Chapter 9: Static Games and Cournot Competition

Learning Objectives:

Students should learn to:

1. The student will understand the ideas of strategic interdependence and reasoning strategically and be able to apply them to economic models of market behavior.
2. The student will understand and be able to apply simple ideas from game theory to economic models. Concepts include:
    a. Rational behavior
    b. Rules of a game
    c. Information sets and the idea of knowing the structure of a game including the preferences and information sets of other players
    d. Simultaneous move games
    e. Sequential move games
    f. Intuitive description of a Nash equilibrium
    g. Dominant strategies (pairwise, strong, and weak)
    h. Dominated strategies
    i. Reaction or "best response" functions
    j. Various practical techniques for solving games (cross out dominated strategies, use arrows to denote direction of improvement, find reaction functions and then find fixed point, etc.)
    k. Formal description of Nash equilibrium
3. The student will understand how to solve a simple Cournot duopoly model with very specific characteristics including:
    a. Constant marginal costs
    b. Identical marginal costs
    c. Linear inverse demand curves and the resulting linear marginal revenue curves
    d. Linear reaction functions
    e. Symmetric Nash equilibrium
4. The student will understand how the simple Cournot model easily generalizes to more realistic models, how to solve these models, and the insights gained from these generalizations.
    a. Different costs between firms
    b. Linear marginal costs (at the instructor's discretion)
    c. More than two firms
    d. Using symmetry to solve more complex models
    e. Welfare implications of the Cournot model vis-a-vis perfect competition and monopoly (cartels)

Suggested Lecture Outline:

Spend three fifty-minute long lectures on this chapter.

Lecture 1:

1. Strategy
2. Rules and structure
3. Simultaneous (normal) and sequential (extensive) form games
4. Dominant and dominated strategies
5. Solution strategies
6. Nash equilibrium as a solution concept
7. Finding Nash equilibrium
8. Best response functions
9. Formal discussion of Nash equilibrium

Lecture 2:

1. Oligopoly models
2. The simple Cournot model
3. Numerical and Graphical Problems on the Cournot Model

Lecture 3:

1. Extensions of the Cournot model
2. Numerical Problems

Suggestions for the Instructors:

1. The following two papers are very useful in teaching strategic behavior and Cournot model:
   a. Gupta, Pal and Sarkar, (1998), A Geometric Solution of a Cournot Oligopoly with Non-identical Firms, Journal of Economic Education
   b. Meister (1999), Oligopoly - An in-class Economic Game, Journal of Economic Education
2. Provide many real world examples of the Cournot model.
3. It is often useful to carry a numerical example along in the Cournot model in addition to the one using letters for parameters. The linear demand curve $P = 320 - 4Q$ with $MC = 74$ has nice results.
4. It is important in presenting the Cournot model to emphasize the best response functions and how these provide information about the strategies of the players. It may be useful to actually trace out the best response function for one player as a function of various quantity levels chosen by the other player.
5. One of the key points in the whole chapter is how the idea of best response functions in game theory and in Cournot (and other) models of oligopoly are one and the same thing.
6. A good interactive learning experience would be to divide the students in teams of 3-4 individuals and have them work out a practice problem in class.

Solutions to the End of the Chapter Problems:

Problem 1

(a) This is a classic matching problem. The easiest way to find the Nash equilibrium is to first eliminate from each row the dominated strategies for Harrison. Harrison has the second payoff in each pair. Looking at the first row, if Tyler chooses small (S), Harrison should also choose small. Thus the point (S,L) in the upper right-hand corner can be eliminated. Looking at the second row, if Tyler chooses large (L), then Harrison should also choose large. Thus the point (L,S) in the lower left-hand corner can be eliminated. Now we move to the dominated strategies for Tyler. If Harrison chooses the first column (S), then Tyler should also choose small. This is already removed and so we gain no information. Unfortunately checking the second column also yields no new information and we are left with the two Nash Equilibria (S,S) and (L,L).

		Harrison	
		Small Party	Large Party
Tyler	Small Party	(1,000, 1,000)	~~(0, 0)~~
	Large Party	~~(0, 0)~~	(500, 500)

(b) The Pareto optimal outcome is (1,000, 1,000) which results from the strategy pair (S,S). At this point there is no way to make either party better off.

Problem 2:

(a) Note that X is the aggregate number of people that all individuals on campus expect to show up. The intercept is twenty since 20 individuals will always show up regardless of expectations. If individuals on campus think that one person will attend, then 21 individuals will show up.

(b) One way to get the intuition here is to assume that each person on campus thinks that 100 people will attend this party. This implies that the aggregate expectation is 100 individuals at the party. Plugging this into the equation implies that attendance is given by

$$A = 20 + 0.6(100) = 80$$

Thus expectations are not correct. If each individual thought no one would attend the party then the attendance would be given by

$$A = 20 + 0.6(0) = 20$$

which again is not a correct expectation. If each individual guesses that 50 people will attend then we obtain

$$A = 20 + 0.6(50) = 50$$

which means expectations are fulfilled. The solution procedure is to find the expected attendance (X) that makes the equation satisfied with X=A. Thus we just plug in X on the right-hand side and solve

$$\begin{aligned} X &= 20 + 0.6(X) \\ \Rightarrow 0.4X &= 20 \\ \Rightarrow X &= 50 \end{aligned}$$

It might be useful to relate this problem to the “multiplier” problem in a simple macro model of consumption where C = a + bY and Y = C+I.

Problem 3

(a)

		Player A	
		Stay	Swerve
Player B	Stay	~~(-6, -6)~~	(2, -2)
	Swerve	(-2, 2)	~~(1,1)~~

The equilibria are the two off-diagonal elements.

(b) To solve this problem we need to use expected values. If player B chooses to Stay his expected payoff is given by the payoffs to staying weighted by the probabilities that player A will Stay or Swerve.

$$\begin{aligned} E(\pi_B|STAY) &= (0.20)(-6) + (0.8)(2) \\ &= -1.2 + 1.6 = 0.4 \end{aligned}$$

If player B chooses to Swerve his expected payoff is given by the payoffs to staying weighted by the probabilities that player A will Stay or Swerve.

$$\begin{aligned} E(\pi_B|SWERVE) &= (0.20)(-2) + (0.8)(1) \\ &= -0.4 + 0.8 = 0.4 \end{aligned}$$

(c) The easiest way to see this is to add the probabilities to the border of game matrix and then compute the joint probabilities in each cell.

		Player A	
		Stay 1/5	Swerve 4/5
Player B	Stay 1/5	(-6, -6) 1/25	(2, -2) 4/25
	Swerve 4/5	(-2, 2) 4/25	(1,1) 16/25

The probability of (Stay, Stay) is 1/25.

Problem 4

To determine my best response function, I equate my marginal revenue with my marginal cost

$$200 - 4Q_1 - 2Q_2 = 8 \Rightarrow Q_1 = \frac{1}{4}[200 - 2Q_2 - 8]$$

Since my rival and I are identical,

$$Q_1^* = Q_2^* = Q^* \Rightarrow \frac{1}{4}[192 - 2Q^*] = Q^* \Rightarrow Q^* = 32 \Rightarrow P = 72$$

My profit is

$$\pi = (72 - 8)32 - 1500 = 548$$

Note: Because of the fixed cost, there are two other asymmetric equilibria. At each, one firm produces its monopoly output and the other produces none. We assume that in this case, a symmetric equilibrium is more reasonable than an asymmetric equilibrium.

Problem 5

Assume that I am firm 1. To determine my best response function, I equate my marginal revenue with my marginal cost.

$$290 - \frac{2}{3}Q_1 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right) = 50 \Rightarrow Q_1 = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right)\right]$$

Since my rivals and I are identical, $Q_i^* = Q^*$ for all i. Therefore,

$$Q^* = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q^*\right)\right] \Rightarrow Q^* = 48 \Rightarrow P^* = 290 - \frac{1}{3}(14)(48) = 66$$

My profit is

$$\pi = (66 - 50)(48) - 200 = 568$$

Problem 6

(a) To determine firm 1's best response function, equate its marginal revenue with marginal cost

$$400 - 4Q_1 - 2Q_2 = 40 \Rightarrow Q_1 = \frac{1}{4}[360 - 2Q_2]$$

Since the firms are identical,

$$Q_1^* = Q_2^* = Q^* \Rightarrow \frac{1}{4}[360 - 2Q^*] = Q^* \Rightarrow Q^* = 60 \Rightarrow P^* = 160$$

Firm 1's profit is

$$\pi_1 = (160 - 40)60 = 7200$$

(b) The monopoly output is $Q_M = \frac{1}{4}[360] = 90$

(45, 45) is a not a solution, because if one firm produces 45, then the other produces $\frac{1}{4}[360 - 2(45)] = 67.5$ to maximize its profit.

Problem 7

To determine firm 1's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_1 - 2Q_2 = 25 \Rightarrow Q_1 = \frac{1}{4}[375 - 2Q_2]$$

To determine firm 2's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_2 - 2Q_1 = 40 \Rightarrow Q_2 = \frac{1}{4}[360 - 2Q_1]$$

Substitute $Q_1 = \frac{1}{4}[375 - 2Q_2]$ into $Q_2 = \frac{1}{4}[360 - 2Q_1]$ yields the equilibrium quantity for firm 2 as $Q_2^* = 57.5$. It is then easy to verify that: $Q_1^* = 65$,

Problem 8

(a) Let the long-run equilibrium number of firms be *N*. For each firm, determine its best response function by equate its marginal revenue with its marginal cost. Since the firms are identical

$$100 - 2q - (N-1)q = 20 \Rightarrow q = \frac{80}{N+1} \Rightarrow P = 100 - \frac{N}{N+1} \cdot 80$$

Each firm's profit must be zero so that no firm has incentive to leave or enter the industry

$$\pi = Pq - C = (100 - \frac{N}{N+1} \cdot 80) \cdot \frac{80}{N+1} - (256 + 20 \cdot \frac{80}{N+1}) = 0 \Rightarrow (N+1)^2 = 25 \Rightarrow N = 4$$

Thus, the long-run equilibrium number of firms is 4.

(b) At the long-run equilibrium, each firm's profit is zero and output is 16, therefore, the industry output is 64, industry price is 36 and industry profit is zero.