2) = \left(\frac{1000}{10}\right)\left[10 - \left(\frac{1}{2}\right)(p_2 - p_1 + 10)\right] = 500 - 50(p_2 - p_1)$$

Hence, Ben's profit is given by

$$\pi_1(p_1, p_2) = (p_1 - 1)(500 + 50(p_2 - p_1))$$

Will's profit is given by

$$\pi_2(p_1, p_2) = (p_2 - 1)(500 - 50(p_2 - p_1))$$

Since Will is the follower, we first maximize π_2 with respect to p_2, to derive Will's reaction function.

$$\frac{\partial \pi_2(p_1, p_2)}{\partial p_2} = (p_2 - 1)(-50) + (500 - 50(p_2 - p_1)) = 0$$

$$\Rightarrow p_2 = \frac{1}{100}[550 + 50p_1] = \frac{1}{2}[11 + p_1]$$

Now, substitute Will's reaction function in to Ben's profit function to get

$$\pi_1(p_1, p_2) = (p_1 - 1)\left(500 + 50\left(\frac{11}{2} + \frac{p_1}{2} - p_1\right)\right) = (p_1 - 1)(775 - 25p_1)$$

We now maximize π_1 with respect to p_1,

$$\frac{\partial \pi_1(p_1, p_2)}{\partial p_1} = (p_1 - 1)(-25) + (775 - 25p_1) = 0$$

$$p_1 = \frac{800}{50} = 16$$

Now, from Will's reaction function, get

$$\Rightarrow p_2 = \frac{1}{2}[11 + p_1] = 13.5$$

Problem 2 (b)

$$p_2 - p_1 = 13.5 - 16 = -2.5$$

Hence, Ben will serve

$$D_1(p_1, p_2) = 500 + 50(p_2 - p_1) = 500 - 50\left(\frac{5}{2}\right) = 375$$

Will serves

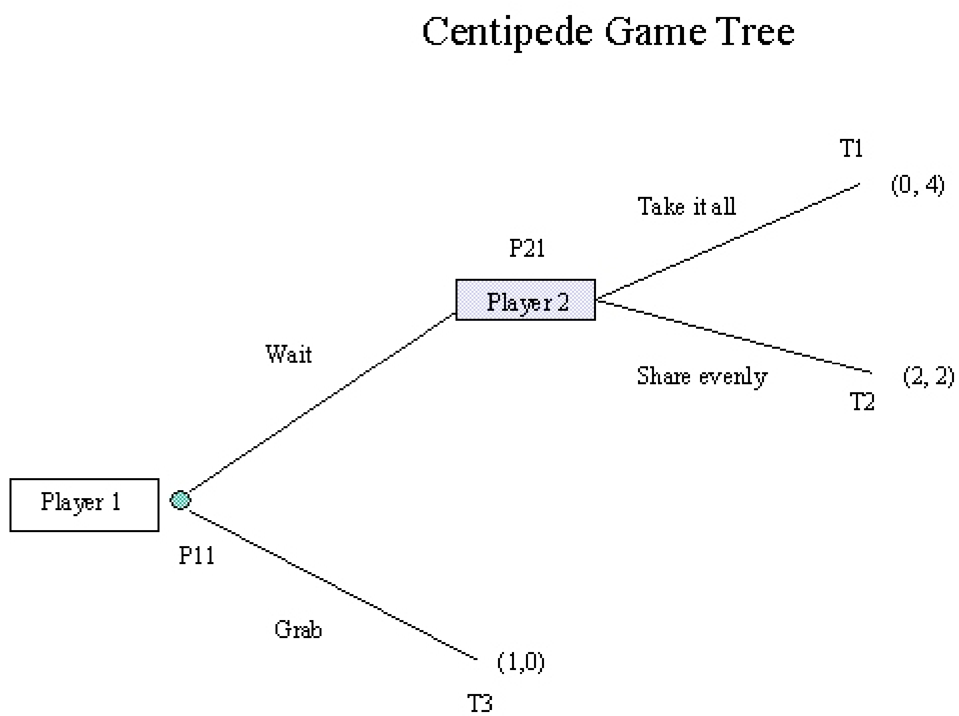
$$D_2(p_1, p_2) = 500 - 50(p_2 - p_1) = 500 + 50\left(\frac{5}{2}\right) = 625$$

Ben's profit = *375 (16 - 1) – 250 = 5375*

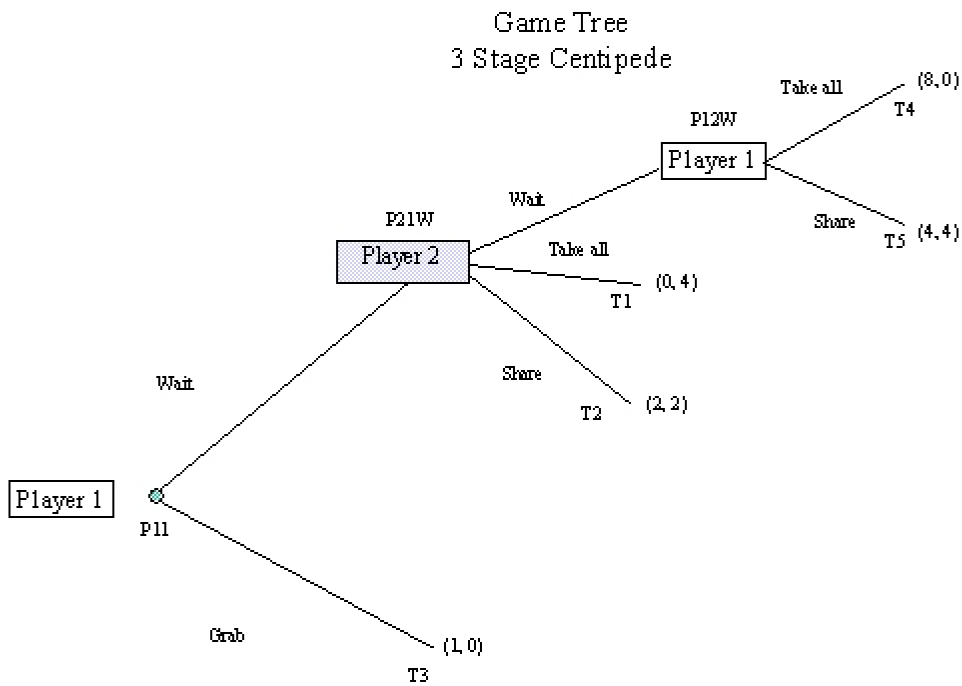
Will's profit = *625 (13.5 – 1) – 250 = 7562.5*

Problem 3

(a)

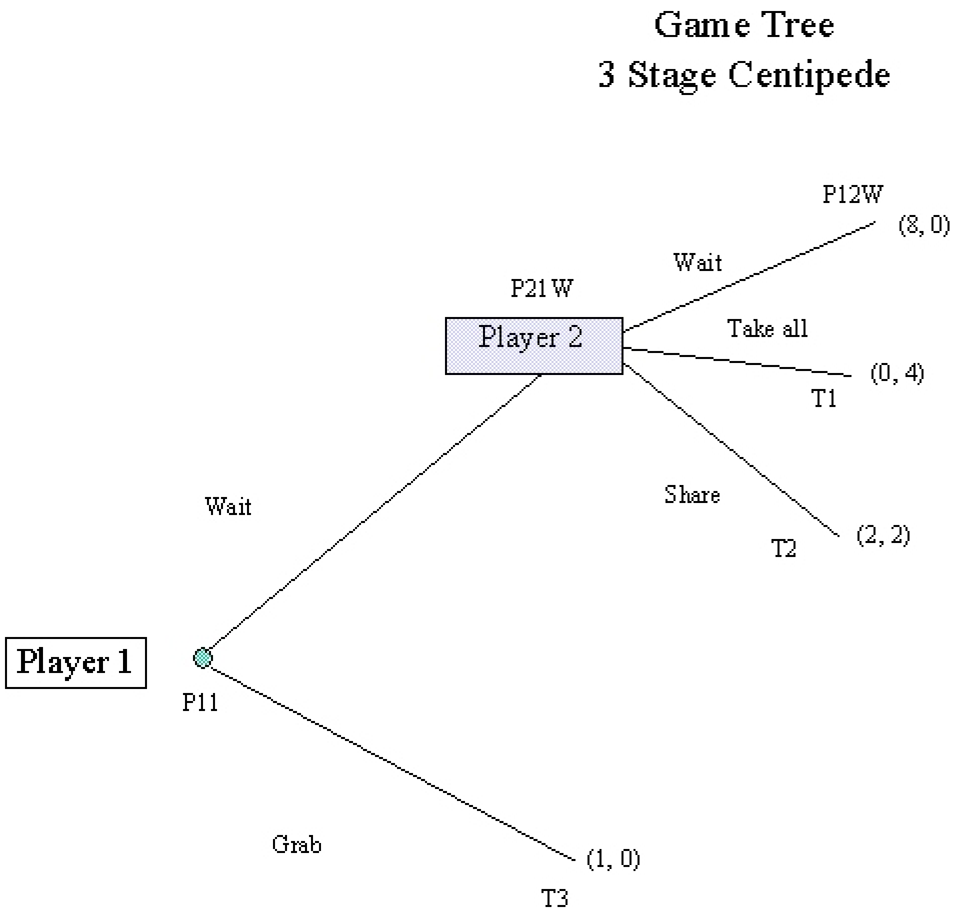


(b) The strategy of splitting the money is never an equilibrium since once the game reaches the point P21, the optimal strategy for the Player 2 is to take the entire $4. Because Player 1 knows this will be the outcome at P2, Player 1 will always choose "Grab" and the outcome will be T3 with Player 1 getting $1 and Player 2 getting nothing.



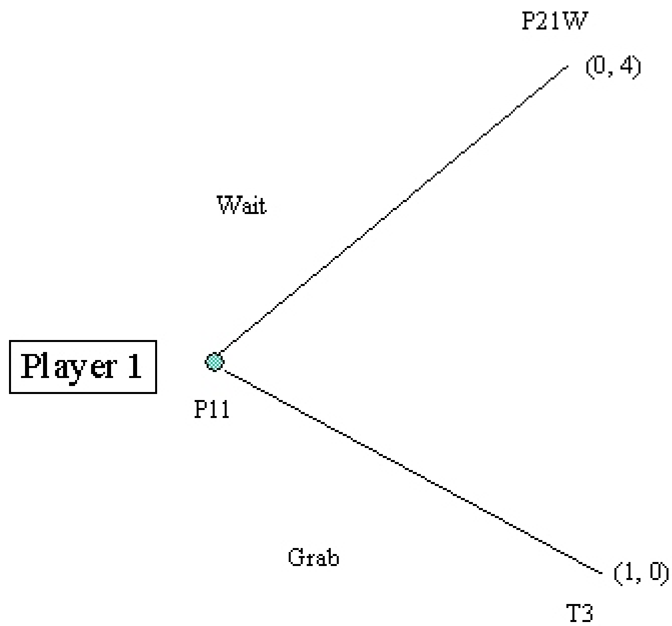
(c)

It is clear that in the third stage Player 1 will choose to keep all the money. Thus we can eliminate this choice from the tree and consider the new game with the final node removed (pruned). This will give



It is now obvious that Player 2 will choose to take it all at node P21W. Thus we can eliminate this node from the tree and replace it with the payoffs to both players when Player 2 chooses to take it all. This will give

Game Tree
3 Stage Centipede



It is now clear that Player 1 will grab the money at the initial node and the final payoff will be (1,0).

Problem 4

		Northern Springs			
		3	4	5	6
	3	(24, 24)	(30, 25)	(36, 20)	(42, 12)
Southern Pelligrino	4	(25, 30)	(32, 32)	(41, 30)	(48, 24)
	5	(20, 36)	(30, 41)	(40, 40)	(50, 36)
	6	(12, 42)	(24, 48)	(36, 50)	(48, 48)

In this case we can use Southern Pelligrino's best response function to find its optimal choice. Denote the best response of Southern Pelligrino as SP(__ NP's choice).

SP(__ 3) = 4 since 25 is the highest first element is column 1.
SP(__ 4) = 4 since 32 is the highest first element is column 2.
SP(__ 5) = 4 since 41 is the highest first element is column 3.
SP(__ 6) = 5 since 50 is the highest first element is column 4.

Now consider the best response function of Northern Springs

NS(3 __) = 4 since 25 is the highest second element is row 1.
NS(4 __) = 4 since 32 is the highest second element is row 2.
NS(5 __) = 4 since 41 is the highest second element is row 3.
NS(6 __) = 5 since 50 is the highest second element is row 4.

The Nash equilibrium is of course where NS(4 __) = 4 and SP(__ 4) = 4 and is the point (4,4). But if Northern Springs must go first and realizes that Southern Pelligrino will go second then Northern Springs has the payoff function defined by the best response function of Southern Pelligrino. The payoffs to Northern Springs are as follows

Payoff_{NS} (NP(3 __)) = Payoff_{NS} (SP(__ 3)) = Payoff_{NS} when SP chooses 4 which is 30.

Payoff_{NS} (NP(4 __)) = Payoff_{NS} (SP(__ 4)) = Payoff_{NS} when SP chooses 4 which is 32.

Payoff_{NS} (NP(5 __)) = Payoff_{NS} (SP(__ 4)) = Payoff_{NS} when SP chooses 4 which is 30.

Payoff_{NS} (NP(6 __)) = Payoff_{NS} (SP(__ 5)) = Payoff_{NS} when SP chooses 5 which is 36.

The equilibrium is now the point (5,6) where NS gets to choose the six first. Both firms are better off in this game because once NP chooses 6 and cannot deviate, the best choice for SP is to choose 5. If NP could now switch it would and go to 4, but then SP would switch and go to 4 and we would be back at the Cournot equilibrium.

(c) It is not an advantage for NP to move first. In a pricing game, the first mover is a sitting target for the firm that moves second. Both do better than in the simultaneous move game, but the second mover does best.

Problem 5

(a)

		Firm 2	
		C	Nothing
Firm 1	A	(8, 8)	(20, 8)
	B	(-3, -3)	(11, 0)
	A, B	(2, -2)	(18, 0)
	Nothing	(0, 10)	(0, 0)

There is a unique Nash equilibrium, where Firm 1 chooses A and Firm 2 chooses C.

(b) Note that A is a dominant strategy for Firm 1. Therefore, even if Firm 1 can commit before Firm 2, the answer does not change.

Problem 6

Find three examples of different ways individual firms or industries can make the strategy "This offer is good for a limited time only" a credible strategy.

i. Make the price applicable to stock on hand when there is a clear time lag in ordering additional stock or the items are one of a kind so that there can be no additional sales.

ii. Announce the price on a "special" purchase where the items are not the items normally stocked and there is a limited supply.

iii. Develop a reputation over time.

Problem 7

(a) Implied inversed demand is: $P = 3{,}000 - 0.08Q$. With $MC = 0$, implied monopoly outcome for Gizmo is: $P = \$1{,}500$; $Q = 18{,}750$; and Profit = \$28,125,000. We interpret the assumption that the metric can supply half the market to mean that it competes as a symmetric duopolist in quantities. In this case, the post-entry equilibrium is: $P = \$1{,}000$; $Q = 25{,}000$. Each firm produces $q_i = 12{,}500$ units and earns an operating profit of \$12,500,000. If the cost of entry is just \$10,000,000, then entry is profitable.

(b) Gizmo's profit falls from \$28,125,000 to \$12,500,00 or by \$15,625,000. If spending \$5 million could deter entry and prevent the \$15,625,000 loss it would surely be worth it. However, it is not clear that buying additional capacity achieves this result. The firm's current capacity of 25,000 is already more than it needs.