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11

Dynamic Games and First and Second Movers

Since its introduction in the 1970s, Boeing's 416-seat 747 jet aircraft has dominated the jumbo jet market. Like McDonnell–Douglas before it, Airbus has for some time sought to develop a challenge to this long-reigning champion. After many delays and disappointments, Airbus may have finally come up with such a challenge. Its new A380 plane with anything between 555 and 800 seats is scheduled to make its regular passenger service debut in 2007 for Singapore Airlines. Whether the plane will win the market share and profit that Airbus hopes for remains unclear. What is clear is that Airbus made a deliberate and careful decision to respond to Boeing's earlier move.

The strategic interaction between the production decisions of the world's two principal manufacturers of commercial aircraft is sequential. First Boeing took an action, and then after that action was taken and observed Airbus chose its action. This is quite different from the static or simultaneous games that we studied in the previous two chapters. Games in which the players take their actions sequentially are dynamic games, and dynamic games are the focus of this chapter. In principle, these games can have many rounds of play, which are often called stages. Here, we concentrate mostly on games with just two stages and, for convenience, just two firms. Typically, one firm will play in the first round, the first mover, and the other will play in the second round, the second mover.

Popular business literature is replete with stories about first mover advantages and often gives advice as to how firms can establish a leadership position by moving first.[1] A classic example of first mover advantage is found in the prepared soup industry. In the late nineteenth century, Campbell was the first entrant into the prepared soup market in the U.S. In the early twentieth century, Heinz was the first entrant in the U.K. market. Campbell entered the U.K. market after Heinz, and similarly Heinz entered the U.S. market after Campbell. Yet the first mover in each market continues to dominate. Campbell has roughly 63% of the US market, but only 9% of the U.K. market, whereas Heinz has a 41% market share in the U.K. and a relatively minor market share in the U.S.[2]

The observation that early entry into a market can confer substantial advantages relative to later entrants raises a further possibility of great interest to industrial economists. The issue

[1] Lieberman and Montgomery (1988).
[2] See, for example, Sutton (1991).

is whether the initial entrant's advantages are so great that it would be impossible for any subsequent firm to enter at all. Remember, entry is a key part of the competitive market's success story as an allocative mechanism. Entry is the policing mechanism that insures a market will return to competitive pricing whenever firms in an industry are earning substantial economic profits. If entry does not occur then the market may not be working very well.

In the next two chapters, we explore the entry process in detail when the firms—the entrants and the incumbents—are strategic players in the market place. This is an important part of oligopoly. At this juncture, the point to realize is that entry is a sequential process—some firms enter early and some enter late. So, developing an understanding of dynamic games is good groundwork for our later investigation of entry and entry deterrence in oligopoly markets.

We first examine quantity and price competition when firms move sequentially rather than simultaneously. We will discover again that price and quantity competition are different, and depending on the kind of competition, there can be first mover or second mover advantages. This raises the interesting question of whether and how a firm can become either a first- or second-mover. Often the key to achieving the desired position and the associated higher profits is the ability of the firm to make a credible commitment to its strategy when the market opens for trade. We examine what credibility means in game theory and how it affects our equilibrium solution concept for dynamic models.

Simultaneous games, such as the traditional Cournot or Bertrand model, describe a once-and-for-all market interaction between the rival firms. In some sequential games as well there is only one market period where trade takes place, although this might occur in several stages. However, the more likely scenario is that rival firms interact and trade today in the market and then interact again in the future. Moreover, the competing firms understand the likelihood of future interaction today. Repeating the market interaction over and over again gives rise to a somewhat different type of dynamic game, usually called a repeated game. We defer our discussion of repeated games until Chapter 14.

11.1 THE STACKELBERG MODEL OF QUANTITY COMPETITION

The duopoly model of Stackelberg (1934) is similar to the Cournot model except for one critically important difference. Both firms choose quantities but now they do so *sequentially* rather than *simultaneously*. The firm, which moves first and chooses its output level, first, is the leader firm. The firm that moves second is the follower firm. The sequential choice of output is what makes the game dynamic. However, the firms trade their goods on the market only once and their interaction yields a "once-and-for-all" market-clearing outcome.

Let market demand again be represented by a linear inverse demand function $P = A - BQ$. Firm 1 is the leader who moves first and firm 2 is the follower who chooses its output *after* the choice of the leader is made. Each firm has the same constant unit cost of production c. Total industry output Q equals the sum of the outputs of each firm, $Q = q_1 + q_2$.

Firm 1 acts first and chooses q_1. How should it make this choice? Both firms are rational and strategic and both firms know this, and know that each other knows this. As a result, firm 1 will make its choice taking into account its best guess as to firm 2's rational response to its choice of q_1. In other words, firm 1 will work out firm 2's best response to each value of q_1, incorporate that best response into its own decision-making and then choose the q_1 option which, given firm 2's best response, maximizes firm 1's profit.

We can solve for firm 2's best response function q_2^* exactly as we did in the Cournot model in Chapter 9. For any choice of output q_1, firm 2 faces the inverse demand and marginal revenue curves:

$$\begin{aligned} P &= (A - Bq_1) - Bq_2 \\ MR_2 &= (A - Bq_1) - 2Bq_2 \end{aligned} \tag{11.1}$$

Setting marginal revenue equal to marginal cost, yields firm 2's best response q_2^* as the solution to the first-order condition:

$$A - Bq_1 - 2Bq_2^* = c \tag{11.2}$$

From which we obtain:

$$q_2^* = \frac{(A - c)}{2B} - \frac{q_1}{2} \tag{11.3}$$

If firm 1 is rational, firm 1 will understand that equation (11.3) describes what firm 2 will do in response to each possible choice of q_1 that firm 1. We can summarize equation (11.3) by $q_2^*(q_1)$. Anticipating this behavior by firm 2, firm 1 can substitute $q_2^*(q_1)$ for q_2 in its demand function so that its inverse demand function may be written as:

$$P = A - Bq_2^*(q_1) - Bq_1 = \frac{A + c}{2} - \frac{B}{2}q_1, \tag{11.4}$$

In turn, this implies that its profit function is:

$$\Pi_1(q_1, q_2^*(q_1)) = \left(\frac{A + c}{2} - \frac{B}{2}q_1 - c\right)q_1 = \left(\frac{A - c}{2} - \frac{B}{2}q_1\right)q_1 \tag{11.5}$$

Note that this substitution results in firm 1's demand and profits being dependent only on its own output choice, q_1. This is because firm 1 effectively sets q_2 as well, by virtue of the fact that q_2 is chosen by firm 2 in response to q_1 according to firm 2's best response function, *and firm* 1 *anticipates this*. In other words, the first-mover correctly predicts the second-mover's best response and incorporates this prediction into its decision-making calculus.

To solve for firm 1's profit maximizing output q_1^* we find the marginal revenue curve associated with firm 1's demand curve in (11.4), that is, $MR_1 = \dfrac{A + c}{2} - Bq_1$, and find the output q_1^* at which marginal revenue is equal to marginal cost. Alternatively, we could derive and solve the first order condition for profit maximization of (11.5), using the calculus technique of differentiation, setting $\dfrac{d\Pi(q_1^*, q_2^*(q_1^*))}{dq_1} = 0$, and solving for q_1^*. Either way we find that:

$$q_1^* = \frac{(A - c)}{2B} \tag{11.6}$$

Given this output choice by firm 1, firm 2 selects its best response as given by equation (11.3), which yields:

$$q_2^* = \frac{(A - c)}{4B} \tag{11.7}$$

Together, equations (11.6) and (11.7) describe the Stackelberg–Nash equilibrium production levels of each firm. Note that the leader's output is exactly equal to the level of output chosen by a simple uniform-pricing monopolist. This is a well-known feature of the Stackelberg model when demand is linear and costs are constant.

The total industry production is of course the sum of the two outputs shown in equations (11.6) and (11.7). This sum is: $Q^S = \frac{3(A - c)}{4B}$. Compare this market output with the earlier Cournot–Nash equilibrium industry output $Q^C = \frac{2(A - c)}{3B}$. Clearly, the Stackelberg model yields a greater industry output. Accordingly, the equilibrium price is lower in the Stackelberg analysis than it is in the Cournot analysis. The price and output results are illustrated in Figure 11.1.

A central feature of the Stackelberg model is the difference in the relative outcome of the two firms. Recall that from the standpoint of both consumer preferences and production techniques, the firms are identical. They produce identical goods and do so at the same, constant unit cost. Yet, because one firm moves first, the outcome for the two firms is different. Comparing q_1^* and q_2^* reveals that the leader gets a far larger market share and earns a much larger profit than does the follower. Moving first clearly has advantages. Alternatively, entering the market late has its disadvantages.

An interesting additional aspect of the disadvantaged outcome for firm 2 in the Stackelberg model is that this outcome occurs even though firm 2 has full information regarding the output choice of q_1. Indeed, firm 2 actually observes that choice before selecting q_2. In the Cournot duopoly model, firm 2 did not have such concrete information. Because the Cournot model is based upon simultaneous moves, each firm could only make a (rational) guess as to its rival's output choice. Paradoxically, firm 2 does worse when it has complete

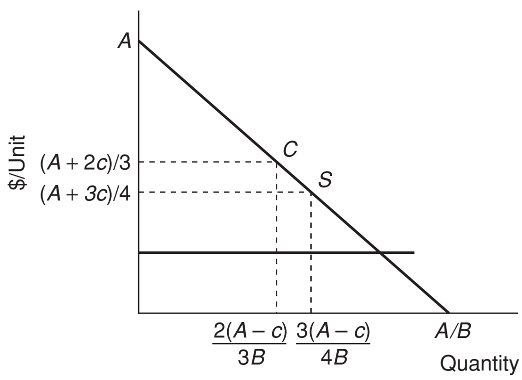

Figure 11.1 The Cournot and Stackelberg outcomes compared
C = Cournot equilibrium; S = Stackelberg equilibrium.

information about firm 1's choice (the Stackelberg case) than it does when its information is less than perfect (the Cournot case). This is because saying the information is concrete amounts to saying that firm 1's choice—at the time that firm 2 observes it—is irreversible. In the Stackelberg model, by the time firm 2 moves, firm 1 is already fully committed to $q_1 = \frac{(A - c)}{2B}$. In the Cournot context $q_1 = \frac{(A - c)}{2B}$ is *not* a best response to the choice $q_2 = \frac{(A - c)}{4B}$ and so firm 2 would not anticipate that firm 1 would produce that quantity. In contrast, in the Stackelberg model we do not derive firm 1's choice as a best response to $q_2 = \frac{(A - c)}{4B}$. Instead, we derived firm 1's output choice as the profit-maximizing output when firm 1 correctly anticipates that firm 2's decision rule is to choose its best value of q_2 *conditional upon* the output choice already made by firm 1. It is this fact, which reflects the underlying assumption of sequential moves that distinguishes the Stackelberg model.

Stackelberg's modification to the basic Cournot model is important. It is a useful way to capture the observed phenomenon that one firm often has a dominant or leadership position in a market. The Stackelberg model reveals that moving first can have its advantage and therefore could be an important aspect of strategic interaction.

11.1 **Practice Problem**

Consider the following game. Firm 1, the leader, selects an output q_1, after which firm 2, the follower, observes the choice of q_1 and then selects its own output q_2. The resulting price is one satisfying the industry demand curve $P = 200 - q_1 - q_2$. Both firms have zero fixed costs and a constant marginal cost of 60.

a. Derive the equation for the follower firm's best response function. Draw this equation on a graph with q_2 on the vertical axis and q_1 on the horizontal axis. Indicate the vertical intercept, horizontal intercept, and slope of the best response function.
b. Determine the equilibrium output of each firm in the leader-follower game. Show that this equilibrium lies on firm 2's best response function. What are firm 1's profits in the equilibrium?
c. Now let the two firms choose their outputs simultaneously. Compute the Cournot equilibrium outputs and industry price. Who loses and who gains when the firms play a Cournot game instead of the Stackelberg one?

11.2 SEQUENTIAL PRICE COMPETITION

What if the two firms, the leader and follower, in this dynamic game competed in price instead of quantity? If the firms are identical, that is, they produce the same product at the same costs then the outcome to the sequential price-setting game is not much different from simultaneous price game of the previous chapter. Prices again fall to marginal cost.

To see this, let us rework the Stackelberg quantity-setting model with instead each firm choosing the price it will charge. Firm 1 is again the leader and sets its price first and firm 2 is the follower setting its price second. Otherwise, the model is exactly the same as before. Each firm produces an identical good at the same, constant marginal cost, c, and consumers will purchase the good at the lower priced firm. If they set the same prices then each firm will serve half the market.

In setting its price, firm 1 must of course anticipate firm 2's best response. Clearly firm 2 will have an incentive to price slightly below firm 1's price whenever firm 1 sets a price greater than unit cost c and less than or equal to the monopoly price. In that case, by undercutting, firm 2 will serve the entire market and earn all the potential profits. On the other hand, if firm 1 sets a price less than unit cost c, then firm 2 will not match or undercut firm 1's price because firm 2 has no interest in making any sales when each unit sold loses money. Finally, if firm 1 sets a price equal to unit cost c firm 2's best response is to match it. The anticipated behavior of firm 2 in stage 2 puts firm 1 in a tight bind. Any price greater than unit cost c results in zero sales and there is no sense in setting a price less than c. The best firm 1 can do then is set a price equal to unit cost c. Firm 2's best response in the next stage is to match firm 1's price.

Matters are very different, however, if the two firms are not selling identical products. In this case, not all consumers buy from the lower priced firm. Product differentiation changes the outcome of price competition quite a bit. To illustrate the nature of price competitio with differentiated products, recall the spatial model of product differentiation that we developed previously. The setup is the following. There is a product spectrum of unit length along which consumers are uniformly distributed. Two firms supply this market. One firm has the address or product design $x = 0$, on the line whereas the other has location**,** $x = 1$. Each of the firms has the same constant unit cost of production c.

A consumer's location in this market is that consumer's most preferred product, or style. "Consumer x" is located distance x from the left-hand end of the market. Consumers differ regarding which variant or location of the good they consider to be the best, or their ideal product, but are identical in their reservation price V for their most preferred product and we assume that the reservation price V is substantially greater than the unit cost of production c. Each consumer buys at most one unit of the product. If consumer x purchases a good that is not her ideal product she incurs a utility loss of tx if she consumes good 1 (located at $x = 0$), and $t(1 - x)$ if she consumes good 2 (located at $x = 1$).

The two firms compete for customers by setting prices, p_1 and p_2, respectively. However, unlike the simple Bertrand model, firm 1 sets its price p_1 first, and then firm 2 follows by setting p_2. In order to find the demand facing the firms at prices p_1, and p_2 we proceed as in the previous chapter by identifying the marginal consumer x^m, who is indifferent between buying from either firm 1 or firm 2. Indifference means that the consumer x^m gets the same consumer surplus from either product and so satisfies the condition:

$$V - p_1 - tx^m = V - p_2 - t(1 - x^m) \tag{11.8}$$

From equation (11.8) we find that the address of the marginal consumer x^m is:

$$x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t} \tag{11.9}$$

At any set of prices, p_1 and p_2, all consumers to the left of x^m buy from firm 1, and all those to the right of x^m buy from firm 2. In other words, x^m is the fraction of the market buying from firm 1 and $(1 - x^m)$ is the fraction buying from firm 2. If the total number of consumers is denoted by N and they are uniformly distributed over the product spectrum the demand function facing firm 1 at any price combination (p_1, p_2) is:

$$D^1(p_1, p_2) = x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t}N \tag{11.10}$$

Similarly, firm 2's demand function is:

$$D^2(p_1, p_2) = (1 - x^m(p_1, p_2)) = \frac{(p_1 - p_2 + t)}{2t}N \tag{11.11}$$

Firm 1 acts first and sets its price p_1. In doing so, Firm 1 anticipates firm 2's best response to the price p_1 that firm 1 sets. In other words, firm 1 works out firm 2's best response to each possible price p_1, and then chooses its profit-maximizing price p_1 given firm 2's best response to that price. We can solve for firm 2's best response function p_2^* exactly as we did in section 4 of Chapter 11. It is

$$p_2^* = \frac{p_1 + c + t}{2} \tag{11.12}$$

Firm 1 knowsthat equation (11.12) describes what firm 2 will do in response to each price p_1 that firm 1 could set. We can summarize equation (11.12) by $p_2^*(p_1)$. Firm 1 knows that if it sets first a price p_1 then firm 2 will set a price $p_2^*(p_1)$. As a result, firm 1's demand (11.10) becomes:

$$D^1(p_1, p_2^*(p_1)) = \frac{(p_2^*(p_1) - p_1 + t)}{2t}N = \frac{N}{4t}(c + 3t - p_1) \tag{11.13}$$

In turn, this implies that firm 1's profit can be described by:

$$\Pi^1(p_1, p_2^*(p_1)) = \frac{N}{4t}(p_1 - c)(c + 3t - p_1) \tag{11.14}$$

In order to solve for firm 1's optimal price we need to work out how firm 1's profit changes as the firm varies its price p_1. The most straightforward way to do this is to take the derivative of the profit function (11.14) with respect to p_1 and set the derivative equal to zero. That is, solving $\frac{d\Pi(p_1^*, p_2^*(p_1^*))}{dp_1} = 0$ leads us to:

$$p_1^* = c + \frac{3t}{2} \tag{11.15}$$

Given this choice of price by firm 1, firm 2 selects its best response as given by equation (11.12), which yields:

$$p_2^* = c + \frac{5t}{4} \tag{11.16}$$

The profit maximizing prices in (11.15) and (11.16) for the sequential price game differ in important ways from the prices that we found for the simultaneous price game in

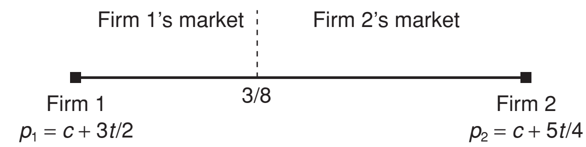

Figure 11.2 Sequential price competition: firm 1 sets its price first anticipating that firm 2 will price below firm 1's price

section 10.3 from the last chapter. One difference is that prices now are higher. In the simultaneous price game the two firms set the same prices $p_1^* = p_2^* = c + t$, whereas in the sequential game firm 1 sets a price in stage 1 that is greater than $c + t$, and firm 2 responds by setting a slightly lower price, but still higher than $c + t$.

A second difference is that the two firms in the sequential price game have different market shares and earn different profits. In the simultaneous price-setting game, each firm served one half the market and earned the same profit equal to $Nt/2$. In the sequential game, on the other hand, firm 1 serves 3/8 of the market, and earns a profit equal to $18Nt/32$, whereas firm 2 serves 5/8 of the market and earns a profit equal to $25Nt/32$. This outcome is described in Figure 11.2.

Finally, note that unlike the Stackelberg output game, the sequential price game just described presents a clear *second* mover advantage. Firm 2 enjoys a larger market share and higher profit than firm 1. Both are better off than in the simultaneous game but firm 2, the second mover, is even better off than firm 1. However, this advantage diminishes as consumer preference for differentiation, as measured in our example by the parameter t, decreases. When the goods are perfect substitutes there is no second mover advantage.

11.2 **Practice Problem**

Let there be 2 hair salons located on Main Street, which is 1 mile long. One is located at the east end of town, $x = 0$, and the other is located at the west end, $x = 1$. There are 100 potential customers who live along the mile stretch, and they are uniformly spread out along the mile. Consumers are willing to pay \$50 for a haircut done at their home. If a consumer has to travel there and back to get a haircut then a travel cost of \$5 per mile is incurred. Each salon has the same unit cost equal to \$10 per haircut.

a. Suppose the east end salon posts its price for a haircut first and then the west end salon posts its price for a haircut. What prices will the two salons set? How many customers does each salon serve? What are the profits?
b. Compare the prices to the ones we found when the two salons set their prices simultaneously, (see Chapter 10). Explain why prices changed in the way they did.

Firms generally seem to do better when they compete sequentially in prices than when they compete sequentially in output. The average price is higher and both firms earn higher profits when price competition is sequential rather than simultaneous. In contrast, the industry price falls and only one firm earns higher profit when quantity competition becomes sequential rather than simultaneous. This difference is related to another distinction. Whereas it is the first mover who has the clear advantage in the quantity game, in the price game, it is the firm who moves last that does best.

The fact that one firm has an advantage over the other firm in either the quantity or the price sequential game is due largely to the fact that the first mover's initial play is irrevocable by the time the second player moves. This may make some sense in the output game if the first mover actually has completed its production and incurred its costs before firm 2

Reality Checkpoint

First Mover Advantage in the TV Market: More Dishes and Higher Prices

When a firm markets a new good or service its consumers are likely to be aware of the fact that it may not work that well. In particular, it may take time to learn how to use the good properly or to use it in such a way that one gets full use of all the features that the product or service contains. Think, for example, of such goods and services as personal computers, personal digital assistants, cellular phones, DVD players, online auctions. It takes experience using a Palm Pilot or an Apple Computer or purchasing a product on e-Bay before one really can get the most out of these goods products. Gabszewicz, Pepall, and Thisse (GPT) (1992) build on this idea to show how consumer learning may confer a first-mover advantage to the first firm to market a new product. Imagine a simple two-stage model. Firm 1 introduces its version of the new product and a rival enters in the second stage with its own, differentiated version of the same good. GPT argue that for those consumers who bought firm 1's product in stage 1 they will know how it works but they will not know that for firm 2's new product. As a result, they will tend to prefer firm 1's good even if firm 2 sells at a lower price.

Indeed, GPT show that the pricing implications can be quite novel. When firm 1 introduces its product in stage 1, it foresees the later entry of firm 2. Firm 1 will have an incentive to price very low in the first stage so as to induce a lot of consumers to try and to become experienced with its product before firm 2 enters. This will create a large group of captive consumers for firm 1 who will be willing to pay a higher price for its product in stage 2 now that they know how the product works. Thus when firm 2 enters, firm 1 actually raises its price and still retains a larger number of consumers because they do not want to learn how to work with firm 2's imperfect substitute. The first mover may not only have a large market share but we may actually see that firm raise its prices at the very time that new competition emerges—exactly the opposite of what simple textbook analysis often implies.

Evidence of the first-mover advantage suggested by GPT may come from the television market. Here, the initial new product was cable TV, which has rapidly spread so that now 70 percent of American homes receive cable service. The Telecommunications Act of 1996 essentially deregulated the cable TV industry hoping that new firms, especially telephone companies, would provide competition to the local cable franchises. By and large, however, competition from alternative cable providers has remained weak. Instead, the major competition to cable that has emerged is from direct broadcast satellite (DBS) TV that consumers receive through a satellite dish. Textbook analysis would suggest that DBS competition would lead to lower cable prices. However, Goolsbee and Petrin (2003) find that, to the contrary, penetration of the market by DBS has led, on average, to an increase in the annual cable fee of about \$34.68. The ability of cable firms to raise price as new rivals appear may reflect precisely the first-mover advantage noted by Gabszewicz, Pepall, and Thisse.

Source: Gabszewicz, J., L. Pepall, and J.-F. Thisse, "Sequential Entry with Brand Loyalty Caused by Consumer Learning-by-doing," *Journal of Industrial Economics*, 60 (December 1992), 397–416; and A. Goolsbee and A. Petrin, "The Consumer Gains from Direct Broadcast Satellite and Competition with Cable TV," *Econometrica*, 72 (March 2004), 351–81.

selects its output. For the price game, however, it seems less plausible. Rather than settle for a second best profit, what is to stop the firm 1 from taking an additional move and trying to undercut the price of firm 2? If it is possible that when the market opens firm 1 can still undercut firm 2's price, then it is also clear that firm 2 will anticipate such a price cut by firm 1 and will want to cut its price still further. Yet if firm 1 anticipates that behavior, it will wish to reduce its price even more. Very quickly, this reasoning brings us back to the simultaneous price setting game. In other words, the sequential aspect of the price game requires that firm 1 *not* be able to change its price after it is set. Instead, firm 1 must be committed to that price. In turn, this raises the question as to how firm 1 can commit to its initial price in a manner that is *credible* to firm 2.

The issue of making a *credible commitment* is also crucial in the quantity setting Stackelberg game. If the first mover actually incurs the cost and produces the output before the follower moves then its production decision is irreversible and the credibility question is resolved. Talk on the other hand is cheap. If the leader simply announces an intention to produce the monopoly output, the follower would have good reason to doubt that the firm will follow through with this announcement. The monopoly output is not what firm 1 would choose to produce in response to the output firm 2 would choose if firm 1 produces the monopoly output.

The bottom line is that while dynamic games yield different results than those played simultaneously, those results depend crucially on the credibility of firms' strategies. Since credibility is so important, we should expect that the firms playing dynamic games will also distinguish between credible strategies and non-credible ones. So, we need to understand what makes strategies credible in dynamic games.

In the next section we explore what credibility means in a dynamic game. We do so in the context of dynamic game that has been of great interest to industrial organization economists. It is a market entry game. The firm to move first is a potential entrant to a monopolized market. The firm that moves second is the incumbent firm and the interest here is whether the incumbent can choose a strategy that deters the entrant from entering its profitable market. Before making its initial move, the entrant anticipates the incumbent's subsequent reaction. The question is what reactions are credible ones.

11.3 CREDIBILITY OF THREATS AND NASH EQUILIBRIA FOR DYNAMIC GAMES

We begin by introducing a concept that is critical to all dynamic games, namely, that of a subgame. A subgame is a part of an entire game that can stand alone as a game in itself. A proper subgame is a game within a game. Simultaneous games cannot have subgames, but dynamic games can. An example of a subgame in a two period model is the competition in the second period, which is a one-shot game within the larger two-period game.

Closely related to the notion of subgame is the concept of subgame perfection, first introduced by Nobel Prize winner Reinhard Selten (1978). It is the concept of subgame perfection that permits us to understand whether a firm's strategy is credible in a dynamic game. The term sounds very technical but it is actually quite simple. Basically, subgame perfection means that if a strategy chosen at the start of a game is optimal, it must be optimal to stick with that strategy at every later juncture in the game as play progresses.

It is easier to understand the concept of subgame perfection by seeing its application in practice. Imagine a dynamic game between two software firms, one a giant called Microhard who is the incumbent firm in the market and the other an upstart firm, Newvel, who wishes to enter the market. In this game the potential entrant, Newvel, moves first choosing either to enter Microhard's market or stay out. If Newvel stays out it earns a normal profit from being somewhere else in the economy, say $\Pi = 1$, and Microhard continues to earn a monopoly profit in the software market, say $\Pi = 5$. If Newvel enters the market then Microhard can choose either to accommodate the new entrant and share the market or to fight the new entrant by slashing prices. If Microhard accommodates Newvel's entry then each firm earns a profit $\Pi = 2$. If, on the other hand, Microhard fights then neither firm makes any profit so each firm earns $\Pi = 0$.

Dynamic games with moves in sequence require more care in presentation than single-period, simultaneous games. In a simultaneous game, a firm moves once and simultaneously and so its *action* is the same as its *strategy*. For a dynamic game, a firm's strategy is a complete set of instructions that tell the firm what *actions* to pick at every conceivable situation in the game. Nevertheless, for this simple dynamic game between Microhard and Newvel we can use a payoff matrix of the type introduced in Chapter 9 to gain insight into which strategy pairs yield a Nash equilibrium to this game.

		Microhard	
		Fight	Accommodate
Newvel	Enter	(0, 0)	(2, 2)
	Stay out	(1, 5)	(1, 5)

Start with the combination (Enter, Fight). This *cannot* correspond to an equilibrium. Enter will lead Newvel to come into the market. If Microhard has adopted the Fight strategy, it must respond to such entry very aggressively. Yet, as the payoff matrix makes clear, such an aggressive action is not Microhard's best response to entry by Newvel. Now try (Enter, Accommodate). This *is* a Nash equilibrium in strategies. If Newvel chooses to Enter, and if Microhard has adopted the strategy, Accommodate, the associated outcome is a best response for both Newvel and Microhard. That is, if Microhard has adopted a strategy to Accommodate, then Enter is the best response for Newvel and if Newvel enters accommodating is a best response for Microhard. So, the combination (Enter, Accommodate) is a Nash equilibrium.

What about the combination (Stay Out, Fight)? It also satisfies the Nash definition. If Newvel chooses Stay Out, then the Fight strategy is a best response for Microhard, while if Microhard has chosen its Fight strategy, then Stay Out is a best response for Newvel. Therefore, (Stay Out, Fight) is also a Nash equilibrium in strategies. We leave it for the reader to show that the strategy combination (Stay Out, Accommodate) is not a Nash equilibrium.

Again, it is important to understand that a Nash equilibrium is defined in terms of strategies that are best responses to each other. In the second Nash equilibrium (Stay Out, Fight), Microhard never actually takes or implements a fighting action. Instead, it relies fully on the *threat* to do so as a device to deter Newvel from entering. The Nash equilibrium concept is not based on what actions are actually observed in the market place, but rather upon what thinking or strategizing underlies what we observe. This is what is meant when we say we need to define a Nash equilibrium in terms of firms' strategies.

There are two Nash equilibria to this game. There is, however, something troubling about one of these, namely, the Nash equilibrium (Stay Out, Fight). It is true that if Microhard has fully committed itself to the strategy, "Fight", then Newvel's best strategy is "Stay Out". But Newvel might question whether such a commitment is really possible. By adopting the Fight strategy, Microhard's essentially says to Newvel, "I am going to price high so long as you stay out but, if you enter my market, I will cut my price and smash you." The problem is that this threat suffers a serious *credibility* problem. We already know that once Newvel has entered the market, taking action to fight back is not in Microhard's best interest. It does much better by accommodating such entry. Consequently, Microhard does not have an incentive to carry out its threat. So, why should Newvel believe that threat in the first place?

What we have really just discovered is that any Nash equilibrium strategy combination based on non-credible threats is not very satisfactory. This means that we need to strengthen our definition of Nash equilibrium to rule out such strategy combinations. This is where the notion of subgame perfection, or a subgame perfect Nash equilibrium, becomes important. If Microhard adopts a strategy that includes the threat of a fight if entry occurs then if the strategy is subgame perfect it must be optimal for Microhard to fight in the event that Newvel enters. However, this is not the case. Accordingly, the Fight strategy is not subgame perfect.

A Nash equilibrium is said to be subgame perfect or perfect if at that point in the game when a player is called upon to make good on a promise or a threat, doing exactly that and fulfilling the promise or threat is what would be the player's best response. In other words, if any promises or threats are made in one period, carrying them out is still part of a Nash equilibrium in a later period should the occasion arise to do so.

The reason we originally found two Nash equilibria in the game above is that we did not apply this notion of subgame perfection. Strategies that employ threats over future actions can be more difficult to identify in the matrix representation of the game and hence it is more difficult to test for subgame perfection using this representation of the game. It is for this reason that for dynamic games we prefer instead to use an extensive or tree representation of the game.

The extensive form of a game is comprised of dots, branches and vectors of payoffs. The dots are called nodes and describe where we are in the game. They are labeled by which firm makes the move at that position—N for Newvel and M for Microhard, in our case. The branches that are drawn from a node represent the choice of actions available to the player at that node. Each branch points either to another node, where further action takes place, or to a vector of payoffs (Newvel's payoff shown first), which means that this particular choice of action has ended the game. Finally, at any node players know about the course of play that has led to that node. The extensive form of the Microhard-Newvel game is shown in Figure 11.3.

When we represent a sequential game in extensive form it is easy to identify a subgame. A subgame is defined as a single node and all the actions that flow from that node. In the extensive game illustrated in Figure 11.3, there are two subgames. There is the full game

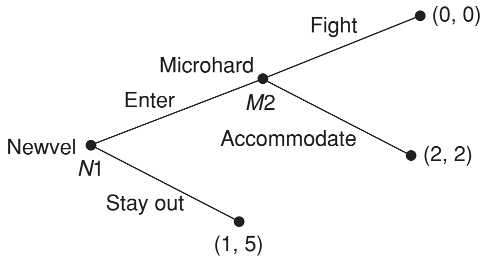

Figure 11.3 The extensive form of the Microhard–Newvel game

starting from node $N1$ (the full game is always a subgame). Then there is the subgame starting at node $M2$, and including all subsequent actions that flow from this node. A strategy combination is subgame perfect if the strategy for each player is a best response against the strategies of the other players for every subgame of the entire game. In the case at hand, it is readily apparent that for the subgame beginning at node 2, the best response strategy for Microhard is Accommodate and *not* Fight. Hence, the strategy combination (Stay Out, Fight) cannot correspond to a subgame perfect equilibrium. The only such equilibrium in this case is that of (Enter, Accommodate).

There is an important technique for solving games with a finite number of nodes. In such games, the simplest way to identify the subgame perfect equilibria is to work backwards. This takes advantage of the property that a subgame perfect equilibrium strategy combination must be a Nash equilibrium in each subgame. In our example, we first calculate the equilibrium for the subgames starting at node $M2$. This gives the unique strategy combination (Enter, Accommodate) and the associated payoff (2,2). We then use these payoffs to determine Newvel's payoffs *if* the game proceeds either to $N1$ or $M2$, effectively creating a shorter game. In our example, this is the full game starting at node $N1$, at which Newvel moves. Obviously, it will choose Enter. In other words, this procedure has eliminated the combination (Stay Out, Fight) as a perfect Nash equilibrium.

There is an important technique for solving games with a finite number of nodes. In such games, the simplest way to identify the subgame perfect equilibria is to work backwards eliminating branches that will not be taken until we have reduced the game tree to having a single branch from each node. This takes advantage of the property that a subgame perfect equilibrium strategy combination must be a Nash equilibrium in each subgame. In our example, we start at node $M2$. We have already seen that we can eliminate the "Fight" branch, leaving only the single "Accommodate" branch from node $M2$. Now pass down the tree to node $N1$. Newvel now knows that Stay Out leads to a payoff of 1, while Enter leads to $M2$ and to Accommodate by Microhard, giving Newvel a payoff of 2. So the Stay Out branch can be eliminated. The game tree now has a single branch from $N1$ and a single branch from $M2$, so we have solved the game. Newvel chooses Enter and Microhard chooses to Accommodate. In other words, this procedure has eliminated the combination (Stay Out, Fight) as a perfect Nash equilibrium.

11.3

Practice Problem

Centipede is a well-known variant of games involving a chance to "grab a dollar." The game is played as between two players, as follows. A neutral third party, call it Nature, puts \$1 on the table. Player 1 can either "grab" this dollar or "wait." If player 1 takes the dollar, the game is over and player 1 gets \$1 and player 2 obviously gets nothing. However, it is completely understood that, if player 1 waits, Nature will *triple* the amount on the table to \$3. At that point, it becomes player 2's turn to move. Her options are as follows. She can either take the entire \$3 for herself or, share the money equally with player 1.

a. Construct the 2×2 payoff matrix for this game taking player 1's actions to be either Grab, or Wait, and player 2's actions to be either Grab (the whole \$3), or Share. Assume the payoffs are equal to the amount of money the player receives.
b. Draw the game in its extensive form.
c. Suppose that player 2 promises player 1 that she will take the action, Share, if player 1 waits. Is this promise credible? Why or why not?

11.4 THE CHAIN STORE PARADOX

In the Microhard and Newvel game there is just one market and one potential entrant, and fighting the entrant was not an optimal response to entry. However, what if Microhard faced more than one entrant? Perhaps fighting one entrant builds a reputation for aggressive behavior that will scare off later entrants. The consideration of the reputational effects of fighting may change Microhard's optimal strategy. Taking predatory action against a rival—costly though it is—could be useful ***if*** it serves to make the threat credible against *other* rivals, either those in other markets or those who may appear later in time. If we introduce this possibility into our example, could Microhard's threat to fight become credible because the subsequent gains in other markets from establishing a reputation as a fighter are sufficiently large? In other words, could reputation effects make Fight credible and the strategy combination (Stay Out, Fight) subgame perfect?

The fact that extension of the above game to many markets (distributed over time or space) and to other rivals may *not* lead to a different outcome is a famous result dubbed by Selten as *The Chain Store Paradox*.[3] To see the logic of this puzzling result, consider a situation in which Microhard has established operating units in each of 20 markets, perhaps 20 different cities. In each city, Microhard faces potential entry by a single, small competitor. At the moment, none of these potential competitors has the capital to start operations. However, as time goes on, one after another will raise the necessary funds. To make matters simple, assume that the payoffs in each of the 20 markets are just as in the payoff matrix of the previous section. The question facing Microhard is how to react to this sequence of potential entrants. In particular, should Microhard adopt an aggressive response to the first entrant and drive it out of business? Will this tactic earn Microhard a reputation for ruthlessness such that subsequent entrants in its other markets will get the message and choose not to enter?

Again, working backwards can help us identify a subgame perfect strategy. So, let's start with one possible scenario in which Microhard is facing the last potential entrant in the final, twentieth market. It is possible that Microhard has followed through on its threat to cut price and drive out any entrant not just in the first market, but also in all previous nineteen markets. This is a possible path in the game and we are interested if such an aggressive response to entry can convince the last potential entrant to stay out, so that Microhard would be spared a fight in this final case.

However, consider the viewpoint of the entrant to the twentieth market. This firm will realize that because there are no subsequent entrants, it is playing a game that is exactly the one-period game we discussed in the previous section. So, using the argument of the previous section, this last entrant will understand that Microhard has an incentive to accommodate its entry. Microhard's profit is greater if it follows a "live and let live" strategy in this last case[4] because it cannot gain from any further demonstration of its ruthlessness. There are no other entrants left to impress! Since Microhard's only possible reason to respond to entry with aggressive price cutting is to establish a reputation for toughness, and because, after the twentieth market battle, having such a reputation does it absolutely no good, Microhard has to accommodate the entrant in this last market. The entrant will understand this and of course enter. Note that the threat to fight in the twentieth market is ***not*** credible even though

[3] Selten (1978). We have obviously limited ourselves here to consideration of finitely repeated games only. Infinitely repeated games are considered in the next chapter.

[4] Implicit here is the presumption that accommodating an entrant is in the short run more profitable than engaging in a price war.

Microhard has already done so in 19 prior cases! The threat to fight is no more credible by Microhard fighting only in eighteen markets, or seventeen, or just one. If a record of 19 previous rounds of aggressive price-cutting does not convince the final entrant, nothing will.

One might think that this simply implies that Microhard cannot credibly threaten the potential entrant in its final market because there are no more entrants, but it is still possible to deter the entry of earlier rivals by means of a threat. To see why this is not possible, consider the potential entrant in the nineteenth rather than the twentieth market. Once again, let's take the extreme case in which Microhard has taken predatory or fighting action in the prior eighteen markets. Now the potential entrant in the nineteenth market can reason as well as we can. As a result, this firm will work out the logic of the preceding case and rightfully conclude that Microhard will not fight in the twentieth market. The entrant in the nineteenth or next-to-last market will then reason as follows: "Microhard will let the last rival firm survive because it is pointless to cut price at that point to gain a tough reputation. Since I know that the entry of the last rival will not be challenged, there is, in fact, no reason for Microhard to act tough on me. Its only reason to do so would be to convince the entrant in the next market. Since this is not possible, the only justification for fighting in market nineteen has been removed." Once again, Microhard's promise to fight if entry occurs is not credible. It gains Microhard nothing by way of a demonstration to the next rival. Absent such a reputation effect, Microhard's best response to entry in the nineteenth market is again to accommodate. Knowing this, the potential entrant in market nineteen will enter.

We can continue in this fashion repeatedly, bringing us back all the way to the initial market. At every stage, we will find that a strategy to fight after entry occurs is not subgame perfect and accordingly not credible. This will be just as true in the first market as in the last. There is no way for the incumbent to threaten credibly an aggressive low-price response to entry.

At this point the only subgame perfect Nash strategy equilibrium is one in which entry occurs and fighting never happens. If this were the end of the story, our interest in the predatory conduct would certainly be very low. Why should we worry about an event that presumably never occurs? The answer is that there may be ways to make the threat to fight credible other than actual fighting, itself. A firm's predatory efforts, no matter what form it takes, will work only if they are *credible* to actual and potential rivals and so influence their beliefs about competing in the market.[5] Predatory conduct that is credible is what we will investigate in the next two chapters.

Summary

Sequential market games are different from simultaneous ones. Moreover, the effect of changing from simultaneous to sequential play differs depending on whether the strategic variable of choice is quantity or price. The basic sequential quantity game, typically referred to the Stackelberg model, confers a large advantage to the firm that chooses production first. In the linear demand and cost case, the first mover in a Stackelberg game produces the monopoly output. The follower produces only half this much. Prices are lower than in the basic Cournot model but the large market share of the first mover gives that firm an increase in profit over what it would earn in the simultaneous production game.

In contrast, a sequential price game with differentiated products yields higher profits for both firms than either would earn if prices were set simultaneously. Moreover, in this case, it is the firm that sets price last that does best. Sequential price games can confer a second-mover as opposed to a first-mover advantage. This advantage to the

[5] Schelling (1960) contains early and lasting contributions to developing equilibrium notions for dynamic games. See also Tirole (1988) and Rasmusen (2007).

second mover diminishes as the products become closer substitutes for each other.

Crucial to any sequential game is the issue of commitment. How do firms establish themselves as leaders or followers? How can a firm commit to its initial choice of output or price in a way that a rival finds credible? This issue is best explored by considering the game in its extended form and identifying strategy combinations that are subgame perfect, i.e., strategies that call for actions at later points in the game in which those actions continue to be optimal when the time comes to take them given the history of play up to that date.

Threats and promises of later punishments and rewards are particularly important in games in which one firm is trying to prevent another from entering its market (or perhaps trying to induce it to leave). The question again is whether such threats and promises can be made credible. If they can then incumbent firms may be able to maintain their dominant position in an industry and not fear competitive entry. This is the subject of our next chapter.

Problems

1. Consider a Stackleberg game of quantity competition between two firms. Firm 1 is the leader and firm 2 is the follower. Market demand is described by the inverse demand function $P = 1{,}000 - 4Q$. Each firm has a constant unit cost of production equal to 20.
   a. Solve for Nash equilibrium outcome.
   b. Suppose firm 2's unit cost of production is $c < 20$. What value would c have so that in the Nash equilibrium the two firms, leader and follower, had the same market share?

2. Let's return to Tuftsville (Chapter 10) where everyone lives along Main Street, which is 10 miles long. There are 1,000 people uniformly spread up and down Main Street, and each day they each buy fruit smoothie from one of the two stores located at either end of Main Street. Customers ride their motor scooters to and from the store and the motor scooters use \$0.50 worth of gas per mile. Customers buy their smoothies from the store offering the lowest price, which is the store's price plus the customer's travel expenses getting to and from the store. Ben owns the store at the west end of Main Street and Will owns the store at the east end of Main Street. The marginal cost of a smoothie is constant and equal to \$1 for both Ben and Will. In addition, each of them pays Tuftsville \$250 per day for the right to sell smoothies.
   a. Ben sets his price p_1 first and then Will sets his price p_2. After the prices are posted consumers get on their scooters and buy from the store with the lowest price including travel expenses. What prices will Ben and Will set?
   b. How many customers does each store serve and what are their profits?

3. In Centipede[6] there are two players. Player 1 moves first, player 2 moves second. After at most two moves, the game ends. The game begins with \$1 sitting on a table. Player 1 can either take the \$1 or wait. If player 1 takes the \$1 the game is over, and player 1 gets to keep the \$1. If player 1 waits the \$1 quadruples to \$4. Now it is player 2's turn. Player 2 can either take the entire \$4 or split the \$4 evenly with player 1.
   a. Draw the extensive form for the game of Centipede.
   b. What is the equilibrium to this game? Can player 2's strategy of splitting the money ever be a part of an equilibrium outcome to the game?
   c. Now suppose that Centipede has three moves. Player 2 can now either wait, split the money or take the \$4. If player 2 waits then the money on the table quadruples again and player 1 can either take it all or split it. Draw the extensive form for the new game and solve for the equilibrium outcome.

4. Dry Gulch has two water suppliers. One is Northern Springs whose water is crystal clear but not carbonated. The other is Southern Pelligrino whose water is naturally carbonated but also somewhat "hard." The marketing department of each firm has worked out the following profit matrix depending on the

[6] This game was first introduced by Rosenthal (1982).

price per 2-gallon container charged by each firm. Southern Pellegrino's profits are shown as the first entry in each pair.

Southern Pelligrino's price	*Northern Springs price* 3	4	5	6
3	24,24	30,25	36,20	42,12
4	25,30	32,32	41,30	48,24
5	20,36	30,41	40,40	50,36
6	12,42	24,48	36,50	48,48

a. What is the Nash equilibrium if the two firms set prices simultaneously?
b. What is the Nash equilibrium if Northern Springs must set its price first, and stick with it, and Southern Pelligrino is free to respond as best it can to Northern Springs' price?
c. Show that choosing ***price*** first is a disadvantage for Northern Springs? Why is this the case?

5. Suppose that firm 1 can choose to produce either good *A* or good *B* or both goods or *nothing*. Firm 2, on the other hand, can produce only good *C* or *nothing*. Firms' profits corresponding to each possible scenario of goods for sale are described in the following table:

Product selection	*Firm 1's profit*	*Firm 2's profit*
A	20	0
A,B	18	0
A,B,C	2	–2
B,C	–3	–3
C	0	10
A,C	8	8
B	11	0

a. Set up the normal form game for when the two firms simultaneously choose their product sets. What is the Nash equilibrium (or equilibria)?
b. Now suppose that firm 1 can commit to its product choice before firm 2. Draw the extensive form of this game and identify its subgame perfect Nash equilibrium. Compare your answer to (a) and explain.
c. The game is like the one in (b) only now suppose that firm 1 can reverse its decision after observing firm 2's choice and this possibility is common knowledge. Does this affect the game? If so, explain the new outcome? If not, explain why not.

6. Find three examples of different ways individual firms or industries can make the strategy "This offer is good for a limited time only" a credible strategy.

7. The Gizmo Company has a monopoly on the production of gizmos. Market demand is described as follows: at a price of $1,000 per gizmo 25,000 units will be sold whereas at a price of $600 30,000 will be sold. The only costs of production are the initial sunk costs of building a plant. Gizmo Co. has already invested in capacity to produce up to 25,000 units.
a. Suppose an entrant to this industry could capture 50% of the market if it invested in $10 million to construct a plant. Would the firm enter? Why or why not?
b. Suppose Gizmo could invest $5 million to expand its capacity to produce 40,000 gizmos. Would this strategy be a profitable way to deter entry?

References

Gabszewicz, J., L. Pepall, and J.-F. Thisse. 1992. "Sequential Entry with Brand Loyalty Caused by Consumer Learning-By-Doing." *Journal of Industrial Economics* 60 (December): 397–416.

Goolsbee, A. and A. Petrin. 2004. "The Consumer Gains from Direct Broadcast Satellite and Competition with Cable TV." *Econometrica* 72 (March): 351–81.

Lieberman, Marvin B. and David B. Montgomery. 1998. "First Mover Advantages." *Strategic Management Journal* 19 (December): 1111–25.

Rasmusen, E. 2007. *Games and Information*. 4th edition. Cambridge, MA: Blackwell.

Rosenthal, R. W. 1981. "Games of Perfect Information, Predatory Pricing, and the Chain Store Paradox." *Journal of Economic Theory* 25 (August): 92–100.

Schelling, T., 1960. *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

Selten, R. 1978. "The Chain Store Paradox." *Theory and Decision* 9 (April): 127–59.

Stackelberg, H. von 1934. *Marktform und Gleichgewicht*. Berlin and Vienna. English translation by A. T. Peacock, 1952, London: William Hodge.

Sutton, J. 1991. *Sunk Cost and Market Structure*. Cambridge, MA: MIT Press.

Tirole, J. 1988. *The Theory of Industrial Organization*. Cambridge, MA: MIT Press.