10

Price Competition

The two largest makers of x86-based microprocessors for use in computers are Intel and Advanced Micro Devices (AMD). While Intel is far larger with nearly 75 percent of the market, AMD's roughly 25 percent share gives it ample market clout and both firms are keenly aware of each other's presence. The strategic interaction that results often appears as a price war in which each firm tries to win customers from the other by offering steep price cuts. The latest such event was played out in 2006 and into 2007. AMD fired the first shot in May 2006, when it dropped the price of its dual-core Athlon 64 X2 5,000+ to $301 from $696. In July of that year, AMD then cut the price of the Athlon 64 X2 4,600+ by 57 percent to $240 from $558. A few days later, Intel marked down the price of its Intel Pentium D processor 960 by 40 percent to $316. Intel also cut prices on many older processors from 50 to 60 percent. Modest price cuts by both firms followed until April of 2007. In that month, AMD slashed prices on its Athlon processors by 20 to 50 percent in anticipation of coming Intel price reductions. Those anticipations were realized later that month when Intel launched its new Core 2 Duo processor line with discounts of 40 to 50 percent.

In the market for high-speed processors the major buyers are computer producers such as Dell, Compaq, and Gateway. These are savvy consumers who know quality and who, if the product is good, will buy from the lowest-priced provider. Intel and AMD post their prices and then try to adjust their production to the demand those prices elicit. This is the way competition works in many markets, including restaurants, electricians, moving companies, consulting firms, and financial services. However, it is quite different from the way competition works in the Cournot model. There each competing firm independently produces an amount of output so that production occurs before the consumer makes a purchase. It is only afterwards that the price adjusts so that consumers will buy the total output that the firms produced. This is what is meant by the phrase "the price adjusts so that the market clears," and it is perhaps an apt description of how the market works in the automobile, aircraft and other manufacturing industries.

In a monopolized market, it would of course make no difference whether the firm initially set a price and then produced whatever amount consumers demanded at that price or, instead, first chose its production and let the price settle at whatever level was necessary to sell that output. When a profit-maximizing monopolist optimally sets price, that choice will imply, via the demand curve, an output level, which is precisely the same amount the monopolist would have chosen if instead it had initially chosen the profit-maximizing amount to produce.

However, once we leave the world of monopoly the equivalence of price and output strategies vanishes. In oligopolistic markets it matters very much whether firms compete in terms of quantities, as in Cournot, or like the high-speed Internet providers in terms of price. The nature of the competition is markedly different. To understand these differences we begin by turning the Cournot model on its head, and look at the same market in which two firms produce identical products but now compete by first setting prices instead of production levels. This is known as the Bertrand model. Later in the chapter we allow the products to be less than perfect substitutes, or to be differentiated. As in Chapter 9, we also focus on static or simultaneous models of price competition limited to a single market period.

10.1 THE BERTRAND DUOPOLY MODEL

The standard Cournot duopoly model, recast in terms of price strategies rather than quantity strategies, is typically referred to as the Bertrand model. Joseph Bertrand was a French mathematician who in 1883 reviewed and critiqued Cournot's work nearly fifty years after its publication in an article in the *Journal des Savants*. Bertrand was critical of mathematical modeling in economics, and to prove his point he analyzed the Cournot model in terms of prices rather than quantities. The legacy of Bertrand is not, however, his criticism of what he termed "pseudo-mathematics" in economics. Instead, Bertrand's contribution was the recognition that using price as a strategic variable is different from using quantity as the strategic variable, and that this difference is worth investigating.

Let us rework the Cournot duopoly model with each firm choosing the price it will charge rather than the quantity it will produce. Otherwise, the model and the assumptions are exactly the same as before. There are two firms who choose their strategies simultaneously. Each produces the identical good at the same, constant marginal cost c. Each firm knows the structure of market demand. In the Cournot model we described demand by a linear inverse demand function $P = A - BQ$. When firms choose prices, rather than quantities, it is more convenient to rewrite the demand function and have total output as the dependent variable.[1] Therefore, we have:

$$Q = a - bP; \quad \text{where } a = \frac{A}{B} \text{ and } b = \frac{1}{B} \tag{10.1}$$

Consider the pricing problem first from firm 2's perspective. In order to determine its best price response to its rival firm 1, firm 2 must first work out the demand for its product *conditional* on both its own price, denoted by p_2, and firm 1's price, denoted by p_1. Rationally speaking, firm 2's reasoning would go as follows. If $p_2 > p_1$, firm 2 will sell no output. The product is homogenous so that consumers always buy from the cheapest source. Setting a price above that of firm 1 therefore means that firm 2 will serve no customers. The opposite is true if $p_2 < p_1$. When firm 2 sets the lower price, it will supply the entire market, and firm 1 will sell nothing. Finally, if $p_2 = p_1$, the two firms will split the market evenly. When both firms charge identical prices the same number of customers patronizes both producers.

[1] When firms choose quantities (as in Cournot's model) it is often easier to work with the inverse demand curve and treat price as the dependent variable. When firms select prices, as in Bertrand's analysis, it is often best to let quantity be the dependent variable.

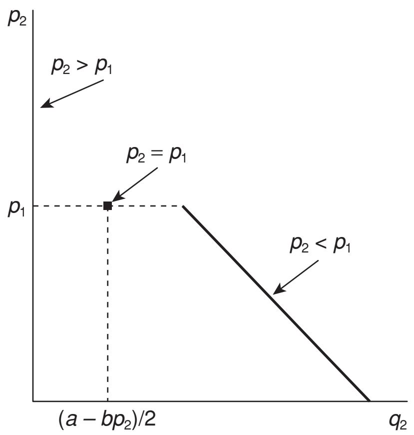

Figure 10.1 Firm 2's demand curve in the Bertrand model
Industry demand equal to $a - bp_2$ is the same as firm 2's demand for all p_2 less than p_1. If $p_2 = p_1$, then the two firms share equally the total demand. For $p_2 > p_1$, Firm 2's demand falls to zero.

The foregoing reasoning tells us that demand for firm 2's output, q_2, may be described as follows:

$$q_2 = 0 \qquad \textit{if } p_2 > p_1$$

$$q_2 = \frac{a - bp_2}{2} \qquad \textit{if } p_2 = p_1$$

$$q_2 = a - bp_2 \qquad \textit{if } p_2 < p_1$$

As Figure 10.1 shows, this demand function is *not* continuous. For any p_2 greater than p_1, demand for q_2 is zero. But when p_2 falls and becomes exactly equal to p_1, demand jumps from zero to $\frac{a - bp_2}{2}$. When p_2 then falls still further so that it is below p_1, demand then jumps again to $a - bp_2$.

This discontinuity in firm 2's demand curve was not present in the quantity version of the Cournot model, and it turns out to make a crucial difference in terms of firms' strategies. The discontinuity in demand carries over into a discontinuity in profits. Firm 2's profit, Π_2, as a function of p_1 and p_2 is

$$\Pi_2(p_1, p_2) = 0 \qquad \textit{if } p_2 > p_1$$

$$\Pi_2(p_1, p_2) = (p_2 - c)\frac{a - bp_2}{2}. \qquad \textit{if } p_2 = p_1$$

$$\Pi_2(p_1, p_2) = (p_2 - c)(a - bp_2) \qquad \textit{if } p_2 < p_1$$

To find firm 2's *best response* function, we need to find the price p_2 that maximizes firm 2's profits $\Pi_2(p_1, p_2)$ for any given choice of p_1. For example, suppose firm 1 chooses a very

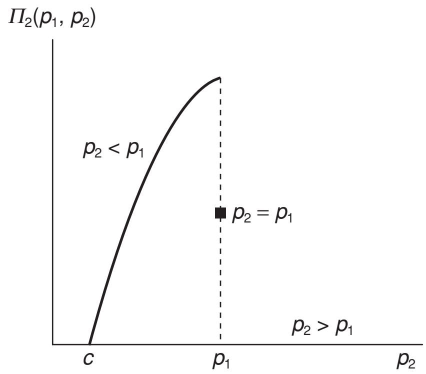

Figure 10.2 Firm 2's profits as a function of P_2 when firm 1 prices above cost but below the pure monopoly price
Firm 2's profits rise continuously as its price rises from the level of marginal cost, c, to just below firm 1's price. When p_2 equals p_1, firm 2's profits fall relative to those earned when p_2 is just below p_1. For p_2 greater than p_1, firm 2 earns zero profits.

high price—higher even than the pure monopoly price, which in this case is $p^M = \frac{a + bc}{2b}$.[2] Since firm 2 could capture the entire market by selecting any price lower than p_1, its best response would be to choose the pure monopoly price p^M, and thereby earn the pure monopoly profits.

Conversely, what if firm 1 set a very low price, say one below its unit cost c? This would be an unusual choice, but if we wish to construct a complete *best response* function for firm 2, we must determine its value for *all* the possible values p_1 can take. If $p_1 < c$, firm 2 is best setting its price at some level above p_1. This will mean that firm 2 will sell nothing and earn zero profits. The alternative of setting $p_2 < p_1 < c$ will lead to *negative* profits: firm 2 sells a positive amount of output, but at a price below unit cost so that firm 2 will loses money on each unit sold.

What about the more likely case in which firm 1 sets its price above marginal cost c but either equal to or below the pure monopoly price p^M? How should firm 2 optimally respond in these circumstances? The simple answer is that it should set a price *just a bit less than* p_1. The intuition behind this strategy is illustrated in Figure 10.2, which shows firm 2's profit given a price p_1, satisfying the relationship $\frac{a + bc}{2b} \geq p_1 > c$.

Note that firm 2's profits rise continuously as p_2 rises from c to just below p_1. Whenever p_2 is less than p_1, firm 2 is the only company that any consumer buys from. However, when p_1 is less than or equal to p^M, the monopoly power that firm 2 obtains from undercutting p_1 is constrained. In particular, the firm cannot sell at the pure monopoly price, p^M and earn the associated profit because at that price, firm 2 would lose all its customers. Still, the firm will wish to get as close to that result as possible. It could, of course, just match firm 1's price exactly. But whenever it does so it shares the market equally with its rival. If, instead of

[2] This is, of course, the same monopoly price as we showed in Chapter 8 for the quantity version of the model with the notational change that $a = A/B$, and $b = 1/B$.

setting $p_2 = p_1$, firm 2 just *slightly* reduces its price below the p_1 level, it will double its sales while incurring only an infinitesimal decline in its profit margin per unit sold. This is a trade well worth the making as Figure 10.2 makes clear. In turn, the implication is that for any p_1 such that $p^M \geq p_1 > c$, firm 2's best response is to set $p_2^* = p_1 - \varepsilon$, where ε is an arbitrarily small amount.

The last case to consider is the case in which firm 1 prices at cost so that $p_1 = c$. Clearly, firm 2 has no incentive to undercut this value of p_1. To do so, would only lead to losses for firm 2. Instead, firm 2 will do best to set p_2 either equal to or above p_1. If it prices above p_1, firm 2 will sell nothing and earn zero profits. If it matches p_1, it will enjoy positive sales but break even on every unit sold. Accordingly, firm 2 will earn zero profits in this latter case, too. Thus, when $p_1 = c$, firm 2's *best response* is to set p_2 either greater than or equal to p_1.

Our preceding discussion may be summarized with the following description of firm 2's best price response

$$
\begin{aligned}
&p_2^* = \frac{a + bc}{2b} && \textit{if } p_2 > \frac{a + bc}{2b} \\
&p_2^* = p_1 - \varepsilon && \textit{if } c < p_1 \leq \frac{a + bc}{2b} \\
&p_2^* \geq p_1 && \textit{if } c = p_1 \\
&p_2^* > p_1 && \textit{if } c > p_1 \geq 0
\end{aligned}
$$

By similar reasoning, firm 1's best response p_1^* for any given value of p_2 would be given by:

$$
\begin{aligned}
&p_1^* = \frac{a + bc}{2b} && \textit{if } p_1 > \frac{a + bc}{2b} \\
&p_1^* = p_2 - \varepsilon && \textit{if } c < p_2 \leq \frac{a + bc}{2b} \\
&p_1^* \geq p_2 && \textit{if } c = p_2 \\
&p_1^* > p_2 && \textit{if } c > p_2 \geq 0
\end{aligned}
$$

We may now determine the Nash equilibrium for the duopoly game when played in prices. We know that a Nash equilibrium is one in which neither firm has an incentive to change its strategy. For example, the strategy combination $[p_1 = \frac{a + bc}{2b}, p_2 = \frac{a + bc}{2b} - \varepsilon]$ *cannot* be an equilibrium. This is because in that combination, firm 2 undercuts firm 1's price and sells at a price just below the monopoly level. However, in such a case, firm 1 would have no customers and earn zero profit. Since firm 1 could earn substantial profit by lowering its price to just below that set by firm 2, it would wish to do so. Accordingly, this strategy cannot be a Nash equilibrium. To put it another way, firm 2 could never expect firm 1 to set the monopoly price of $p_1 = (a + c)/2b$ precisely because firm 1 would know that so doing would lead to zero profit as firm 2 would undercut that price by a small amount ε and steal all firm 1's customers.

Reality Checkpoint
Flat Screens and Flatter Prices

Perhaps one of the most dramatic examples of Bertrand competition comes from the market for flat screen TVs. Such screens use one of three basic technologies. These are: liquid crystal display (LCD), digital light processing (DLP), and plasma. Initially, the technologies were such that LCD worked best on small screens, plasma worked best on medium-sized screens, and DLP worked best with large screens. In addition, DLP screens were not as flat. However, over time, the differences between the three types have diminished. The result has been the eruption of a sever price war. From mid-2003 to mid-2005, prices for new TVs based on these technologies fell by an average of 25 percent per year. Fifty-inch plasma TVs that sold for $20,000 in 2000 were selling for $4,000 in 2005. Nor has this pressure let up. In November 2006, Syntax-Brillian cut the price on its 32-inch LCD TV by 40 percent. Sony and other premium brands were forced to follow suit. Prices on all models fell further. Indeed, when Sony was rumored to be thinking of further reducing its 50-inch price to $3,000, James Li, the chief executive of Syntax-Brillian, was quoted as saying, "If they go to $3,000, I will go to $2,999." Bertrand would have been proud.

Source: D. Darlin, "Falling Costs of Big-Screen TV's to Keep Falling" and "The No-Name Brand Behind the Latest Flat-Panel Price War," *New York Times*, August 20, 2005, p. C1 and February 12, 2007, p. C1.

As it turns out, there is one and only one Nash equilibrium for the Bertrand duopoly game described above. It is the price pair, ($p_1^* = c$, $p_2^* = c$).[3] If firm 1 sets this price in the expectation that firm 2 will do so, and if firm 2 acts in precisely the same manner, neither will have an incentive to change. Hence, the outcome of the Bertrand duopoly game is that the market price equals marginal cost. This is, of course, exactly what occurs under perfect competition. The only difference is that here, instead of many small firms, we have just two firms each of which is large relative to the market.

It is no wonder that Bertrand made note of the different outcome obtained when price replaces quantity as the strategic variable. Far from being a cosmetic or minor change, this alternative specification has dramatic impact. It is useful, therefore, to explore the nature and the source of this powerful effect more closely.

Practice Problem 10.1

Let the market demand for carbonated water be given by $Q^D = 100 - 5P$. Let there be two firms producing carbonated water, each with a constant marginal cost of 2.

a. What is the market equilibrium price and quantity when each firm behaves as a Cournot duopolist choosing quantities? What are firms' profits?
b. What is the market equilibrium price and quantity when each firm behaves as a Bertrand duopolist choosing price? What are firms' profits?

[3] If prices cannot be set continuously but only in say whole dollar amounts, then, two other possible Nash equilibria also exist. One is when firm 1 sets $p_1 = c$, and firm 2 sets its price $1 above marginal cost. The other is with Firm 2 setting $p_2 = c$, and now firm 1 pricing $1 above marginal cost. Profit is zero for each firm in all three cases.

10.2 BERTRAND RECONSIDERED

Like its Cournot cousin, the Bertrand analysis of a duopoly market is not without its critics. One chief source of criticism with the Bertrand model is its assumption that *any* price deviation between the two firms leads to an immediate and complete loss of demand for the firm charging the higher price. It is, of course, this assumption that gives rise to the discontinuity in both firms' demand and profit functions. It is also this assumption that underlies our derivation of each firm's best response function.

There are two very sound reasons why a firm's decision to charge a price higher than its rival would not result in the complete loss of all its customers. One reason is that typically the rival firm does not have the capacity to serve all of the customers who demand the product or service at its low price.[4] The second is that consumers many not view the two products as perfect substitutes.

To see the importance of capacity constraints, consider the fictional case of a small New England area with two ski resorts, Pepall Ridge and Snow Richards, each located on different sides of Mount Norman. Skiers regard the services at these resorts to be the same and will choose when possible to ski at the resort that quotes the lowest lift ticket price. Pepall Ridge is a small resort that can accommodate 1,000 skiers per day. Snow Richards is slightly bigger and can handle 1,400 skiers a day. Skiing on Mount Norman has this year become extremely popular. The demand for skiing services on Mount Norman is estimated to be $Q = 6{,}000 - 60P$, where P is the price of a daily lift ticket and Q is number of skiers per day.

The two resorts compete in price. Suppose that the marginal cost of providing lift services is the same at each resort and is equal to $10 per skier. However, the outcome in which each resort sets a price equal to marginal cost *cannot* be a Nash equilibrium. Demand when the price of a lift ticket is equal to $10 would be equal to 5,400 skiers, far exceeding the total capacity of the two resorts. To be sure, if each resort had understood the extent of demand, each might have built additional lifts, ski runs, and parking facilities and had greater capacity. Nevertheless, it is still not likely that the Nash equilibrium will end up with each resort setting its price of lift ticket equal to the marginal cost of $10 per skier. Why? Think of it this way. If Pepall Ridge sets a price of $11, does it make sense for Snow Richards to set a price of $10.90? This makes sense only if Snow Richards can serve all the skiers who would come at this lower price and it cannot! Should it try and build that much capacity? That would be fairly short-sighted behavior for Snow Richards. For if Pepall Ridge is serving no skiers at a price of $11 while Snow Richards is serving all the skiers at a price of $10.90, Pepall Ridge will have an incentive to lower its price to $10.80 and get back as many customers as it can. However to serve all the customers Pepall Ridge would also need an increased capacity.

This reasoning suggests that the pressure for each firm, Pepall Ridge and Snow Richards, to cut price to marginal cost rests on each having sufficient capacity to serve the entire market demand at the competitive price. However, if each had that capacity then in equilibrium when each charges the competitive price of $10, the market is split and each serves only 2,700. It is unlikely that each resort would want to build capacity of 5,400 if each will only serve 2,700 in equilibrium. If that is the case, there is little pressure on price to fall to the marginal cost of $10.

[4] Edgeworth (1897) was one of the first economists to investigate the impact of capacity constraints on the Bertrand analysis.

More generally, denote as Q^C, the competitive output or the total demand when price is equal to marginal cost, i.e., $Q^C = a - bc$. If neither firm has the capacity to produce Q^C but instead can each produce only a smaller amount, then the Bertrand outcome with $p_1 = p_2 = c$ will *not* be the Nash equilibrium. In a Nash equilibrium, it must be the case that each firm's choice is a *best response* to the strategy of the other. Consider then the original Bertrand solution with prices equal to marginal cost c and profit at each firm equal to zero. When there is a capacity constraint so that neither firm can serve the *entire* market at the competitive price, firm 2 can contemplate *raising* its price. If firm 2 sets p_2 above marginal cost, and hence above p_1 it would surely lose some of its customers. But it would not lose all of them. Firm 1 *does not have the capacity* to serve them. Some customers would remain with firm 2. Yet firm 2 is now earning some profit from each such customer ($p_2 > c$) implying that its total profit is now positive whereas before it was zero. It is evident therefore that $p_2 = c$, is not a best response to $p_1 = c$. Accordingly, the strategy combination ($p_1 = c$, $p_2 = c$) cannot be a Nash equilibrium if there are capacity constraints.

When capacity constraints come into play, the game between the two firms really becomes a two-stage one. In the first stage, the two firms choose capacity levels. In the second, they then compete in price. Examining the outcomes corresponding to the strategic combinations in such games is tricky. However, neither firm is likely to acquire enough capacity in stage one to serve the entire market when pricing at marginal cost in stage two. Yet if neither acquires that large amount of capacity then the Bertrand solution of each charging a price equal to marginal cost *cannot* be a Nash equilibrium. We will return to the issue of capacity choice in Chapter 12. It is worth noting at this point that the equilibrium in a model of price competition *with* capacity constraints takes us away from the simple or standard Bertrand outcome, and closer to the outcome in the Cournot model.[5]

To see this point more clearly let's return to the ski resort competition between Pepall Ridge and Snow Richards. We assume that at any price at which a resort has demand beyond its maximum capacity, the skiers that the resort serves are those skiers who are the most eager and who have the highest willingness to pay. For example, if each resort sets a lift price of \$50, total market demand is 3,000. This is beyond the total capacity of 2,400 and, therefore, each resort will need somehow to ration or choose which skiers will actually ski. Our assumption, sometimes called the efficient rationing assumption, is that the resorts will do this by serving customers in order of their willingness to pay. Pepall Ridge will choose those 1,000 potential skiers with the 1000 highest willingesses to pay. Given efficient rationing we can derive the residual demand curve facing Snow Richards at any price.

A price of particular interest is \$60. Suppose then that both resorts have set $p_1 = p_2 = \$60$. At these prices, total demand is equal to 2,400, which is just equal to the total capacity of the two resorts. Is this a Nash Equilibrium? We can answer this question by using the logic above to determine the demand function facing Snow Richards when Pepall Ridge sets a price equal to \$60. Under our assumption of efficient rationing, this is shown in Figure 10.3. It is the original demand curve shifted to the left by 1,000 units, i.e. it is $Q = 5{,}000 - 60P$ (or, in inverse form, $P = 83.333 - Q/60$. The marginal revenue curve facing Snow Richards when Pepall Ridges charges a price of \$60 is also shown there.

Note though that while changes in its price also change its quantity demanded, Snow Richards is always constrained to serve no more than its capacity of 1,400. In this light, consider again, the situation in which Snow Richards sets a price just equal to the \$60 that Pepall Ridge is

[5] This result is formally modeled in a two-stage game in Kreps and Scheinkman (1983).

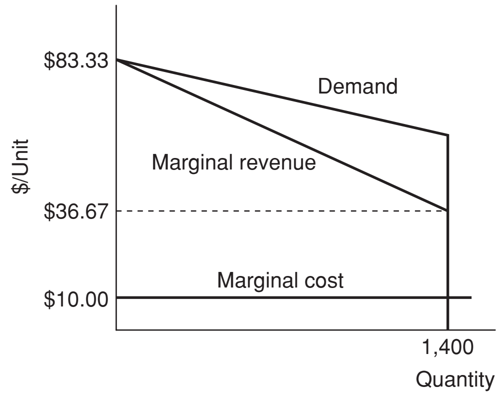

Figure 10.3 Snow Richards residual demand curve

charging. Is this a best response? We check this by asking whether Snow Richards has an incentive to change its price. The answer is no. Lowering its price will not lead to any more customers since Snow Richards is at capacity. Yet raising its price is not an attractive option either. This will lower its demand below capacity of 1,400. Since marginal revenue exceeds marginal cost, losing customers also loses profit. Accordingly, Snow Richards has no incentive either to lower or to raise its price from \$60, assuming that Pepall Ridge is also setting that price. By a similar logic, we can also show that Pepall Ridge has no incentive to change its price from \$60 given that Snow Richards is charging that amount. Therefore, $p_1 = p_2 = \$60$ is the Nash equilibrium for this game.

As noted earlier, the logic of the above example is quite general. Firms competing in prices selling identical products will rarely choose the capacity necessary to serve the total market demand forthcoming at competitive prices. As a result, both output and capacity will be less than the competitive level. In turn, this implies that prices must rise to a level at which demand equals the total industry capacity—a level that is necessarily above marginal cost. Thus, the efficiency property of the Bertrand solution can break down when firms are capacity constrained.

Practice Problem 10.2

Suppose now market demand for skiing increases to $Q^D = 9{,}000 - 60P$. However, because of environmental regulation the two resorts cannot increase their capacities and serve more skiers. What is the Nash equilibrium outcome for this case? That is, what are the profit-maximizing prices set by Pepall Ridge and Snow Richards?

10.3 BERTRAND IN A SPATIAL SETTING

There is a second reason why the simple Bertrand efficient outcome of price equal to marginal cost may not occur. The two firms often do not, as Bertrand assumed, produce identical products. Think of hair salons, for example. No two hair stylists cut and style hair in exactly the same way. Nor will the salons have exactly the same sort of equipment or furnishings. Also, so long as the two firms are not side-by-side, they will differ in their locations. As we saw in Chapter 7, this is often sufficient by itself to generate a preference by some consumers for one salon or the other even when different prices are charged. In short, differences in locations, furnishings, or cutting styles can each be sufficient to permit one salon to price somewhat higher than its rival without immediately losing all of its customers.

We presented a spatial model of product differentiation in Chapter 7. There our aim was to understand the use of such differentiation by a monopoly firm to extract additional surplus from its customers. The same model, however, may also be used to understand the nature of price competition when competing firms market differentiated products. Let's review the basic set-up presented earlier. There is a line of unit length (say one mile) along which consumers are uniformly distributed. This market is supplied by two stores. This time though, the same company does not operate the two stores. Rival firms operate them. One firm—located at the west end of town—has the address $x = 0$. The other—located at the east end of town—has the location, $x = 1$. Each of the firms has the same, constant unit cost of production c.

We define a consumer's "location" in this market to be that consumer's most preferred product, or style. Thus "consumer x" is located distance x from the left-hand end of the market, where distance may be geographic in a spatial model or measured in terms of characteristics in a more general product differentiation sense. While consumers differ regarding which variant or location of the good they consider to be the best, or their ideal product, they are identical in their reservation price V for their most preferred product. We further assume that the reservation price V is substantially greater than the unit cost of production c. Each consumer buys at most one unit of the product. If consumer x purchases a good that is not her ideal product she incurs a utility loss. Specifically, consumer x incurs the cost $t.x$ if she consumes good 1 (located at $x = 0$), and the cost $t(1 - x)$ if she consumes good 2 (located at $x = 1$). If she buys good 1 at price p_1 she enjoys consumer surplus $V - p_1 - t.x$ and if she buys good 2 at price p_2 she enjoys consumer surplus $V - p_2 - t(1 - x_2)$. Of course, she will purchase the good that offers her the greater consumer surplus provided that this is greater than zero. Figure 10.4 describes this market setting.

It bears emphasizing that the concept of location that we have introduced here serves as a metaphor for all manner of qualitative differences between products. Instead of having two stores geographically separated we can think of two products marketed by two different firms that are differentiated by some characteristic, such as sugar content in the case of soft drinks, or fat content in the case of fast food, or fuel efficiency in the case of automobiles. Our unit line in each case represents the spectrum of products differentiated by this characteristic and each consumer has a most preferred product specification on this line. For the case of soft drinks our two firms could be Pepsi and Coca-Cola. For the case of fast food, our two firms could be McDonald's and Burger King, whereas for automobiles our two firms could be Ford and GM.

As in the simple Bertrand model, the two firms compete for customers by setting prices p_1 and p_2, respectively. These are chosen simultaneously, and we want to solve for a Nash equilibrium solution to the game. If $V > c$ then in equilibrium it must be the case that both firms have a positive market share—otherwise it would mean that at least one firm's price was set so high that it had zero market share and, therefore zero profits. But a firm could

Figure 10.4 The Main Street spatial model once again

always obtain positive profits by cutting its price. Thus, the zero market share situation cannot be part of a Nash equilibrium. We focus here on the Nash equilibrium outcome when the entire market is served. That is, when the market outcome is such that every consumer buys exactly one unit of the product from either firm 1 or firm 2.[6] The entire market will be served so long as each consumer's reservation price V is sufficiently large. When V is large, firms have an incentive to sell to as many customers as possible because such a high willingness-to-pay implies that each customer can be charged a price sufficiently high to make each such sale profitable.

When the entire market is served then it must be the case that there is some consumer, called the marginal consumer x^m, who is indifferent between buying from either firm 1 or firm 2. That is, she enjoys the same consumer surplus either way. Algebraically, this means that for consumer x^m:

$$V - p_1 - tx^m = V - p_2 - t(1 - x^m) \tag{10.2}$$

Equation 10.2 may be solved to find the address or the location of the marginal consumer, x^m. This is:

$$x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t} \tag{10.3}$$

At any set of prices, p_1 and p_2, all consumers to the left of x^m buy from firm 1. All those to the right of x^m buy from firm 2. In other words, x^m is the fraction of the market that buys from firm 1 and $(1 - x^m)$ is the fraction that buys from firm 2. If the total number of consumers is N and they are uniformly distributed over the market space the demand function facing firm 1 at any price combination, (p_1, p_2) in which the entire market is served is[7]:

$$D^1(p_1, p_2) = x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t}N \tag{10.4}$$

Similarly, firm 2's demand function is:

$$D^2(p_1, p_2) = (1 - x^m(p_1, p_2)) = \frac{(p_1 - p_2 + t)}{2t}N \tag{10.5}$$

These demand functions make sense in that each firm's demand is decreasing in its own price but increasing in its competitor's price. Notice also that, unlike the simple Bertrand duopoly model in section 10.1, the demand function facing either firm here is continuous in both p_1 and p_2. This is because when goods are differentiated, a decision by say firm 1 to set p_1 a little higher than its rival's price p_2 does not cause firm 1 to lose all of its customers. Some of its customers still prefer to buy good 1 even at the higher price simply because they prefer that version of the good to the style (or location) marketed by firm 2.[8]

[6] Refer to Figure 7.3 in Chapter 7 for a discussion of this point.

[7] We are using N here to refer to the *number of consumers* in the market.

[8] Our assumption that the equilibrium is one in which the entire market is served is critical to the continuity result.

The continuity in demand functions carries over into the profit functions. Firm 1's profit function is:

$$\Pi^1(p_1, p_2) = (p_1 - c)\frac{(p_2 - p_1 + t)}{2t}N \tag{10.6}$$

Similarly, firm 2's profits are given by:

$$\Pi^2(p_1, p_2) = (p_2 - c)\frac{(p_1 - p_2 + t)}{2t}N \tag{10.7}$$

In order to work out firm 1's best response pricing strategy we need to work out how firm 1's profit changes as the firm varies price p_1 in response to a given price p_2 set by firm 2. The most straightforward way to do this is to take the derivative of the profit function (10.6) with respect to p_1. When we set the derivative equal to zero we can then solve for the firm's best response price p_1^* to a given price p_2 set by firm 2.[9]

However, careful application of the alternative solution method of converting firm 1's demand curve into its inverse form and solving for the point at which marginal revenue equals marginal cost will also work. From (10.4), we can write firm 1's inverse demand curve for a given value of firm 2's price p_2 as $p_1 = p_2 + t - \frac{2t}{N}q_1$. Hence firm 1's marginal revenue curve is $MR_1 = p_2 + t - \frac{4t}{N}q_1$. Equating firm 1's marginal revenue with its marginal cost gives the first-order condition for profit maximization, $p_2 + t - \frac{4t}{N}q_1 = c$. Solving for the optimal value of firm 1's output, again given the price chosen by firm 2, we then obtain:

$$q_1^* = \frac{N}{4t}(p_2 + t - c) \tag{10.8}$$

When we substitute the value of q_1^* from equation (10.8) into firm 1's inverse demand curve, we find the optimal price for firm 1 to set given the value of the price set by firm 2. This is by definition firm 1's best response function:

$$p_1^* = \frac{p_2 + c + t}{2} \tag{10.9}$$

where t is the per unit distance transportation or utility cost incurred by a consumer. Of course, we can replicate this procedure for firm 2. Because the firms are symmetric, the best response function of each firm is the mirror image of that of its rival. Hence, firm 2's best price response function is:

$$p_2^* = \frac{p_1 + c + t}{2} \tag{10.10}$$

[9] Setting $\partial\Pi^1(p_1, p_2)/\partial p_1 = 0$ in equation (5.31) yields immediately: $p_1^* = (p_2 + c + t)/2$.

The best response functions described in (10.9) and (10.10) for the two firms are illustrated in Figure 10.5. They are upward sloping. The (Bertrand-)Nash equilibrium set of prices is, of course, where these best response functions intersect. In other words, the Nash equilibrium is a pair of prices (p_1^*, p_2^*) such that p_1^* is firm 1's best response to p_2^*, and p_2^* is firm 2's best response to p_1^*. Thus, we may replace p_1 and p_2 on the right-hand-side of the equations in (10.9) and (10.10) with p_1^* and p_2^*, respectively. Solving jointly for the Nash equilibrium pair (p_1^*, p_2^*) yields:

$$p_1^* = p_2^* = c + t \tag{10.11}$$

In equilibrium, each firm charges a price that is equal to the unit production cost *plus* an amount t, the utility cost per unit of distance a consumer incurs in buying a good that is at some distance from her preferred good. At these prices, the firms split the market. The marginal consumer is located at the address $x = 1/2$. The profit earned by each firm is the same and equal to $(p_i^* - c)N/2 = tN/2$.

Consider the two hair salons located one mile apart on Main Street. All the potential customers live along this stretch of Main Street and they are uniformly spread out. Each consumer is willing to pay at most $50 for a haircut done at the consumer's home. However if a consumer has to travel to get her haircut she incurs a round-trip travel cost of $5 per mile. Each of the hair salons can cut hair at a constant unit cost of $10 per cut, and each wants to set a price per haircut that maximizes the salon's profit. Our model predicts that the equilibrium price of a haircut in this town will be $15, a price that is greater than the marginal cost of a haircut.

Two points are worth making in connection with these results. First, note the role that the parameter t plays. It is a measure of the value each consumer places on obtaining her most preferred version of the product. The greater is t, the less willing the consumer is to buy a product "far away" from her favorite location or product or style. That is, a high t value indicates consumers have strong preferences for their most desired product and incur a high utility loss from having to consumer a product that is less than ideal. The result is that neither firm has much to worry about when charging a high price because consumers prefer to pay that price rather than buy a low-price alternative that is "far away" from their preferred style. When t is large, the price competition between the two firms is softened. In other words, a large value of t means that product differentiation makes price competition much less intense.

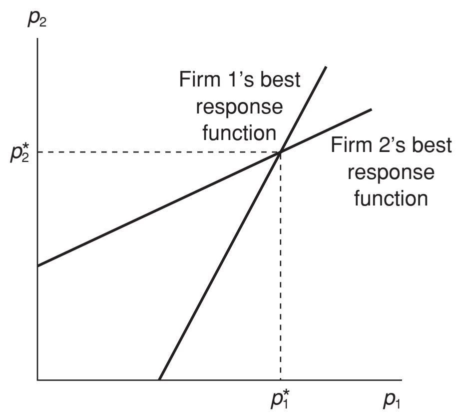

Figure 10.5 Best-reponse functions for price competition with imperfect substitutes.

Reality Checkpoint
Unfriendly Skies: Price Wars in Airlines Markets

Following general deregulation in 1977, the profitability of the airline industry in 1977, has generally deteriorated and also become much more volatile. An important source of these developments has been the continued outbreak of price wars. Morrison and Winston (1996) define such conflicts as any city-pair route market in which the average airfare declines by 2 percent or more within a single quarter. Based on this definition, they estimate that over 8l percent of airline city-pair routes experienced such wars in the 1979–95 time period. In the wars so identified, the average fare in fact typically falls by over 37 percent and sometimes falls by as much as 79 percent. These wars appear to be triggered by unexpected movements in demand and the entrance of new airlines on a route, especially low-cost airlines like Southwest. Morrison and Winston (1996) also find that the effect of such fare wars on industry profits is important. On average, they estimate that the intense price competition cost airlines $300 million in foregone profits in each of the first 16 years following deregulation. This amounts to over 20 percent of total net income over these same years. Of course, to the extent that this profit loss simply reflects movement toward the Bertrand outcome of marginal cost pricing it shows up as a gain to consumers and a net improvement in efficiency. Judging from their comments in the press however, airline executives appear to take little comfort in such gains.

Sources: S. Morrison and C. Winston, "Causes and Consequences of Airline Fare Wars," *Brookings Papers on Economic Activity, Microeconomics, 1996* (1996), 85–124; M. Maynard, "Yes, It Was a Dismal Year for Airline: Now for the Bad News," *New York Times*, December 16, 2002, p. C2.

However, as t falls consumers place less value on obtaining their most preferred styles, but rather are attracted by lower prices. This intensifies price competition. In the limit, when $t = 0$, product differentiation is of no value to consumers. They treat all goods as essentially identical. Price competition becomes fierce and, in the limit, drives prices to marginal cost just as in the original Bertrand model.

The second point concerns the location of the firms. We simply assumed that the two firms were located at either end of town. However, the location or product design of the firm is also part of a firm's strategy. Allowing the two firms to choose simultaneously *both* their price and their location strategies makes the model too complicated to solve here. Still, the intuition behind location choice is instructive. There are two opposing forces affecting the choice of price and location. On the one hand, the two firms will wish to avoid locating at the same point because to do so eliminates all differences between the two products. Price competition in this case will be fierce as in the original Bertrand model. On the other hand, each firm also has some incentive to locate near the center of town. This enables a firm to reach as large a market as possible. Evaluating the balance of these two forces is what makes the solution of the equilibrium outcome so difficult.[10]

[10] There is a wealth of literature on this topic with the outcome often depending on the precise functional forms assumed. See, for example, Eaton (1976), D'Aspremont et al. (1979), Novshek (1980), and Economides (1989).

10.3 **Practice Problem**

Imagine that the two hair salons located on Main Street no longer have the same unit cost. In particular, one salon has a constant unit cost of $10 whereas the other salon has a constant unit cost of $20. The low-cost salon, call it Cheap-Cuts, is located at the east end of town, $x = 0$. The high-cost salon, The Ritz, is located at the west end, $x = 1$. There are 100 potential customers who live along the mile stretch, and they are uniformly spread out along the mile. Consumers are willing to pay $50 for a haircut done at their home. If a consumer has to travel to get a haircut then there a travel cost of $5 per mile is incurred. Each salon wants to set a price for a haircut that maximizes the salon's profit.

a. The demand functions facing the two salons are not affected by the fact that now one salon is high-cost and the other is low-cost. However the salons' best response functions are affected. Compute the best response function for each salon. How does an increase the unit cost of one salon affect the other salon's best response?
b. Work out the Nash equilibrium in prices for this model. Compare these prices to the ones derived in the text for the case when the two salons had the same unit cost equal to $10. Explain why prices changed in the way they did. It may be helpful in your explanation to draw the best response functions when the salons are identical and compare them to those when the salons have different costs.

10.4 STRATEGIC COMPLEMENTS AND SUBSTITUTES

Best response functions in simultaneous-move games are extremely useful tools for understanding what we mean by a Nash equilibrium outcome. But an analysis of such functions also serves other useful purposes. In particular, examining the properties of best response functions can aid our understanding of how strategic interaction works and how that interaction can be made "more" or "less" competitive.

Figure 10.6 shows both the best response functions for the standard Cournot duopoly model and the best response functions for the Bertrand duopoly model with differentiated products. One feature in the diagram is immediately apparent. The best response functions for the Cournot

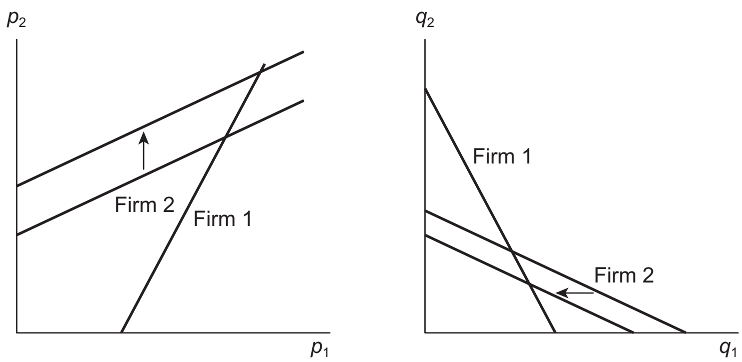

Figure 10.6 Best-response functions for the Cournot (quantity) case and the Bertrand (price) case
A rise in firm 2's cost shifts its response function inwards in the Cournot model but outwards in the Bertrand model. Firm 1 reacts aggressively to increase its market share in the Cournot case. It reacts mildly in the Bertrand price by *raising* its price.

quantity model are *negatively* sloped—firm 1's best response to an increase in q_2 is to *decrease* q_1. But the best response functions in the Bertrand price model are *positively* sloped. Firm 1's best response to an increase in p_2 is to increase p_1, as well.

Whether the best response functions are negatively or positively sloped is quite important. The slope reveals much about the nature of competition in the product market. To see this, consider the impact of an increase in firm 2's unit cost c_2. Our analysis of the Cournot model indicated that the effect of a rise in c_2 would be to shift *inward* firm 2's best response curve. As Figure 10.6 indicates, this leads to a new Nash equilibrium in which firm 2 produces less and firm 1 produces more than each did before c_2 rose. That is, in the Cournot quantity model, firm 1's response to firm 2's bad luck is a rather aggressive one in which it seizes the opportunity to expand its market share at the expense of firm 2.

Consider now the impact of a rise in c_2 in the context of the differentiated goods Bertrand model. The rise in this case shifts firm 2's best response function *upwards*. Given the rise in its cost, firm 2 will choose to set a higher p_2 than it did previously in response to any given value of p_1. How does firm 1 respond? Unlike the Cournot case, firm 1's reaction is less aggressive. Firm 1—seeing that firm 2 is now less able to set a low price—realizes that the price competition from firm 2 is now less intense. Hence, firm 1 now reacts by raising p_1.

When the best response functions are upward sloping, we say that the strategies (prices in the Bertrand case) are *strategic complements*. When we have the alternative case of downward sloping best response functions, we say that the strategies (quantities in the Cournot case) are *strategic substitutes*. This terminology comes from Bulow, Geanakopolos, and Klemperer (1985) and reflects similar terminology in consumer demand theory. When a consumer reacts to a rise (fall) in the price of one product by buying less (more) of it and more (less) of another, we say that the two goods are substitutes. When a consumer reacts to a change in the price of one good by buying either more or less of *both* that good and another product, we say that the two goods are complements. This is the source of the similarity. Quantities in Cournot analysis are strategic substitutes because a rise in c_2 induces a fall in q_2 but a rise in q_1. Prices in a Bertrand model are strategic complements because a rise in c_2 induces an increase in p_2 and also in p_1.

The different nature of the strategic interaction and the different equilibria makes clear that the choice of whether to use price or quantity as the strategic variable to model market competition is an important one. What factors influence this choice? In those industries in which firms set their production schedules far in advance of putting the goods on the market for sale, there is a good case to assume that firms compete in quantities. Examples include the world energy market, coffee-growers, and automobile producers. In many service industries, such as banking, insurance, and air travel, it is much more natural to think in terms of price competition. In certain manufacturing industries, such as cereal and detergents, the price competition for customers is a stronger factor then the setting of production schedules, and so Bertrand price competition may be the more appropriate model.

10.5 EMPIRICAL APPLICATION Brand Competition and Consumer Preferences—Evidence from the California Retail Gasoline Market

Gasoline is typically produced by refiners and then shipped to a central distribution point. The gasoline is then bought either by an unbranded independent retailer such as RaceTrac, or by service stations selling a branded product such as an Exxon or a Chevron station. In the latter case, a special additive unique to the brand has to be added. For example, to sell "Chevron" gasoline, a station has to have added Techronas™ to the fuel. Thus, each specific brand is differentiated by the use of its own additive. Independent stations, however, simply sell the basic gasoline without any additive. Here, we briefly describe a paper by Justine Hastings (2004) that examines the nature of price competition in the retail gasoline market in the southern California.

The background to the study is as follows. In June of 1997, the Atlantic Richfield Company (ARCO), a well-known refiner and retail brand, acquired control of about 260 gasoline stations that formerly had been operated by the independent retailer, Thrifty, in and around Los Angeles and San Diego. ARCO then converted these to ARCO stations—a process that was essentially completed by September of that same year. Thus, the ARCO-Thrifty acquisition resulted in the exit of a large number of independent service stations in Southern California as these were replaced part by ARCO sellers.

Hastings (2004) asks what effect the ARCO–Thrifty deal had on retail gasoline prices. In principle, the effect could be either positive or negative, depending on consumer preferences. If consumers identify brands with higher quality and independents with lower quality, then conversion of the unbranded (low-quality) stations to the ARCO brand would mean that these stations now sell a closer substitute to the other branded products. This would intensify price competition and *lower* branded gasoline prices. However, if a large pool of consumers is unresponsive to brand labels because their willingness to pay for higher quality is limited and they only want to buy gasoline as cheaply as possible, then the loss of the Thrifty stations removes this low-cost alternative and *raises* gasoline prices.

To isolate the effect of the ARCO–Thrifty merger, Hastings (2004) looked at how prices charged by gasoline stations in the Los Angeles and San Diego areas differed depending on whether they competed with a Thrifty or not. Her data cover the prices charged by 699 stations measured at four different times: February 1997, June 1997, October 1997, and December, 1997. Notice that the first two dates are for prices before the conversion while the last two dates are for prices after the conversion. She then defines submarkets in which each station's competitors are all the other stations within one mile's driving distance. A simple regression that might capture the effect of the merger would be:

$$p_{it} = Constant + \alpha_i + \beta_1 X_{it} + \beta_2 Z_{it} + e_{it} \tag{10.12}$$

where p_{it} is the price charged by station i at time t; α_i is a firm-specific dummy that lets the intercept be different for each service station; X_{it} is a dummy variable that has the value 1 if station i competes with an independent (Thrifty) at time t and 0 otherwise; likewise Z_{it} is 1 if a competitor of station i has become a station that is owned by a major brand as opposed to a station that operates as a franchisee or lessee of a major brand, and 0 otherwise. This last variable, Z_{it} is meant to capture the impact of any differential effects depending on the contractual relationship between a major brand and the station that sells that brand. The key variable of interest however is X_{it}. We want to know whether the estimated coefficient β_1 is negative, which would indicate that having independent rivals generally leads to lower prices—or is positive, which would indicate that the presence of independents softens competition and raises prices.

However, there is a potentially serious problem with estimating equation (10.12). The problem is that over the course of 1997, gasoline prices were rising generally throughout southern California. Equation (10.12) does not allow for this general rising trend. Consider our key variable X_{it}. In the data, this will be 1 for a lot more stations before the merger in

February and June, than it will be in October and December. As a result, the coefficient β_1 will likely be negative because prices were lower in February and June (when there were a lot more independents) than in September and December (after the merger removed the Thrifty stations). That is, β_1 will be biased because it will pick up time effects as well as the effects of independents.

In order to isolate the price effects that are purely due to independent rivals alone, Hastings (2004) puts in location specific time dummies for February, June, and September. (The effect of December is of course captured in the regression constant). That is, she estimates an equation something like:

$$p_{it} = Constant + \alpha_i + \beta_1 X_{it} + \beta_2 Z_{it} + \beta_3 T_i + e_{it} \tag{10.13}$$

where T_i or time is captured not as a continuous variable but, again, by time-specific dummies. Her results, both with and without the time dummies (but suppressing the firm specific intercepts) are shown in Table 10.1.

Consider first the column of results for the equation that includes the location time dummies. Here, the estimate of β_1, the coefficient on having a Thrifty or independent rival in a station's local market, implies that this led the station to lower its price by about five cents per gallon. The standard error on this estimate is very small, so we can be very confident of this measure. Note too how this contrasts with the effect measured in the regression results shown in the first column that leaves out the time effects. That estimate suggests a much larger effect of ten cents per gallon decline when a station has independent rivals. Again, this is because in leaving out the time effects, the regression erroneously attributes the general rise in gasoline prices throughout the region to the merger when in fact prices were clearly rising for other reasons as well. We should also note that the coefficient estimate for β_2 is not significant in either equation. So, the type of ownership by a major brand does not seem to be important for retail gasoline prices.

One picture is often worth a large number of words. Figure 10.7 illustrates the behavior of Southern California gasoline prices over the period covered by Hastings's data for each of two groups: (1) the treatment group that competed with a Thrifty station; and (2) the control group of stations that did not.

Table 10.1 Brand competition and gasoline prices

Variable	*Without location-time dummies Coefficient (standard error)*	*With location-time dummies Coefficient (standard error)*
Constant	1.3465 (0.0415)	1.3617 (0.0287)
X_{it}	–0.1013 (0.0178)	–0.0500 (0.0122)
Z_{it}	–0.0033 (0.0143)	–0.0033 (0.0101)
LA*February		0.0180 (0.0065)
LA*June		0.0243 (0.0065)
LA*December		0.1390 (0.0064)
SD*February		–0.0851 (0.0036)
SD*June		–0.0304 (0.0036)
SD*December		0.0545 (0.0545)
R^2	0.3953	0.7181

Dependent Variable = price per gallon of regular unleaded

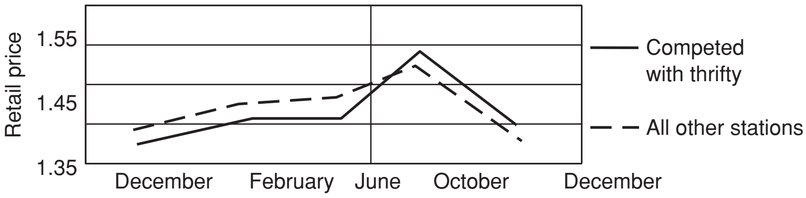

Figure 10.7 "Thrifty" competition and gasoline prices in southern California.

Notice the general rise in prices in both groups through October. Clearly, this is a phenomenon common to the gasoline market in general and not the result of the merger per se. However, a close look at the data does reveal that the merger did have some impact. In the months before the merger, stations that competed with a Thrifty had prices that were two to three cents *lower* than those in the control group. Starting about the time of the merger in June, however, and continuing afterwards, these same stations had prices two to three cents *higher* than those in the control group. It is this roughly five-cent effect that is being picked up in the final column of the preceding table. For both groups, those that initially competed with a Thrifty prior to the merger and those that did not, prices differ between the beginning of 1997 and the end. To isolate the effects of the merger, we need to look at how these differences over time were different between the two groups. If we recall that for the treatment group stations $X_{it} = 1$, at first but 0 after the merger, while it is always zero firms in the control group, the price behavior for the two groups is:

	Before merger	*After merger*	*Difference*
Treatment group:	$\alpha_i + \beta_1$	α_i + time effects	$-\beta_1$ + time effects
Control group:	α_j	α_j + time effects	time effects

Thus, β_1 in our regression reflects the difference between the difference over time in the treatment group and that in the control group. For this reason, β_1 is often referred to as a *difference-in-differences* estimator.

Summary

In the Bertrand model firms compete in prices. In the simple model of Bertrand competition, prices are pushed to marginal cost even if there are just two firms. By contrast, when there is quantity or Cournot competition, prices remain substantially above marginal cost so long as the number of firms is not large. High cost firms can survive in Cournot competition. However, high cost firms cannot survive Bertrand competition against a firm with lower costs. In short, the simplest Bertrand model predicts competitive and efficient market outcomes even when the number of firms is quite small.

However, the efficient outcomes predicted by the simple Bertrand model depend upon two key assumptions. The first is that firms have extensive capacity so that it is possible to serve all a rival's customers after undercutting the rival's price. The second key assumption is that the firms produce identical products so that relative price is all that matters to consumers when choosing between brands. If either of these assumptions is relaxed, the efficiency outcomes of the simple Bertrand model no longer obtain. If firms must choose production capacities in advance, the outcome with Bertrand price competition becomes closer to what occurs in the Cournot model. If products are differentiated, prices are again likely to remain above marginal cost. Indeed, given the fierceness of price competition, firms have a real incentive to differentiate their products.

A useful model of product differentiation is the Hotelling (1929) spatial model, which we first introduced in Chapter 4. This model uses geographic location as a metaphor for more general distinctions between different versions of the same product. It thereby makes it possible to consider price competition between firms selling differentiated products. The model makes it clear that Bertrand competition with differentiated products does not result in efficient marginal cost pricing. It also makes clear that the deviation from such pricing depends on how much consumers value variety. The greater value that the typical consumer places on getting her most preferred brand or version of the product, the higher prices will rise above marginal cost.

Ultimately, the differences between Cournot and Bertrand competition reflect underlying differences between quantities and prices as strategic variables. The quantities chosen by Cournot firms are strategic substitutes—increases in one firm's production lead to decreases in the rival's output. In contrast, the prices chosen by Bertrand competitors are strategic complements. A rise in one firm's price permits its rival to raise price, too.

Models of price competition based on the spatial model have provided an extremely useful framework for empirical work. Many policy makers are interested in investigating how a change in market structure—through entry or mergers or regulatory policy—will affect price competition. One key underlying issue in price competition is how it is affected by consumer preferences for the different brands. The spatial model of differentiation captures consumers' preferences for both variety (horizontal product differentiation) and quality (vertical product differentiation) and is used extensively in empirical work on competition policy. In this respect, it is important to identify which type of differentiation applies. Depending on the nature of consumer preferences, a merger between a high-quality and low-quality firm that results in the transformation of the low-quality firm outlets to high-quality ones could either weaken competition because it removes a low-quality competitor or intensify competition because it adds to the high-quality supply.

Problems

1. Suppose firm 1 and firm 2 each produce the same product and face a market demand curve described by $Q = 5{,}000 - 200P$. Firm 1 has a unit cost of production c_1 equal to 6 whereas firm 2 has a higher unit cost of production c_2 equal to 10.
   a. What is the Bertrand–Nash equilibrium outcome?
   b. What are the profits of each firm?
   c. Is this outcome efficient?

2. Suppose that market demand for golf balls is described by $Q = 90 - 3P$, where Q is measured in kilos of balls. There are two firms that supply the market. Firm 1 can produce a kilo of balls at a constant unit cost of \$15 whereas firm 2 has a constant unit cost equal to \$10.
   a. Suppose firms compete in quantities. How much does each firm sell in a Cournot equilibrium? What is the market price and what are firms' profits?
   b. Suppose firms compete in price. How much does each firm sell in a Bertrand equilibrium. What is market price and what are firms' profits?

3. a. Would your answer in 2b change if there were 3 firms, one with unit cost = \$20 and two with unit cost = \$10? Explain why or why not.
   b. Would your answer in 2b change if firm 1's golf balls were green and endorsed by Tiger Woods, whereas firm 2's are plain and white? Explain why or why not.

4. In Tuftsville everyone lives along Main Street that is 10 miles long. There are 1,000 people uniformly spread up and down Main Street, and each day they each buy fruit smoothie from one of the two stores located at either end of Main Street. Customers ride their motor scooters to and from the store and the motor scooters use \$0.50 worth of gas per mile. Customers buy their smoothies from the store offering the lowest price, which is the store's price plus the customer's travel expenses getting to and from the store. Ben owns the store at the west end of Main Street

and Will owns the store at the east end of Main Streets.

a. If both Ben and Will charge $1 per smoothie how many will each of them sell in a day? If Ben charges $1 per smoothie and Will charges $1.40 how many smoothies will each sell in a day?
b. If Ben charges $3 per smoothie what price would enable Will to sell 250 smoothies per day? 500 smoothies per day? 750 smoothies per day? 1,000 smoothies per day?
c. If Ben charges p_1 and Will charges p_2 what is the location of the customer who is indifferent between going to Ben's and going to Will's? How many customers go to Will's store and how many go to Ben's store? What are the demand functions that Ben and Will face?
d. Rewrite Ben's demand function with p_1 on the left-hand side. What is Ben's marginal revenue function?
e. Assume that he marginal cost of a smoothie is constant and equal to $1 for both Ben and Will. In addition each of them pays Tuftsville $250 per day for the right to sell smoothies. Find the equilibrium prices, quantities sold and profits.

5. Return to Main Street in Tuftsville. Now suppose that George would like to open another store at the midpoint of Main Street. He too is willing to pay Tuftsville $250 a day for the right to sell smoothies.
   a. If Ben and Will do not change their prices what is the best price for George to charge? How much profit would he earn?
   b. What do you think would happen if George did open another store in the middle of Main Street? Would Ben and Will have an incentive to change their prices? Their locations? Would one or both leave the market?

6. Suppose that there are two firms, firm B and firm N, produce complementary goods, say bolts and nuts. The demand curve for each firm is described as follows:

$$Q_B = Z - P_B - P_N \text{ and } Q_N = Z - P_N - P_B$$

   For simplicity, assume further that each firm faces a constant unit cost of production, $c = 0$.
   a. Show that the profits of each firm may be expressed as $\Pi^B = (P_B)(Z - P_B - P_N)$ and $\Pi^N = P_N(Z - P_B - P_N)$.
   b. Show that each firm's optimal price depends on the price chosen by the other as given by the optimal response functions: $P_B^* = (Z - P_N)/2$ and $P_N^* = (Z - P_B)/2$.
   c. Graph these functions. Show that the Nash equilibrium prices are: $P_B = P_N = Z/3$.
   d. Describe the interaction between two monopolists selling separate but complementary goods, which we presented in Chapter 9 as a game.

7. Assume that two firms sell differentiated products and face the following demand curves:

$$q_1 = 15 - p_1 + 0.5p_2 \text{ and } q_2 = 15 - p_2 + 0.5p_1$$

   a. Derive the best response function for each firm. Do these indicate that prices are strategic substitutes or strategic complements?
   b. What is the equilibrium set of prices in this market? What profits are earned at those prices?

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