# Chapter 10: Price Competition

**Learning Objectives**

Students should learn to:

1. Understand the logic behind the Bertrand model of price competition, the idea of discontinuous reaction functions, how to solve a simple Bertrand duopoly model, and the fundamental differences between Bertrand and Cournot models.
2. See some of the shortcomings of the simple Bertrand model and how these can be addressed in a simple but compelling fashion.
3. Analyze how capacity constraints modify and enrich the Bertrand model
4. Analyze the application of the Bertrand competition to the simple location model when there are two firms in the model
5. Differentiate strategic substitutes and strategic complements.
6. Understand that assumptions about the nature of product differentiation have implications for the impact of changes in the market environment, as well as recognize the possibility of sometimes testing such assumptions.

**Suggested Lecture Outline:**

Spend two fifty-minute long lectures on this chapter.

**Lecture 1:**

1. The Bertrand Model
2. Numerical problems on the Bertrand model
3. Bertrand model with capacity constraints and/or non-identical firms

**Lecture 2:**

1. Applications of Bertrand competition to location models
2. Numerical problems / examples
3. Strategic substitutes / complements

**Suggestions for the Instructor:**

1. Some examples for the capacity constraint case include restaurants, dentists, feed companies, car dealerships in the short run, and power plants.
2. As with the Cournot case it may be useful to carry along a numerical example for the Bertrand model.
3. Stress that the solution in the extended Bertrand model is just another case of the use of response functions to find a Nash equilibrium.
4. Spend lots of time to go over the location model

**Solutions to the End of the Chapter Problems:**

**Problem 1**

**(a)** At equilibrium $p_1^* = p_2^* = 10$, assuming that if both firms charge the same price, then the firms split the market evenly.

**(b)** The higher cost firm makes zero profit, whereas the lower cost firm's profit is $\left(p_1^* - c_1\right)Q_1 = (10-6)(5000-200(10)) = 12000$

**(c)** No, this outcome is not efficient.

**Problem 2**

**(a)** Note that the inverse demand function is $P = 30 - \frac{1}{3}Q$. Then the Cournot quantities are:

$$Q_1^* = \frac{(30 - 2(15) + 10)}{3(1/3)} = 10, Q_2^* = \frac{((30 - 2(10) + 15)}{3(1/3)} = 25$$

The market price is $P = 30 - \frac{1}{3}Q = 30 - \frac{1}{3}(10 + 25) = 18.33$

Profit of Firm 1 = $(18.33 - 15)(10) = 33.3$

Profit of Firm 2 = $(18.33 - 10)(25) = 208.25$

**(b)** At a Bertrand equilibrium, $p_1^* = p_2^* = 15$, assuming that if both firms charge the same price, then the consumers buy from the lower priced firm.

Total sales = 90 – 3(15) = 45. Firm 1 sells zero and earns zero profit. Firm 2 sells 45 units and earns (15 – 10) (45) = 225

**Problem 3**

**(a)** Yes, the outcome will change. The two lower cost firms will charge $10 and share the market equally.

**(b)** The answer may change depending on how much premium the consumers are willing to pay for the green balls endorsed by Tiger Woods.

**Problem 4**

**Note:** Suppose that a consumer travels one mile to go to a store. Since the consumer needs to return home after purchase, it will cost her 2 ($0.50) = $1 to travel. Assume that V is very high.

**(a)** If both of them charge $1**, e**ach will serve 500 in a day. If Ben charges $1 and Will charges $1.40, suppose the customer at the distance t from Ben's store is indifferent to buy fruit smoothie from each store, then since

$$2(0.5)x + 1 = 2(0.5)(10 - x) + 1.4 \Rightarrow x = 5.2$$

Ben will sell 520 and Will will sell 480 per day.

**(b)** If Ben charges $3, then $8.00 will enable Will to sell 250. $3.00 will enable him to sell 500, no positive price can enable him to sell more than 650. So, no positive price by Will permits him to reach a volume of either 750 or 1000.

**(c)** Suppose Ben charges $p_1$ and Will charges $p_2$. Let a consumer at a distance *x* from Ben is indifferent between the two firms. Therefore,

$$p_1 + x = p_2 + (10 - x) \Rightarrow 2x - 10 = p_2 - p_1 \Rightarrow x = 5 + \frac{p_2 - p_1}{2}$$

Therefore, the demand faced by Ben is $x_1(p_1) = 500 + 50(p_2 - p_1)$

Demand faced by Will is $x_2(p_2) = 1000 - x_1(p_1) = 500 - 50(p_2 - p_1)$

**(d)** $p_1 = 10 + p_2 - \frac{x_1}{50}$ Ben's marginal revenue function is $MR_1 = 10 + p_2 - \frac{x_1}{25}$

**(e)** Ben's profit is given by

$$\Pi_1 = (p_1 - 1)x_1(p_1) = (p_1 - 1)(500 + 50(p_2 - p_1))$$

Ben chooses his price to maximize his profit.

$$\frac{\partial \Pi_1}{\partial p_1} = (p_1 - 1)50(-1) + (500 + 50(p_2 - p_1)) = 0$$

Now, by symmetry, Ben and Will charge the same price in equilibrium. Therefore,

$$-p_1 + 1 + 10 = 0 \Rightarrow p_1^* = p_2^* = 11$$

Hence, the profit earned by each of them = (11 – 1) (500) - 250 = 5000 – 250 = 4750

**Problem 5**

George locates at the center. Let consumers at a distance of x (on both sides) are indifferent between buying from George and his rival. Consequently, George's market length is 2x.

**(a)** To find x, observe that a consumer located at $5 - x$ is indifferent between buying from Ben and George. Therefore, $11 + (5 - x) = p_{George} + x \Rightarrow x = \frac{(16 - p_{George})}{2}$

So, George chooses his price to maximize $\Pi_{George} = (p_{George} - 1)2x = (p_{George} - 1)(16 - p_{George})$

$$\frac{\partial \Pi_{George}}{\partial p_{George}} = ((p_{George} - 1)(-1) + (16 - p_{George})) = 0 \Rightarrow p_{George} = \frac{17}{2} = \$8.5$$

George's market length is $2x = 16 - \frac{17}{2} = \frac{15}{2} \Rightarrow \Pi_{George} = (7.5)(1000)\frac{15}{20} - 250 = 5375$

**(b)** Yes, Ben and Will have incentives to change their locations and prices. Otherwise, each of them makes a loss. Even after the adjustments, at the new equilibrium, both Ben and Will will make a loss and leave the market.

**Problem 6**

**(a)** Since unit costs are zero, profit is equal to revenue. And revenue is equal to price times quantity. For each firm the revenue is then given by the above expressions since the total quantity of nuts or bolts sold by the firm is equal to the market quantity.

**(b)** Since the choice variable in this model is quantity, it is useful to write the above profit expressions in terms of quantities. This is done by noting that from the demand equations

$$Q_B = Z - P_B - P_N \Rightarrow P_B = Z - P_N - Q_B$$
$$Q_N = Z - P_N - P_B \Rightarrow P_N = Z - P_B - Q_N$$

Revenue is then given by

$$R_B = P_B Q_B = (Z - P_N - Q_B)Q_B = ZQ_B - P_N Q_B - Q_B^2$$
$$R_N = P_N q_N = (Z - P_B - Q_N)Q_N = ZQ_N - P_B Q_N - Q_N^2$$

Marginal revenue is given by

$$MR_B = Z - P_N - 2Q_B$$
$$MR_N = Z - P_B - 2Q_N$$

Setting these equal to marginal cost (=0) and solving gives

$$MR_B = Z - P_N - 2Q_B = 0$$
$$\Rightarrow Q_B = \frac{Z - P_N}{2}$$
$$MR_N = Z - P_B - 2Q_N = 0$$
$$\Rightarrow Q_N = \frac{Z - P_B}{2}$$

Given the levels of quantity we can get the level of price from the price equations.

$$P_B = Z - P_N - \left(\frac{Z - P_N}{2}\right) = \frac{Z - P_N}{2}$$
$$P_N = Z - P_B - \left(\frac{Z - P_B}{2}\right) = \frac{Z - P_B}{2}$$

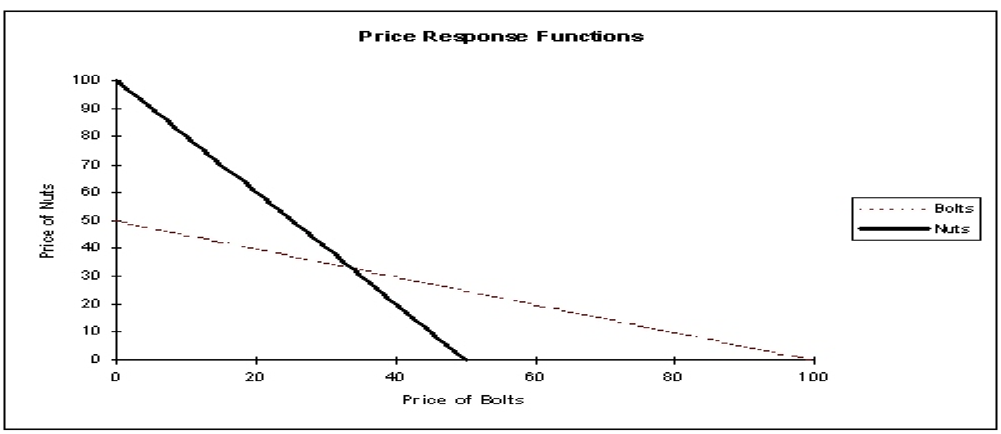
**(c)** Find the Nash equilibrium prices by solving the two equations simultaneously as follows

$$P_B = \frac{Z - P_N}{2} = \frac{Z - \left(\frac{Z - P_B}{2}\right)}{2} \Rightarrow P_B = \frac{1}{4}Z + \frac{1}{4}P_B \Rightarrow P_B = \frac{1}{3}Z$$

Similarly for $P_N$ we obtain

$$P_N = \frac{Z - P_B}{2} = \frac{Z - \left(\frac{Z}{3}\right)}{2}$$
$$\rightarrow P_B = \frac{1}{2}Z - \frac{1}{6}Z$$
$$\Rightarrow P_B = \frac{1}{3}Z$$

The graphs look like this for Z = 100




**(d)** While this could be viewed as a coordination problem for two firms as in this problem, it could also be viewed as a joint product problem for a multiproduct monopolist. If the monopolist were to take into account the joint nature of the purchasing decision and sell "nut and bolt pairs", a higher level of production of both goods would occur. This will result in a lower price than if the firm (or two monopolists) did not coordinate the production and sales. When two monopolists sell complementary goods in separate markets, the Nash equilibrium prices for the two goods are higher than what the two monopolists would charge if they coordinated their pricing. Coordination or cooperation leads in this case to lower prices! This is because the goods are complementary so that the best response functions are downward sloping as is clear from the figure in part c.

**Problem 7**

**(a)** Without loss of generality, suppose the cost is zero, then profit for each firm is given by

$$\pi_1 = p_1 q_1 = (15 - p_1 + 0.5 p_2) p_1$$

$$\pi_2 = p_2 q_2 = (15 - p_2 + 0.5 p_1) p_2$$

Each firm chooses its price to maximize its profit

$$\frac{\partial \pi_1}{\partial p_1} = 15 - 2p_1 + 0.5 p_2 = 0 \Rightarrow p_1 = \frac{15 + 0.5 p_2}{2}$$

$$\frac{\partial \pi_2}{\partial p_2} = 15 - 2p_2 + 0.5 p_1 = 0 \Rightarrow p_2 = \frac{15 + 0.5 p_1}{2}$$

They are the best response functions. Prices are strategic complementary.

**(b)** From the best response functions, derive the equilibrium set of prices

$$p_1^* = \frac{1}{2}\left(15 + 0.5\left(\frac{15 + 0.5 p_1^*}{2}\right)\right) \Rightarrow p_1^* = 10$$

$$p_2^* = \frac{1}{2}(15 + 0.5 \times 10) \Rightarrow p_2^* = 10$$

The equilibrium set of prices in this market is 10 for each firm. Profits earned at these prices are

$$\pi_1^* = p_1^* q_1^* = (15 - p_1^* + 0.5 p_2^*) p_1^* = (15 - 10 + 0.5 \times 10) \times 10 = 100$$

$$\pi_2^* = p_2^* q_2^* = (15 - p_2^* + 0.5 p_1^*) p_2^* = (15 - 10 + 0.5 \times 10) \times 10 = 100$$