Chapter 5: Price Discrimination and Monopoly: Linear PricingLearning Objectives:

Students should learn to:

1. Define and explain the differences between uniform pricing and price discrimination.
2. Explain the conditions that must exist for a firm to practice price discrimination.
3. Give examples of markets in which arbitrage is more easily accomplished than others and explain why.
4. Give many examples of third degree price discrimination.
5. Solve third degree price discrimination problems graphically and algebraically. The student will understand the difference between horizontal and vertical summation of demand and marginal revenue curves. The student will be able to state simple results about price elasticities and relative prices in separated markets.
6. Understand third degree price discrimination in the context of differentiated products.
7. Discuss the issues related to welfare and third degree price discrimination.

Suggested Lecture Outline:

Spend two fifty-minute long lectures on this chapter.

Lecture 1:

1. Monopoly pricing MR = MC
2. Uniform versus non-uniform pricing
3. Feasibility of Price Discrimination
4. Third degree price discrimination
5. Graphical and algebraic examples / problems of third degree price discrimination

Lecture 2:

1. More algebraic examples / problems of third degree price discrimination
2. Third degree price discrimination for differentiated products
3. Social welfare and third degree price discrimination
4. Algebraic examples on social welfare and third degree price discrimination

Suggestions for the Instructor:

1. There are many good examples of third degree price discrimination. Have the students try to come up with examples on their own in class. The resulting discussion is a good lead-in to the topic.
2. Arbitrage (or lack thereof) is especially important for third degree price discrimination. Discuss why arbitrage may often fail to equilibrate the market in such cases or how the firm effectively prevents any arbitrage by the use of restrictions. A good example is the use of two one-way tickets as opposed to a round-trip airfare.
3. Show how to add demand and marginal revenue horizontally both graphically and algebraically. This involves inverting the curves and can be easily done for examples where the coefficient on price is one. It is important to show where kinks may occur. One might also consider the competitive solution when there is a single supply curve and two consumers with different demands.

4. Be sure when discussing elasticities to note that demand elasticities are negative but we often refer to them in terms of their absolute value. In particular, η in the text, is assumed to be a positive number.
5. Stress the intuition about the welfare results of price discrimination.

Solutions to End of the Chapter Problems:

Problem 1

False. If the aggregate quantity does not increase, then third degree price discrimination lowers welfare.

Problem 2

We don't know because we don't know the cost of different pizzas.

Problem 3

(a) The demand functions for the two consumer groups are

$X_1 = 200 - P$ if $P \le 200$, and $X_1 = 0$ if P $\ge$ 200.

$X_2 = 50 - \frac{1}{2}P$ if $P \le 100$, and $X_2 = 0$ if P $\ge$ 100

First consider the case when P $\ge$ 200. In this case, $X_1 = 0$ and $X_2 = 0$, implying $X_1 + X_2 = 0$

Now, consider $100 < P \le 200$. In this case, $X_1 = 200 - P$, and $X_2 = 0$, implying $X_1 + X_2 = 200 - P$.

Next, consider $0 \le P \le 100$. In this case, $X_1 = 200 - P$ and $X_2 = 50 - \frac{1}{2}P$, implying

$X_1 + X_2 = 250 - \frac{3}{2}P$

(b) First consider the case when P $\ge$ 200. In this case, $X_1 + X_2 = 0$, implying $\pi = 0$.

Now, consider $100 < P \le 200$. In this case, $X_1 + X_2 = 200 - P$. Therefore,

$$\begin{aligned}
\pi &= PX - 40X \\
&= (200 - X)X - 40X \\
&= 200X - X^2 - 40X \\
\frac{d\pi}{dX} &= 200 - 2X - 40 = 0 \\
\Rightarrow 2X &= 160 \\
\Rightarrow X &= 80 \\
\Rightarrow P &= 120 \\
\Rightarrow \pi &= (120)(80) - (40)(80) \\
&= 6{,}400
\end{aligned}$$

Next, consider $0 \le P \le 100$. In this case, $X_1 + X_2 = 250 - 1.5P$. Therefore,

$$\pi = PX - 40X$$
$$= (166.66 - \frac{2}{3}X)X - 40X$$
$$= 166.66X - \frac{2}{3}X^2 - 40X$$
$$\frac{d\pi}{dX} = 166.66 - \frac{4}{3}X - 40 = 0$$
$$\Rightarrow \frac{4}{3}X = 126.66$$
$$\Rightarrow X = 95$$
$$\Rightarrow P = 103\frac{1}{3}$$
$$\Rightarrow \pi = (103.33)(95) - (40)(95)$$
$$= 6{,}016.66$$

But this solution violates the assumption that P is less than 100, so is not viable. If P is 100, then X = 100 and profits will be 6000. This is less than 6400. The maximum is then for the second case with no sales to group 2. Total profits are equal to 6400.

Consumer surplus is obtained by using the first market only. It is given by

$$CS = \int_{120}^{200} (200 - x)\, dx$$
$$= \left(200x - \frac{x^2}{2} \right) \Big|_{120}^{200} = \left((40{,}000 - 20{,}000) - (24{,}000 - 7{,}200) \right)$$
$$= 20{,}000 - 16{,}800 = 3{,}200$$

Problem 4

(a) For the first market

$$\pi_1 = P_1X_1 - 40X_1$$
$$= (200 - X_1)X_1 - 40X_1$$
$$= 200X_1 - X_1^2 - 40X_1$$
$$\frac{d\pi}{dX_1} = 200 - 2X_1 - 40 = 0$$
$$\rightarrow 2X_1 = 160$$
$$\Rightarrow X_{1_1} = 80$$
$$\rightarrow P = 120$$
$$\Rightarrow \pi_1 = (120)(80) - (40)(80)$$
$$= 6{,}400$$

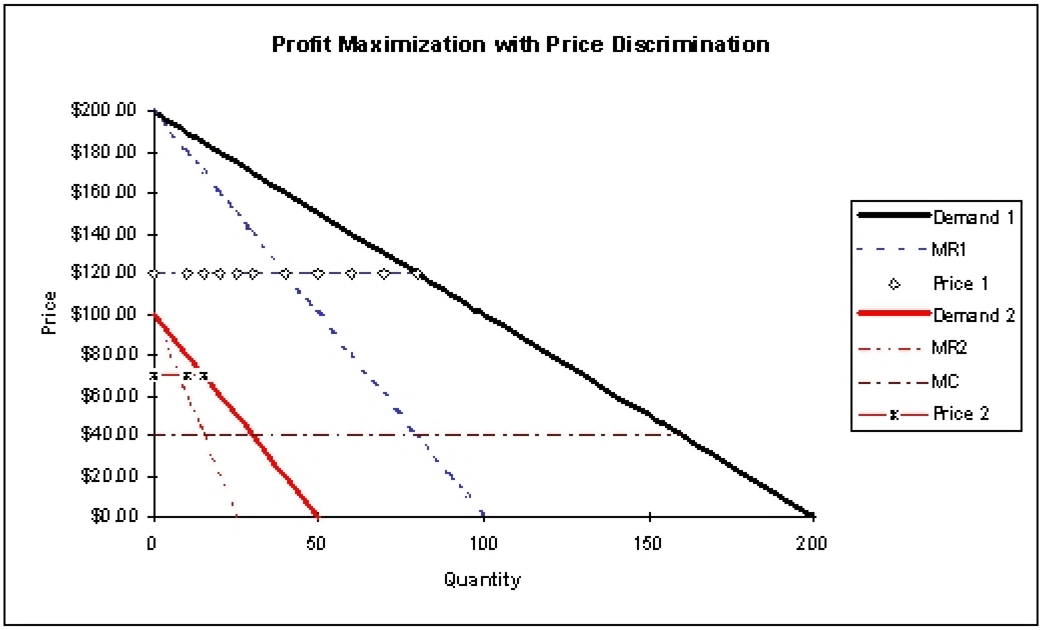
For the second market:

$$
\begin{aligned}
\pi_2 &= P_2X_2 - 40X_2 \\
&= (100 - 2X_2)X_2 - 40X_2 \\
&= 100X_2 - 2X_2^2 - 40X_2 \\
\frac{d\pi}{dX_2} &= 100 - 4X_2 - 40 = 0 \\
\Rightarrow 4X_2 &= 60 \\
\Rightarrow X_2 &= 15 \\
\Rightarrow P &= 70 \\
\Rightarrow \pi_2 &= (70)(15) - (40)(15) \\
&= 450
\end{aligned}
$$

Consumer Surplus in the second market is given by:

$$
\begin{aligned}
CS &= \int_{70}^{100} (50 - 0.5x)\, dx \\
&= \left(50x - \frac{x^2}{4} \right) \Big|_{70}^{100} = \left((5{,}000 - 2{,}500) - (3{,}500 - 1{,}225) \right) \\
&= 2{,}500 - 2{,}275 = 225
\end{aligned}
$$

Consumer Surplus in the first market is obtained from part (b) in Problem 3, which is 3200. Total profits are then 6850 and total consumer surplus is 3425.



(b) Price discrimination has increased total surplus. This is because, without price discrimination, one market is not served.

Problem 5

(a) With price discrimination

$$Q_C = 200 - 2P_C \quad \Rightarrow \quad P_C = 100 - \frac{1}{2}Q_C \quad \Rightarrow \quad MR_C = 100 - Q_C$$

Now, equate marginal revenue for a cold day with marginal cost.

$$100 - Q_C = 20 \;\Rightarrow\; Q_C = 80 \Rightarrow \; P_C = 100 - \frac{1}{2}(80) = 60$$

$$Q_H = 300 - 2P_H \quad \Rightarrow \quad P_H = 150 - \frac{1}{2}Q_H \quad \Rightarrow \quad MR_H = 150 - Q_H$$

Now, equate marginal revenue for a hot day with marginal cost.

$$150 - Q_H = 20 \;\Rightarrow\; Q_H = 130 \Rightarrow \; P_H = 150 - \frac{1}{2}(130) = 85$$

Coca-Cola will charge 60 cents on a cold day and 85 cents on a hot day.

(b) Without price discrimination

If half of the days are hot and other half are cold, then the expected aggregate demand faced by the Coca-Cola is

$$Q_A = \frac{1}{2}(300 - 2P) + \frac{1}{2}(200 - 2P) = 250 - 2P$$

Hence, the expected marginal revenue is

$$MR_A = 125 - Q_A$$

Now, equate expected marginal revenue with marginal cost.

$$125 - Q_A = 20 \;\Rightarrow\; Q_A = 105 \Rightarrow \; P_A = 72.5$$

(c) Profits with price discrimination = $\frac{1}{2}[(85-20)(130) + (60-20)(80)] = 5825$

Profits without price discrimination = $(72.5 - 20)(105) = 5512.5$

Hence, profit is higher with discrimination.

Problem 6

Profits if it serves only North America

$$Q_N = 100 - P_N \quad \Rightarrow \quad P_N = 100 - Q_N \quad \Rightarrow \quad MR_N = 100 - 2Q_N$$

Now, equate marginal revenue in North America with marginal cost.

$$100 - 2Q_N = 20 \;\Rightarrow\; Q_N = 40 \Rightarrow \; P_N = 100 - (40) = 60$$

Hence, $\pi_N = (P_N - 20)Q_N = (60 - 20)(40) = 1600$

Profits if it serves both Sub-Saharan Africa and North America

Aggregate Demand is $Q_A = (1+\alpha)100 - 2P$

In that case, marginal revenue is $MR_A = (1+\alpha)50 - Q_A$

Equate marginal cost with marginal revenue

$(1+\alpha)50 - Q_A = 20 \;\Rightarrow\; Q_A = (1+\alpha)50 - 20$

Hence, $P_A = (1+\alpha)50 - \frac{1}{2}\left[(1+\alpha)50 - 20\right] = (1+\alpha)25 + 10$

Therefore, $\pi_A = (P_A - 20)Q_A = 2\left[(1+\alpha)25 - 10\right]^2$

Now, equate $\pi_A = \pi_N$, or equivalently

$$2\left[(1+\alpha)25 - 10\right]^2 = 1600 \;\Rightarrow\; \alpha = \left(\left(\frac{40}{\sqrt{2}} + 10\right)/25\right) - 1 = 0.531$$

Problem 7

(a) With price discrimination

$P_T = 18 - Q_T \;\Rightarrow\; MR_T = 18 - 2Q_T$

Now, equate marginal revenue in Toronto with marginal cost.

$18 - 2Q_T = 3 \;\Rightarrow\; Q_T = 7.5 \;\Rightarrow\; P_T = 10.5$

where the marginal cost of producing and delivering cough syrup to each town is 3, since the total cost is 2 + 3Q_i where $i = T, M$.

$P_M = 14 - Q_M \;\Rightarrow\; MR_M = 14 - 2Q_M$

Now, equate marginal revenue in Montreal with marginal cost.

$14 - 2Q_M = 3 \;\Rightarrow\; Q_M = 5.5 \;\Rightarrow\; P_M = 8.5$

Thus, the optimal price of Buckley's cough medicine is 10.5 in Toronto and 8.5 in Montreal if the two markets are separate.

(b) Without price discrimination

If Toronto and Montreal are treated as a common market, then the aggregate demand faced by Buckley is

$Q_A = Q_T + Q_M = (18 - P) + (14 - P) = 32 - 2P \;\Rightarrow\; MR_A = 16 - Q_A$

Now, equate marginal revenue with marginal cost

$16 - Q_A = 3 \;\Rightarrow\; Q_A = 13 \;\Rightarrow\; P = \$9.5$

Thus, the optimal price if Buckley's medicine for a common market is $9.5

Problem 8

If the MSAC charges the same price per hour regardless of type of player, then the aggregate demand faced by MSAC is

$Q = 1000Q_A + 1000Q_N = 1000(6 - P) + 1000(3 - P/2) = 9000 - 1500P \;\Rightarrow$

$P = 6 - Q/1500 \;\Rightarrow\; MR = 6 - Q/750$

Now, equate marginal revenue with marginal cost

$6 - Q/750 = 0 \;\Rightarrow\; Q = 4500 \;\Rightarrow\; P = 3$

Thus, the price is 3 it should charge to maximize club revenue.