Chapter 7: Product Variety and Quality under Monopoly

Learning Objectives:

Students should learn to:

1. Distinguish vertical from horizontal product differentiation.
2. Interpret physical space and distance as distance in product space in defining a spatial model of product differentiation.
3. Set up and solve graphically and algebraically a simple one dimensional spatial model with the following characteristics:
    a. Single firm
    b. One outlet located in the middle of the product space
    c. Identical good except for distance
    d. Transportation cost that is proportional to distance
    e. Identical consumers who are spaced uniformly on the product line
    f. Consumers demand only one unit of the good per period with constant reservation price V
    g. Constant marginal cost of serving each customer
    h. All consumers in the economy are served
4. Explain the extension of the model with one shop, optimally located in the center of the product space, to a model with two shops, each located 1/4 of the distance towards the center from the respective ends of the product space and then to the case of *n* located symmetrically around the center.
5. Explain why the location of a firm in product (geographic) space is an important part of the profit maximization decision.
6. Discuss issues related to the optimal number of shops operated such as the impact of fixed set-up costs, transportation costs, the number of consumers, and so forth.
7. Explain when it is socially optimal to add an additional shop; that is, be able to derive and explain the inequality.
8. Discuss the issue of whether the monopolist produces too much or too little product variety by considering the benefits and costs of adding more shops to the spatial model.
9. Explain how the monopolist might price discriminate in the spatial model. The student will be able to explain *uniform delivered pricing*.
10. Show that the price discriminating monopolist in the spatial model who serves all the customers will produce the socially optimal product variety, that is, the monopoly firm will not produce too much product variety.
11. Extend the analogy of price discrimination in geographic space to price discrimination in product characteristics space.
12. Explain how quality can affect demand and also supply. The student will be able to solve simple problems where there is a choice of both quality and the output level to supply.
13. Explain the difference between social welfare and monopoly welfare. The student will be able to apply this concept to the problem of quality choice.
14. Understand the role of price discrimination in competitive markets and testing for this effect in airline markets.

Suggested Lecture Outline:

Spend three fifty-minute long lectures on this chapter.

Lecture 1:

1. Vertical versus horizontal price differentiation
2. A Spatial Approach to Horizontal Product Differentiation
3. Spatial model of product differentiation (Hotelling)
4. Monopoly pricing in the spatial model without price discrimination
5. Issues of product variety in the spatial model

Lecture 2:

1. Monopoly pricing in the spatial model with price discrimination
2. Numerical examples / problems on spatial models

Lecture 3:

1. Vertical Product Differentiation
2. Numerical examples / problems on vertical product differentiation
3. Offering more than one product in a Vertically Differentiated Market

Suggestions for the Instructor:

1. Go slow!
2. Make sure the students understand the difference between horizontal and vertical differentiation. One way to do this is to consider an economy made up of many individuals each with the same utility function and one where individuals have different utility functions.
3. It may be helpful in presenting the location model to present a numerical example along with the algebraic one. The example in the text with V = 20, N = 20, t = 12, and c = 4 works great. One can then consider n = 1, 2, 3, etc.
4. Make the students draw few graphs on the board.
5. Solve numerical problems / examples in class.
6. Students will find vertical differentiation with many products difficult.

Solutions to the End of the Chapter Problems:

Problem 1

(a) For z = 1, profits for this firm is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (36 - 2Q)(1)Q - 0 - (65)(1^2) \\ &= 36Q - 2Q^2 - 65\end{aligned}$$

Taking the derivative of the profit with respect to Q will yield

$$\begin{aligned}\frac{d\pi}{dQ} &= 36 - 4Q = 0 \\ \Rightarrow 4Q &= 36 \\ \rightarrow Q &= 9 \\ \Rightarrow P &= 36 - 2Q = 18\end{aligned}$$

Profit is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (18)(1)(9) - 65 \\ &= 162 - 65 = 97\end{aligned}$$

(b) Profit when z = 2 is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (36 - 2Q)(2)Q - 0 - (65)(2^2) \\ &= 72Q - 4Q^2 - 260\end{aligned}$$

Taking the derivative of the profit with respect to Q will yield

$$\begin{aligned}\frac{d\pi}{dQ} &= 72 - 8Q = 0 \\ \rightarrow 8Q &= 72 \\ \rightarrow Q &= 9 \\ \Rightarrow P &= 36 - 2Q = 18\end{aligned}$$

Profit is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (18)(2)(9) - 260 \\ &= 324 - 260 = 64\end{aligned}$$

(c) The monopolist will go with low quality.

Problem 2

This is a clear case where the individual firm incentive to increase variety is at odds with the socially optimal level of product variety. The individual firms will try to fill each niche in product space so that they can obtain some revenue from that spot as opposed to letting the revenue go to another firm. Rather than using a price instrument that would also give a lower price to consumers at other spots (where the competition may not be so fierce), they locate a brand (outlet) near the spot. If the cost of adding the brand is less than what consumers paid to travel to the old brand spot, they can offer a lower price and beat the competition. Consider, for example, the case of Wheaties, Total, Corn Total, and Raisin Bran Total, which are produced by General Mills. They compete with Corn Flakes and Raisin Bran and each other in such a way that competitors' products have a hard time finding a unique niche. Also notice that as "truly" new cereals such as Fruit and Fibre or Granola have come on the market, that there has been a rapid filling of the product space around these new competitors.

Problem 3

This is the classic location model from the chapter. The reservation price is given by V = \$5. The number of customers is N = 1,000. The length of the beach is 5 miles. The "cost" to travel from one end of the beach to the other is \$5.00. The marginal cost per crepe is c = \$0.50 and the fixed cost per stall is F = \$40. First consider one shop in the middle of the beach. Demand is given by

$$\begin{aligned} Q(p_1, 1) &= 2N\left(\frac{V - p_1}{t}\right) \\ &= (2)(1{,}000)\left(\frac{5 - p_1}{5}\right) \\ &= 2{,}000\left(\frac{5 - p_1}{5}\right) \\ &= 400(5 - p_1) \\ &= 2{,}000 - 400p_1 \end{aligned}$$

Given that there are 1,000 consumers, we can find the price that will allow this one stall to sell to all of them.

$$\begin{aligned} Q(p_1, 1) &= 2{,}000 - 400p_1 \\ 1{,}000 &= 2{,}000 - 400p_1 \\ \Rightarrow 400p_1 &= 1{,}000 \\ p_1 &= \$2.50 \end{aligned}$$

This makes sense, the customers at the ends of the beach must travel 2.5 miles (10 quarter miles) to the stall. At a cost of \$0.25 per 1/4 mile, the ten 1/4 miles gives a cost of \$2.50. This plus the price of the crepe at \$2.50 is just their reservation price. This will give profit from this one shop of

$$\begin{aligned} \pi &= (2.50)(1{,}000) - (0.50)(1{,}000) - 40 \\ &= 2{,}500 - 500 - 40 \\ &= \$1{,}960 \end{aligned}$$

If instead of supplying the whole market with this one shop, the firm were to restrict output, the optimal output level is determined by setting marginal revenue equal to marginal cost. We find marginal revenue by inverting the demand function and then using the "twice the slope" rule.

$$\begin{aligned} Q &= 2N\left(\frac{V - p}{t}\right) \\ \Rightarrow Qt &= 2NV - 2Np \\ \Rightarrow p &= \frac{2NV - Qt}{2N} \\ &= V - \frac{t}{2N}Q \\ &= 5 - \frac{5}{2{,}000}Q \\ MR &= 5 - \frac{10}{2{,}000}Q \\ &= 5 - 0.005Q \end{aligned}$$

Setting this equal to marginal cost of $0.50 we obtain

$$MR = 5 - 0.005Q = 0.50 = MC$$
$$\rightarrow 4.50 = 0.005Q$$
$$\Rightarrow Q = 900$$

Price is then given by

$$P = V - \frac{t}{2N}Q$$
$$= 5 - \frac{5}{2,000}(900)$$
$$= 5 - 2.25$$
$$= 2.75$$

So with only one stall, the market is not fully served. We can see this directly using the equation in the text which says that if V < c + t/n, only part of the market should be served, i.e.

$$c + \frac{t}{n} < V$$
$$0.50 + \frac{5}{1} = 5.50$$
$$5.50 \nless 5.00.$$

If there are two stalls, the entire market will be served as can be seen from

$$c + \frac{t}{n} < V$$
$$0.50 + \frac{5}{2} = 3.00$$
$$3.00 < 5.00.$$

Two stalls will be located 1/4 and 3/4 of the way along the beach. Each will sell to the maximum number of customers, i.e. 500. In order to sell to 500 customers, they must charge a price of $3.75 as can be seen below.

$$Q(p_1, 1) = 2,000 - 400p_1$$
$$500 = 2,000 - 400p_1$$
$$\Rightarrow 400p_1 = 1,500$$
$$p_1 = \$3.75$$

Joint profits for the two stalls can be computed as

$$\pi = (3.75)(1,000) - (0.50)(1,000) - 80$$
$$= 3,750 - 500 - 80$$
$$= \$3,170$$

Three stalls will be located 1/6, ½, and 5/6 of the way along the beach. Each will sell to the maximum number of customers, i.e. 333 1/3. In order to sell to 333 1/3 customers, they must charge a price of $4.166 as can be seen below.

$$Q(p_1, 1) = 2,000 - 400p_1$$
$$333\frac{1}{3} = 2,000 - 400p_1$$
$$\Rightarrow 400p_1 = 1,666\frac{2}{3}$$
$$p_1 = \$4.16\overline{6}$$

Joint profits for the three stalls can be computed as

$$\begin{aligned}\pi &= (4.166)(1{,}000) - (0.50)(1{,}000) - 120\\ &= 4{,}166.6\overline{6} - 500 - 120\\ &= \$3{,}546.6\overline{6}\end{aligned}$$

So three stalls dominates two stalls. We can proceed in a similar fashion with four stalls each serving 250 consumers.

$$\begin{aligned}Q(p_1, 1) &= 2{,}000 - 400p_1\\ 250 &= 2{,}000 - 400p_1\\ \Rightarrow 400p_1 &= 1{,}750\\ p_1 &= \$4.375\end{aligned}$$

Joint profits for four stalls can be computed as

$$\begin{aligned}\pi &= (4.375)(1{,}000) - (0.50)(1{,}000) - 160\\ &= 4{,}375 - 500 - 160\\ &= \$3{,}715\end{aligned}$$

We could proceed in this fashion or use the equations in the text for profits with N consumers and n stalls.

$$\pi(N, n) = N\left(V - \frac{t}{2n} - c\right) - nF$$

$$\pi(N, n+1) = N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F$$

Profit with n+1 firms will be higher than with n firms if

$$\pi(N, n+1) > \pi(N, n)$$

$$\Rightarrow N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F > N\left(V - \frac{t}{2n} - c\right) - nF$$

$$\rightarrow N\left(\frac{t}{2n}\right) - N\left(\frac{t}{2(n+1)}\right) + nF - (n+1)F > 0$$

$$\rightarrow N\left(\frac{t}{2n}\right) - N\left(\frac{t}{2(n+1)}\right) - F > 0$$

$$\Rightarrow N\left(\frac{t}{2n} - \frac{t}{2(n+1)}\right) - F > 0$$

$$\Rightarrow N\left(\frac{t(n+1)}{2n(n+1)} - \frac{tn}{2n(n+1)}\right) - F > 0$$

$$\Rightarrow N\left(\frac{t}{2n(n+1)}\right) - F > 0$$

$$\Rightarrow N\left(\frac{t}{2n(n+1)}\right) > F$$

$$\rightarrow \frac{tN}{F} > 2n(n+1)$$

$$\Rightarrow \frac{tN}{2F} > n(n+1)$$

For this problem the left-hand side of this inequality is

$$\frac{tN}{2F} = \frac{(5)(1{,}000)}{(2)(40)} = \frac{5{,}000}{80} = 62.5$$

With four stalls, n(n+1) = (4)(5) = 20. With seven stalls, n(n+1) = (7)(8) = 56, while with eight stalls, n(n+1) = (8)(9) = 72. So the firm should increase from seven to eight stalls, but not from eight to nine. So the optimal number of stalls is eight. To see this explicitly, compare profits with eight and nine stalls.

First, for eight stalls.

$$\begin{aligned}
Q(p_1, 1) &= 2{,}000 - 400p_1 \\
125 &= 2{,}000 - 400p_1 \\
\rightarrow 400p_1 &= 1{,}875 \\
p_1 &= \$4.6875 \\
\pi &= (4.6875)(1{,}000) - (0.50)(1{,}000) - 320 \\
&= 4{,}687.5 - 500 - 160 \\
&= \$3{,}867.5
\end{aligned}$$

Then, for nine stalls.

$$\begin{aligned}
Q(p_1, 1) &= 2{,}000 - 400p_1 \\
\frac{1{,}000}{9} &= 2{,}000 - 400p_1 \\
\rightarrow 400p_1 &= \frac{17{,}000}{9} \\
p_1 &= \$4.72\overline{2} \\
\pi &= (4.72\overline{2})(1{,}000) - (0.50)(1{,}000) - 360 \\
&= 4{,}722.2 - 500 - 360 \\
&= \$3{,}862.2\overline{2}
\end{aligned}$$

The table below shows the optimal price, revenue, total variable cost, total fixed cost and profit for various numbers of stalls assuming that all consumers are served in each case.

Stalls	**Price**	**Revenue**	**Variable Cost**	**Fixed Cost**	**Profit**	**n(n+1)**	**tN/2F**	**c + t/n**
1	2.5	2,500	500	40	1,960	2	62.5	5.5
2	3.75	3,750	500	80	3,170	6	62.5	3
3	4.1667	4,166.667	500	120	3,546.67	12	62.5	2.16667
4	4.375	4,375	500	160	3,715	20	62.5	1.75
5	4.5	4,500	500	200	3,800	30	62.5	1.5
6	4.5833	4,583.333	500	240	3,843.33	42	62.5	1.33333
7	4.6429	4,642.857	500	280	3,862.86	56	62.5	1.21429
8	4.6875	4,687.5	500	320	**3,867.5**	72	62.5	1.125
9	4.7222	4,722.222	500	360	3,862.22	90	62.5	1.05556
10	4.75	4,750	500	400	3,850	110	62.5	1
15	4.8333	4,833.333	500	600	3,733.33	240	62.5	0.83333
20	4.875	4,875	500	800	3,575	420	62.5	0.75

Problem 4

Given the standard assumption that the effort costs of the stall-holders are the same as those o the sunbathers, there is no change in the costs faced by consumers and profits faced by stall-holders, since the only change is a transfer from consumers' disutility cost to stall-holders' delivery cost. Therefore, the optimal number of stalls is still eight for profit maximization.

If the incurred effort costs half as much has those of the sunbathers, then in order to have

$$\pi(N,n) = N\left(V - \frac{t}{2n} - c\right) - nF < \pi(N,n+1) = N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F \text{ require}$$

$$\frac{tN}{2F} = n(n+1) \Rightarrow \frac{(2.5)(1000)}{(2)(40)} = n(n+1) \Rightarrow n = 5$$

Then the optimal number of stalls is five.

Problem 5

Since the marginal cost of making both types of computers are identical, the profit maximizing strategy is to offer high performance laptops to both groups.

Problem 6

Profit maximization requires that Dell offers a quality – price combination (z_1, p_1) to the "normal" people and a quality – price combination (z_2, p_2) to the "techies" that works to sort the two groups and permit more surplus extraction. Therefore, to the normal people, Dell can charge a price $p_1 = 1000z_1$. from the discussion in the text, it follows that Dell should offer $z_2 = 3$, which is the highest quality. Now, Dell needs to adjust p_2 so that the techies do not buy a computer of quality z_1. Thus, the highest price it can charge the techies is p_2, where p_2 is such that $2000(z_1 - 1) - p_1 = 4000 - p_2 \Rightarrow p_2 = 6000 - 2000z_1 + p_1$

If the proportions of normal and techies are N_n and N_t, respectively, then Dell's profit is:

$$\pi = N_t(6000 - 2000z_1 + p_1) + N_n(1000z_1) - 500(N_t + N_n)$$

$$\frac{\partial \pi}{\partial z_1} = -2000N_t + 1000N_n$$

From the above equation, it follows that Dell's profit maximizing strategy depends on the proportions of techies and normal people.

Problem 7

(a) An increase in product quality affects demand positively. Observe that

$$\frac{\partial P}{\partial Z} = \frac{\partial}{\partial Z}\left[22 - \frac{Q}{100Z}\right] = \frac{Q}{100Z^2} > 0$$

(b) Find out the total profits associated with Z = 1, 2 and 3.

Note that when Z = 1, $P = 22 - \frac{Q}{100}$ and $MC(Q) = 2 + (1)^2 = 3$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=1)=22-\frac{2Q}{100}=3=MC(Z=1)\Rightarrow Q=950\Rightarrow \pi(Z=1)=9025$$

Note that when Z = 2, $P=22-\frac{Q}{200}$ and $MC(Q)=2+(2)^2=6$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=2)=22-\frac{2Q}{200}=6=MC(Z=2)\Rightarrow Q=1600\Rightarrow \pi(Z=2)=12800$$

Note that when Z = 3, $P=22-\frac{Q}{300}$ and $MC(Q)=2+(3)^2=11$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=3)=22-\frac{2Q}{300}=11=MC(Z=3)\Rightarrow Q=1650\Rightarrow \pi(Z=3)=9075$$

Therefore, Z = 2 is the profit maximizing level of quality for the monopolist.

Problem 8

To maximize the social welfare, is to maximize

$$W=[2000(z-1)-p]N_t+[1000z-p]N_n+(p-500)(N_t+N_n)$$

Therefore, the quality choice is the highest quality, which is z = 3.

The monopolist would maximize its profit under this social optimal quality condition. The price must be greater than its marginal cost 500, and allow both types of consumer to purchase. Therefore, the price it would charge is 3000.