7

Product Variety and Quality Under Monopoly

Most firms sell more than one product. Microsoft offers not only an operating system and an Internet browser but also a number of other products, most notably, the word-processing package Word the spreadsheet software, *Excel* and the presentation package PowerPoint. Photographic firms such as Eastman Kodak sell both cameras and film, and in each case in a wide range of varieties. The telecommunications giant Comcast now offers telephone service, high-speed Internet service and cable TV. A newer giant in the same field, AOL-Time Warner offers a combination of Internet access and entertainment programming. Fashion designers such as Ralph Lauren offer a broad range of apparel from sportswear to haute couture, both for women and men.

The multi-product firm is likely to be the norm. This leads us to the question of exactly now much variety a firm should aim to offer. Take a company such as Kellogg. Is there any flavor, color or texture of breakfast cereal that they do not market? Or take Procter & Gamble, a large consumer product company that offers more than a dozen varieties of their Head and Shoulders shampoo and even more varieties of their Crest toothpaste. Indeed Procter & Gamble asked itself the same question when it reexamined its product strategy in the early 1990s. By 1996 the company had reduced their list of products by one-third as compared with 1991.

A firm's incentive to offer many varieties of what is essentially the same product—breakfast foods, hair or tooth care—is simple enough to understand. It is a way for the firm to reach and sell to consumers with very different tastes. Because consumers often differ regarding their most preferred color, or flavor, or texture, selling successfully to many consumers requires offering something a little different to each of them. Specifically, to induce a consumer to make a purchase the firm must market a product that is reasonably close to the version that the consumer prefers. When a firm offers a variety of products in response to different consumer tastes, it is called *horizontal product differentiation*.

However with respect to certain product features consumers often agree on what makes for a good product. For example, all consumers likely agree that a car with antilock brakes is better than one without such a stopping mechanism. Similarly, all probably agree that while the X-type Jaguar is an attractive car, it pales in comparison to the XJ. Everyone is likely to agree that flying from Boston to San Francisco first class is a better than flying coach. Where consumers differ in these examples is not in what features they consider desirable but, instead, in how much a desired feature is worth to them, i.e., how much they are willing to pay for antilock brakes, a better Jaguar, or first class airfare. When a firm responds to differing consumer willingness to pay for quality of a product by offering different qualities of the same product it is called *vertical product differentiation*.

In this chapter we analyze the horizontal and vertical product differentiation strategies of a monopoly firm. We examine how product differentiation may be used by the firm to increase profitability. We also consider the welfare properties of these strategies.

7.1 A SPATIAL APPROACH TO HORIZONTAL PRODUCT DIFFERENTIATION

There are many situations in which individual consumers have their own preferred brand or variety of product, whether this is breakfast cereal, hair treatment, or an automobile. We begin by considering a market in which consumers differ regarding the features that make the product attractive to them. However, they are more or less alike in terms of their basic willingness to pay for one of these products. For example, all consumers might be willing to pay the same price for a product, a margherita pizza, if it is sold at a shop close to their home. However, not all consumers will be equidistant from the shop. Some will be close and some will be far away. Given the time and effort required to travel, the willingness to pay of those who live far from the shop will be lower. Alternatively, those who live close—and who therefore do not have to incur travel expenses—will be willing to pay a higher price. The fact that a product sold close to home is different from one that is sold far away is a good example of what is referred to as horizontal product differentiation. Such differentiation is characterized by the property that each consumer has her own preferred location of the shop or product, namely, one close to the consumer's own address.

When the consumer market is differentiated by geographic location, a firm can vary its product strategy through its choice of where the product is sold. The firm may choose to sell its product only in one central location to which all shoppers must come: Giorgio Armani, for example, does this. Alternatively, it may decide to offer the product at many locations spaced throughout the city: McDonald's, Dunkin' Donuts and Subway are obvious examples. Customers are not indifferent between these alternative strategies. If the firm sells only at one central location, those who do not live in the middle of town have to incur travel costs to come to the store. These costs are greatest for those living farthest from the center. The alternative strategy of selling at many different locations allows more consumers to purchase the good without going too far out of their way.

When geography is taken into account and traveling is costly, consumers are willing to pay more for a product marketed close to their own geographic location. In this case products are differentiated by the locations at which they are sold. This setting is known as the spatial model of product differentiation, pioneered by Hotelling (1929).[1] Before presenting the model formally, there is an additional point worth emphasizing. While the model is easiest to present in terms of a geographic representation it can easily be more broadly interpreted. With just a little imagination, geographic space can be transformed into a "product" or, more properly "characteristics space." In such a space, each consumer's "location" reflects her most preferred set of product characteristics such as color, style, or other features. Recall our earlier discussion in Chapter 4 of a soft-drink firm offering a product line differing in terms of sugar content. This example made use of precisely this type of horizontal or spatial differentiation.

[1] Hotelling (1929) was concerned with analyzing competition between two stores whereas we consider here the case in which the stores are owned by the same firm and so act cooperatively.

The travel cost of the geographic model can be understood as a psychic or utility cost that the consumer incurs if she must purchase a good whose characteristics are "distant" from her most preferred type. Just as consumers prefer to go to video stores close to their home, so they prefer to buy clothes that are "close" to their individual preferred style, or soft drinks close to their preferred amount of sugar content. In fact Hotelling suggested just this interpretation in his seminal article. As he wrote:

> Distance, as we have used it for illustration, is only a figurative term for a great congeries of qualities. Instead of sellers of an identical commodity separated geographically we might have considered two . . . cider merchants . . . one selling a sweeter liquid than the other. If consumers of cider are thought of as varying by infinitesimal degrees in the sourness they desire, we have much the same situation as before. The measure of sourness now represents distance, while instead of transportation costs there are degrees of disutility resulting from the consumer getting cider more or less different from what he wants. (1929, p. 54)

7.2 MONOPOLY AND HORIZONTAL DIFFERENTIATION

Assume that there is a town spread out along a single road; call it Main Street, of say one mile in length. There are *N* consumers who live spaced evenly along this road from one end of town to the other. A firm that has a monopoly in, for example, fast food, must decide how to serve these consumers at the greatest profit. What this means is that the monopolist must choose the number of retail outlets, or shops, that it will operate, where these should be located on Main Street, and what prices they should charge. In the product differentiation analogy to drinks of different sweetness, the monopolist has to decide how many different drinks it should offer, what their precise degrees of sweetness should be, and what their prices should be. More generally, what range of products should the monopolist bring to the market and how should they be priced? In what follows we use for clarity the geographic interpretation of the model, but we again emphasize that you should always bear in mind its much wider interpretation.

In this section we consider cases in which the monopolist does not price discriminate among the consumers that are served. Consumers travel to a retail outlet in order to buy the product, and they incur transport costs. We assume that the transportation cost is t per unit of distance (there-and-back) traveled. Except for their addresses or their locations, consumers are identical to each other. We assume that in each period each consumer is willing to buy exactly one unit of the product sold by the monopolist provided that the price paid, including transport costs, which we shall call the *full price*, is less than her reservation price, which we denote by V.

Suppose that the monopolist decides to operate only a single retail outlet. Then it makes sense to locate this outlet at the center of Main Street. Now consider the monopolist's pricing decision. The essentials of the analysis are illustrated in Figure 7.1. The easternmost resident (residing at the left end of the diagram) has an address of $z = 0$. The westernmost resident has an address of $z = 1$ and the shop is located at $z = 1/2$.

The vertical axis in Figure 7.1 is, as usual, a measure of price. The value V in this diagram is the reservation price for each consumer. The full price that each consumer pays is comprised of two parts. First, there is the price p_1 actually set by the monopolist. Secondly, there is the additional cost consumers incur in getting to the shop (and back home). Measured per unit of distance there and back, the *full price* actually paid by a consumer who lives a distance x from the center of town is the monopolist's price plus the transport cost

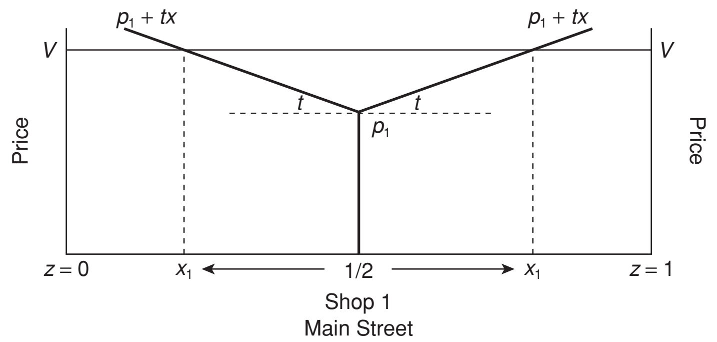

Figure 7.1 The full price at the shop on Main Street
The full price, including transport cost, rises as consumers live farther from the shop.

or $p_1 + tx$. This full price is indicated by the Y-shaped set of lines in Figure 7.1. It indicates that the full price paid by a consumer at the center of town—one who incurs no transport cost—is just p_1. However, as the branches of the Y indicate, the full price rises steadily beyond p_1 for consumers both east and west of the town center. So long as distance from the shop is less than x_1, the consumer's reservation price V exceeds the full price $p_1 + tx$ and such a consumer buys the monopolist's product. However, for distances beyond x_1 the full price exceeds V and these consumers do not buy the product. In other words, the monopolist serves all those who live within x_1 units of the town center. How is the distance x_1 determined? Consumers who reside distance x_1 from the shop are just indifferent between buying the product and not buying it at all. For them, the full price $p_1 + tx_1$ is equal to V so we have:

$$p_1 + tx_1 = V \text{ which implies that } x_1 = \frac{V - p_1}{t} \tag{7.1}$$

Now x_1 is really just a fraction. Since the town is one mile long, x_1 is a fraction of a mile and the retail outlet sells to a fraction $2x_1$ of the whole town—since it sells to consumers to the left and to the right of the market center so long as they live no further than x_1 from the shop. Moreover, there are N consumers evenly distributed over Main Street. Accordingly, there are $2x_1N$ consumers who each are willing to buy one unit of the product if it is priced at p_1. By substituting the expression for x_1 from equation (7.1) into the number of customers served by the monopolist, $2x_1N$ at price p_1, we find that the total demand for the monopolist's product given that it operates just one shop is:

$$Q(p_1, 1) = 2x_1N = \frac{2N}{t}(V - p_1) \tag{7.2}$$

Equation (7.2) says something interesting. Despite our assumption that each consumer buys exactly one unit or none of the monopolist's product, the demand function in the equation shows that aggregate demand increases as the monopolist lowers the price. The reason is illustrated in Figure 7.2. When the shop price is reduced from p_1 to p_2 demand increases because *more consumers are willing to buy the product at the lower price*. Now all consumers within distance x_2 of the shop buy the product.

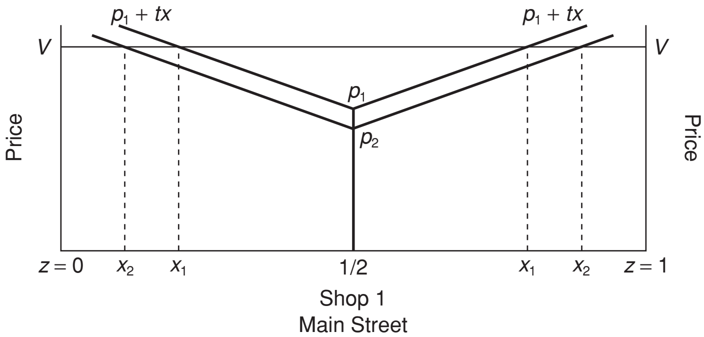

Figure 7.2 Lowering the price at the shop on Main Street
A fall in the shop price brings additional customers from both east and west.

Suppose that the monopolist wants to sell to every customer in town. What is the *highest* price that the monopolist can set and still be able to sell to all N consumers? The answer must be the price at which the consumers who live furthest from the shop, i.e., those who are half a mile away, are just willing to buy. At a shop price p these consumers pay a full price of $p + t/2$ and so will buy only if $p + t/2 \leq V$. What this tells us is that with a single retail outlet at the market center the maximum price that the monopolist can charge and still supply the entire market of N consumers with its one store is $p(N, 1)$ given by:

$$p(N, 1) = V - \frac{t}{2} \tag{7.3}$$

Let the monopolist's costs be c per unit sold and assume that there are set-up costs of F for each retail outlet. These set-up costs could be associated with the cost of buying a site, commissioning the building and so on. In the product differentiation analogy, the set-up costs might be the costs of designing and marketing the new product. Whatever the framework, the monopolist's profit with a single retail outlet that supplies the entire market is:

$$\pi(N, 1) = N[p(N, 1) - c] - F = N\left(V - \frac{t}{2} - c\right) - F \tag{7.4}$$

We are now ready to investigate why this single shop should be located in the center of town. The reason why this is the best location is that makes it easiest to reach all the customers. At the price $p = V - \frac{t}{2}$, a move a little to the east will not gain any new customers on the east end of town (there are no more to gain) but will lose some of those at the extreme west end of town. In other words, if the firm moves from the center, the only way it can continue to serve the entire town is by cutting its price below $V - \frac{t}{2}$. Only by keeping its single location at the center can it reach all customers with a price as high as $V - \frac{t}{2}$.

Now we wish to consider what happens when there are two, or three, or n outlets along Main Street. As before, we continue to assume that unit cost at each shop is c per unit sold

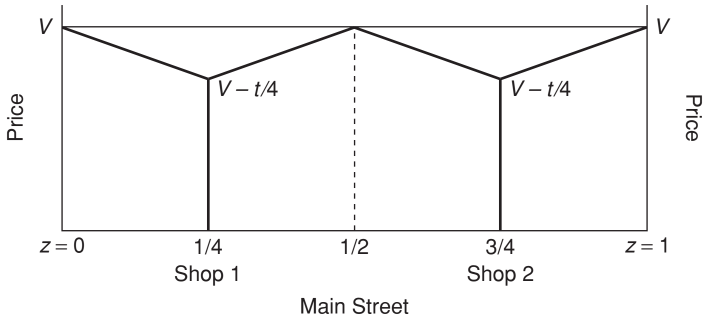

Figure 7.3 Opening two shops on Main Street
The maximum price is higher with two shops than it is with one.

and that the set-up cost for each outlet is F. In other words there are no scope economies from operating multiple outlets. We start by asking what happens if the number of retail outlets is increased to two. Because each segment of the market along Mains Street is the same and each shop has the same costs the monopolist will choose to set the same price at each shop. Moreover the monopolist will want to coordinate the location of these two shops so as to maximize price charged at a shop while still reaching the entire market. In fact, the same intuition that justifies a central location with just one firm can be used to show that the optimal location strategy is to locate one of the two shops 1/4 mile from the left-hand end and the other 1/4 mile from the right-hand end of Main Street as in Figure 7.3.[2]

Consider the maximum price that the monopolist can now charge while still supplying the entire market. The maximum distance that any consumer has to travel to a shop is 1/4 mile—much less than the 1/2 mile when there is only one retail outlet. As a result, the highest price that the monopolist can charge and supply the entire market is:

$$p(N, 2) = V - \frac{t}{4} \tag{7.5}$$

which is higher than the price with a single retail outlet. The monopolist's profit is now:

$$\pi(N, 2) = N\left(V - \frac{t}{4} - c\right) - 2F \tag{7.6}$$

What happens if the monopolist decides to operate three shops? By exactly the same argument as above, these shops should be located symmetrically at 1/6, 1/2, and 5/6 miles from the left-hand end of the market so that each supplies 1/3 of the market, as illustrated in Figure 7.4. The maximum distance that any consumer has to travel now is 1/6 mile, so the price at each shop (again assuming, of course, that all consumers are to be served) is:

$$p(N, 3) = V - \frac{t}{6} \tag{7.7}$$

2 For the interested reader a formal proof of this result is given in Appendix A to this chapter. Appendix B extends the analysis to include the possibility that the two shops have different costs.

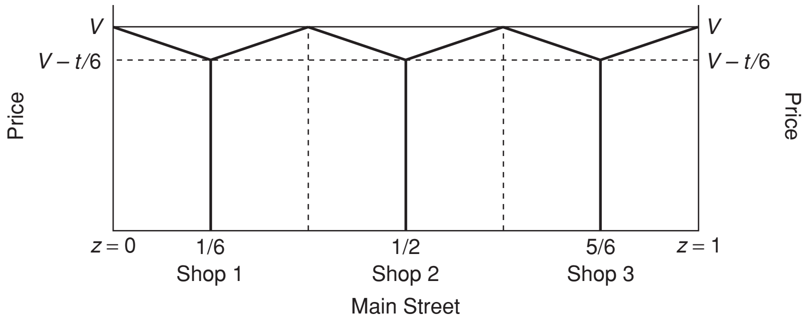

Figure 7.4 Opening three shops on Main Street

While profit is now

$$\pi(N, 3) = N\left(V - \frac{t}{6} - c\right) - 3F \tag{7.8}$$

There is, in fact, a general rule emerging. If the monopolist has n shops to serve the entire market, they will be located symmetrically at distances $1/2n$, $3/2n$, $5/2n$, . . . , $(2i - 1)/2n$, . . . , $(2n - 1)/2n$ from the left-hand end of the market. The maximum distance that any consumer has to travel to a shop is $1/2n$ miles, so the price that the monopolist can charge at each shop while supplying the entire market is

$$p(N, n) = V - \frac{t}{2n} \tag{7.9}$$

At this price, its profit is

$$\pi(N, n) = N\left(V - \frac{t}{2n} - c\right) - nF \tag{7.10}$$

The important feature that emerges from this analysis is that as the number of retail outlets n increases the monopolist's price at each shop gets closer and closer to the consumer's reservation price V. In other words, by increasing the number of shops the monopolist is able to charge each consumer a price much closer to her maximum willingness to pay, V, and thereby appropriate a much greater proportion of consumer surplus.

The moral of the foregoing analysis is clear—especially when we remember to interpret the geographic space of Main Street as a more general product space. Even if no scope economies are present, a monopolist has an incentive to offer many varieties of a good. Doing so allows the monopolist to exploit the wide variety of consumer tastes, charging each consumer a high price because each is being offered a variety that is very close to her most preferred type. It is not surprising, therefore, that we see such extensive product proliferation in real-world markets such as those for cars, soft drinks, toothpastes, hair shampoos, cameras and so on.[3]

[3] See Shapiro and Varian (1999) for a similar argument regarding product variety in e-commerce markets.

However, there must be some factor limiting this proliferation of varieties or outlets. We do not observe a McDonald's on every street corner, or individually tailored Levis, or each person's custom-designed breakfast cereals or soft drinks! We therefore need to think about what constrains the monopolist from adding more and more retail outlets or product variants. Equation (7.10) gives the clue. Admittedly, adding additional retail outlets allows the monopolist to increase its prices. However, the establishment of each new shop or new product variant also incurs the set-up cost. If, for example, the monopolist decides to operate $n + 1$ retail outlets its profit is

$$\pi(N, n + 1) = N\left(V - \frac{t}{2(n + 1)} - c\right) - (n + 1)F \tag{7.11}$$

This additional shop, or variety of drink, or new product variant increases profit only if $\pi(N, n + 1) > \pi(N, n)$, which requires that:

$$\frac{t}{2n}N - \frac{t}{2(n + 1)}N - F > 0$$

This simplifies to:

$$n(n + 1) < \frac{tN}{2F} \tag{7.12}$$

Suppose, for example, that there are 5 million consumers in the market so that $N = 5{,}000{,}000$ and that there is a fixed cost F = \$50,000 associated with each shop. Suppose further that the transport cost t = \$1. Hence, $tN/2F = 50$. Then if n is less than or equal to 6, equation (7.12) indicates that it is profitable to add a further shop. However, once the monopolist sets up $n = 7$ shops equation (7.12) indicates that it is not worthwhile to add any more. (You can easily check that the monopolist should operate exactly 7 shops for any value of $tN/2F$ greater than 42 but less than 56.)

Equation (7.12) actually has a simple and appealing intuition. The monopolist has to balance the increase in price and revenues that results from increased product variety against the additional set-up costs that offering increased variety entails. What this tells us is that we would expect to find greater product variety in markets where there are many consumers (N is large), or where the set-up costs of increasing product variety are low (F is small), or where consumers have strong and distinct preferences regarding product characteristics (t is large).

The first two conditions should be obvious. It tells us why there are many more retail outlets in Chicago than in Peoria; why we see many franchise outlets of the same fast-food chain but not of a gourmet restaurant; and why we see many more Subway outlets in a city than Marriott hotels. What does the third condition mean? For a given number of n shops, equation (7.12) tells us that an additional $(n + 1)$ shop will be increasingly desirable as the transportation cost t becomes greater. Thus, as t increases so will the monopolist's optimal number of outlets or degree of product variety.

The interpretation is that when t is high consumers incur very large costs if they are not being offered their most preferred brand. That is, a large value of t implies that consumers are very strongly attached to their preferred product type or location and are unwilling to purchase products that deviate significantly from this type—or travel very far to buy the

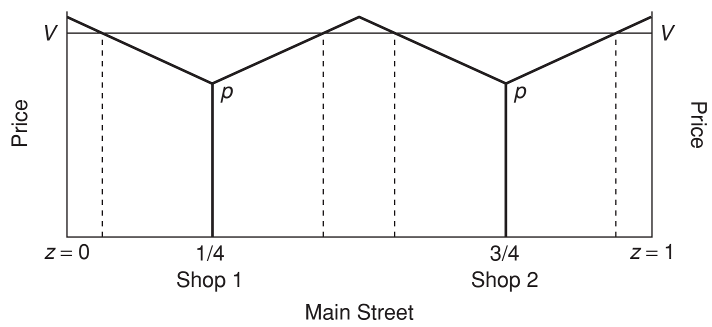

Figure 7.5 Stand-alone retail shops on Main Street

product. If the monopolist is to continue to attract consumers it must tailor its products more closely to each consumer's unique demand, which requires that it offers a wider range of product variants—or operates more retail outlets.

In this kind of market, adding a new shop does not necessarily mean increasing the total supply of the good. Instead, it means replacing some of an existing variety with an alternative variety that more closely matches the specific tastes of some customers. As we have seen, this also allows the firm to charge a higher price. Yet this advantage does not come free. The firm must incur the set-up cost F for each new shop.

However, we have not actually checked whether or not it makes sense and profit for the monopolist to serve the entire market. We need to identify the condition that determines whether the monopolist will prefer to supply only part of the market rather than the whole market. If only part of the market is served then each retail outlet is effectively a "stand alone" shop whose market area does not touch that of the remaining outlets as in Figure 7.5. We show in the inset that the profit-maximizing price when only part of the market is served is $p^* = (V + c)/2$, which does not depend on the number of shops the monopolist has. This leads to a simple rule that determines whether or not the entire market is to be served. Suppose that there are n retail outlets. Then we know from equation (7.9) that the price at which the entire market can be served is $p(N, n) = V - t/2n$. Serving the entire market is therefore better than supplying only part of the market provided:

$$p(N, n) > p^* \text{ which implies } V - \frac{t}{2n} > \frac{V + c}{2} \text{ which implies } V > c + \frac{t}{n} \tag{7.13}$$

We can put this rule another way. What equation (7.13) tells us is that the monopolist's optimal pricing policy can be described as follows:

1. If marginal production cost plus per unit transport cost divided by the number of retail outlets, $c + t/n$, is greater than the consumers' reservation price V, the monopolist should set a price at each shop of $p^* = (V + c)/2$ and supply only part of the market.
2. If marginal production plus per unit transport cost divided by the number of retail outlets, $c + t/n$, is less than the consumers' reservation price V, the monopolist should set a price at each shop of $p(n) = V - t/2n$ and supply the entire market.

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Derivation Checkpoint

Optimal Partial Market Price

Assume that the left-hand shop is located at $^1/_4$ mile. At a price p this shop sells to consumers located within distance r on each side such that $p + tr = V$, or $r = (V - p)/t$. Total demand for this shop is therefore $2rN$. Profit to this shop is therefore:

$$\pi = 2N(p - c)(V - p)/t$$

Differentiating with respect to p gives the first order condition:

$$\frac{\partial \pi}{\partial p} = \frac{2N}{t}(V - 2p + c) = 0$$

Solving for the optimal price p^* then yields

$$p^* = (V + c)/2$$

At this price, profit is:

$$\pi = \frac{N}{2t}(V - c)^2$$

The intuition behind this rule is relatively straightforward. When the consumer reservation price is low relative to production and transportation costs and when there are few outlets, trying to supply the entire market gives the monopolist a very low margin over operating costs and could even lead to selling at a loss. By contrast, when the consumer reservation price is high relative to the cost of production and transportation and there are many outlets, a price that allows the monopolist to supply the entire market offers a reasonable margin over costs. In these latter circumstances, the monopolist will not wish to set a high price that sacrifices any sales. Since the marginal revenue of every unit sold significantly exceeds the production cost, the monopolist will wish to sell all the units it can.

7.3 IS THERE TOO MUCH PRODUCT VARIETY?

The profit-maximizing firm with market power may have an incentive to create a large number of outlets or product varieties so as to provide each consumer with something close to her most preferred location and thereby extract a high price. It is easy to think of real-world firms that, while not pure monopolists, have substantial market power and employ this strategy. For instance, automobile manufacturers market many varieties of compact, midsize, and large, luxury class cars. Franchise operations such as McDonald's or Subway grant exclusive geographic rights so as to space their outlets evenly over an area and avoid competition between neighboring franchises. Telephones come in an increasingly wide variety of styles and colors. Soft drink, cereal, and ice cream companies offer a wide array of minimally differentiated goods.

The sometimes overwhelming degree of product variety that we frequently observe raises the question whether the incentive to offer a variety of product types could be too strong. Alternatively, does the monopolist provide the degree of product variety consistent with maximizing social welfare? Or are the incentives so strong that the monopolist provides too much product variety? To answer this question, we first need to describe the socially optimal degree of product variety. Although the argument can be made in general terms, it is easiest to see the answer for the case in which the entire market is served.

We use the efficiency criterion to determine the optimality of the variety of products offered. This requires that we maximize the total net surplus, i.e., consumer valuation minus cost. In this light, note that once the entire market is served, the total value to consumers of the output is unchanged no matter how many shops the monopolist operates. That is, once all N consumers are buying the product, the total value placed by consumers on this production is $N.V$ no matter how many shops, or product variants, there are. Similarly, once all N consumers are served, the total variable production cost is constant at cN, again regardless of the number of outlets.

Once all N consumers are served, only two factors change as more stores or product varieties are added. One of these is the transport cost incurred by a typical or average consumer. Clearly, as more shops are added more consumers find themselves closer to a store and this cost falls. That's the good news. The bad news is that adding more shops also incurs the additional set-up cost of F per shop. Our question about whether the monopolist provides too many (or too few) shops thus comes down to determining whether it is the good news or the bad news that dominates. More formally, when all consumers are served, the total surplus is the total value NV minus the total production cost cN minus the total transportation and set-up costs. Since the first two terms are fixed independent of the number of shops, maximizing the net social surplus is equivalent to minimizing the sum of transportation and set-up costs. Does the monopolist's strategy achieve this result?

One feature of the monopoly outcome makes answering this question a little easier. It is the fact that the monopolist always spaces its shops evenly along Main Street no matter how many it operates. That is, a single shop is located at the center; two shops are located at 1/4 and 3/4, and so on. This feature greatly facilitates the calculation of total transportation cost that consumers incur for any number of shops n.

Consider Figure 7.6 which shows both the full price and the transportation cost paid by those consumers buying from a particular outlet, shop i. As before, the top Y-shaped figure shows that a consumer located right next to the store has no transport cost and pays a price

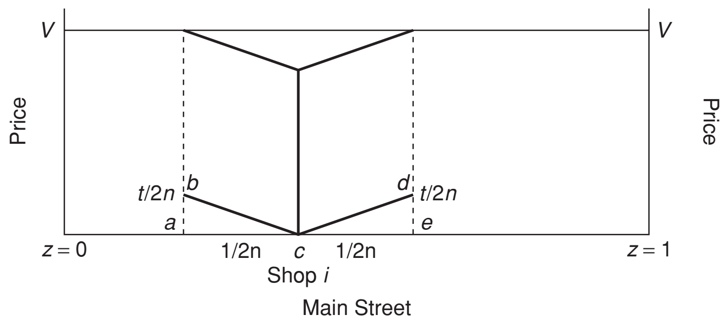

Figure 7.6 Cost of serving customers when there are n shops

of $p = V - \frac{t}{2n}$. As we consider consumers farther from the shop, the branches of the Y show that the full price rises because these consumers incur greater and greater transportation costs. The lower branches in the figure provide a direct measurement of this transportation cost for each such consumer. Again, the transportation cost for a consumer located right next to the store is zero. It rises gradually to $t/2n$—the transportation cost paid by a consumer who lives the maximum distance from the shop. Total transportation cost for the consumers of shop i is the sum of the individual transportation cost of each consumer.

This is indicated by the areas of the symmetric triangles *abc* and *cde* in Figure 7.6. Each of these triangles extends to a height of $t/2n$. Each also has a base $1/2n$. Hence, the area of each is $t/8n^2$. Remember though that the base reflects the fraction of the total N consumers that shop i serves in either direction. This means that to translate this area into actual dollars of transportation costs that the consumers of shop i pay we have to multiply through by N. The result is that the customers who patronize shop i from the east pay a total transportation cost of $tN/8n^2$ as do those who patronize it from the west. The total transportation costs incurred by all consumers of shop i is therefore the sum of these two amounts or $tN/4n^2$. If we now multiply this by the number of shops n, we find that the total transportation costs associated with all n shops is simply $tN/4n$. The same exercise tells us that the total set-up cost for all n shops is nF. Accordingly, the transportation plus set-up costs associated with serving all N customers and operating n shops is:

$$C(N, n) = \frac{tN}{4n} + nF \tag{7.14}$$

By the same argument, total transportation plus set-up costs with $(n + 1)$ shops are

$$C(N, n + 1) = \frac{tN}{4(n + 1)} + (n + 1)F \tag{7.15}$$

Recall that our goal is to minimize this total cost. Therefore, we will always wish to add an additional shop so long as total costs fall. Comparison of equations (7.15) and (7.14) indicates that this will be the case, i.e., $C(n + 1) < C(n)$, if:

$$\frac{tN}{4n} - \frac{tN}{4(n + 1)} > F \tag{7.16}$$

Simplification of this inequality reveals that it will be socially beneficial to add one more shop or one more product variant beyond the n existing ones so long as:

$$n(n + 1) < \frac{tN}{4F} \tag{7.17}$$

Now compare this condition with that of equation (7.12), which describes the condition under which the monopolist will wish to add an additional shop. The denominator of the right-hand-side term is $2F$ in equation (7.12) while it is $4F$ in (7.17). This means that it is less likely for an additional shop to meet the requirement of equation (7.17) and be socially desirable than it is for it to meet the requirement of equation (7.12) and enhance the monopolist's profit. In other words, the monopolist has an incentive to expand product variety even when the social gains from doing so have been exhausted. The monopolist chooses too great a degree of product variety.

Taking the same example that we had earlier in which $t = \$1$, $N = 5{,}000{,}000$, and $F = \$50{,}000$, we have that $tN/4F = 25$. In this case, equation (7.17) implies that the socially optimal number of shops is five. However, we have already shown that the monopolist would like to operate seven shops or offer seven product varieties in this market. In short, the monopolist offers too much product variety.

Casual evidence supports the "too much variety" hypothesis. Look at the myriad of ready-to-eat breakfast cereals offered by the major cereal firms, the multitude of options available on automobiles, and the vast array of finely distinguished perfumes and lipsticks available at department stores around the country. Admittedly, the producers of these goods are not pure monopolists but they do exercise considerable market power and so they are likely subject to many of the same influences that we have just been considering.

The basic reason that a monopolist offers too much variety is because the firm maximizes profit, not total surplus. When deciding to add another shop, the monopolist balances the additional set-up cost against the additional revenues that it can earn from being able to increase prices. However, from the viewpoint of efficiency this additional revenue is not a net gain. It is just a transfer of surplus from consumers to the monopolist. The true social optimum would balance the set-up cost of an extra shop against the reduction in transportation costs that results. Clearly, this criterion will lead to the establishment of fewer shops than will the criterion used by the monopolist.

Operating additional shops is attractive to the monopolist because it permits charging a high price to distant consumers. The monopolist operating just one shop at the center of Main Street could otherwise only sell to those distant consumers by offering a steep price cut that must be extended to all customers. Some variety is good. That is why the optimum in our

Reality Checkpoint

You Will Soon be able to Buy a Sandwich Anywhere

McDonald's has for many years been the model of a successful franchise operation. Approximately 85 percent of McDonald's outlets are operated by franchisees who have also been responsible for more than 60 percent of their annual revenues. Despite some recent problems that have led McDonald's to close more than 700 under-performing restaurants, the company still has some 30,000 outlets worldwide making it one of the largest owners of retail property in the world. There is, however, a threat looming on the horizon. An increased demand for healthier fast-food options has diverted demand to newer restaurant chains offering an alternative to the fat-rich traditional hamburger and fries. For the eleventh time in 15 years Entrepreneur magazine has named Subway as the number one franchise opportunity. Subway now has more than 18,000 locations in 72 countries and has more restaurants than McDonald's in the United States and Canada. Subway aims to open a further 2,000 outlets in the United States this year as well as expanding in the United Kingdom, Eastern Europe, and India.

Sources: *Financial Times*, June 4, 2003 "Healthier Options are in Demand"; "Landmark 18,000th Subway Restaurant Opens" http://www.prnewswire.com.

example is five shops. The monopolist goes beyond this point because while it reduces total surplus it grabs much more of that reduced total for the firm.[4]

Our discussion of excess variety raises another issue that takes us back to the discussion in Chapters 5 and 6. If somehow the firm could charge a price to distant consumers that they are willing to pay *without* lowering the price to nearby ones then reaching these distant consumers from just a few shops or with just a few varieties would be more attractive. That is, if the monopolist could price discriminate her tendency to oversupply variety might be much less strong. We now examine this possibility.

7.4 MONOPOLY AND HORIZONTAL DIFFERENTIATION WITH PRICE DISCRIMINATION

Our discussion so far has assumed that the monopolist does not price discriminate between its customers. This makes sense when customers travel to the shop to purchase the good and so do not reveal their addresses, or who they are to the monopolist. Suppose instead that the monopolist controls delivery of the product, and so will know who is who by their address in the market. What pricing policy might we expect the monopoly firm to adopt?

First, it should be clear that when the monopolist makes the delivery it will charge every consumer the consumer's reservation price V. This is a pricing policy known as *uniform delivered pricing*. A firm adopting such a pricing policy charges all consumers the same prices and absorbs the transportation costs in delivering the product to them. This is discriminatory pricing because even though consumers pay the same price, this price does not reflect the true costs of supplying consumers in different locations. By way of analogy, charging a consumer in San Francisco the same price as a consumer in New York for a product manufactured in New York is just as much discriminatory pricing as charging a different price for this product to two different New York residents.

As in the no-price-discrimination case, we should check whether the monopoly firm actually wants to supply every consumer. Suppose that, as before, the firm operates n retail outlets evenly spaced along Main Street. Then the transportation and production costs that the firm incurs in supplying the consumers located furthest away from a retail outlet are $c + t/2n$. There is profit to be made from such sales provided that:

$$V > c + t/2n \tag{7.18}$$

Notice that this is a weaker condition than equation (7.13) without price discrimination. This is another example of a typical property of price discrimination. It allows the monopolist to serve consumers who might otherwise be left unserved.

Now consider how many shops (or product varieties) the price-discriminating monopolist would choose to operate. Given that the firm is supplying the entire market and is charging every consumer the consumer's reservation price of V, the firm's total revenue is fixed at $N.V$. Total costs are variable production costs, which are fixed at cN, plus the transport costs that the firm absorbs and the set-up costs nF. These two latter costs are just the costs $C(N, n)$ from equation (7.14). So the profit of the price-discriminating monopolist is:

[4] Spence (1976) is a classic analysis of optimal product variety.

$$\pi(N, n) = NV - cN - \left(\frac{tN}{4n} + nF\right) \tag{7.19}$$

How does the monopolist maximize profit in this case? Here, profit maximization is achieved by minimizing the costs $C(N, n)$ since total revenue and production costs are fixed. But this means that *the discriminating monopolist will choose to offer the socially efficient degree of product variety*.

If you recall our discussion of price discrimination in Chapter 5 you should not find this too surprising. We saw in that chapter that a monopolist who engages in first-degree price discrimination will extract all consumer surplus and therefore will want to produce the efficient amount of output. The result just obtained extends that finding to the case of a product differentiated market. In such a market, a firm that can achieve first-degree price discrimination will not only produce the socially efficient output but also the socially efficient amount of variety. With price discrimination the firm can reach far away customers by means of a price reduction (i.e., the firm pays a relatively high delivery charge) rather than by adding on an extra variety. Hence, the incentive to go beyond the socially optimal amount of variety does not exist.

Price discrimination in a geographic spatial model has a clear interpretation—the monopoly firm incurs the delivery cost and doing so charges different net prices for the same good. How do we interpret price discrimination in a product characteristics space rather than a geographic one? Alternatively, how can a monopolist control "delivery" of products that are differentiated by characteristics rather than by location? MacLeod, Norman and Thisse (1988) provide the analogy:

> In the context of product differentiation, price discrimination arises when the producer begins with a "base product" and then redesigns this product to the customers' specifications. This means that the firm now produces a *band* of horizontally differentiated products . . . instead of a single product . . . Transport cost is no longer interpreted as a utility loss, but as an additional cost incurred by the firm in adapting its product to the customers' requirements . . . [So] long as product design is under the control of the producer—equivalent to the producer controlling transportation—he need not charge the full cost of design change. (1988, pp. 442–3)

Consider, for example, buying a Ford Taurus. On the one hand you might choose one of the standard variants. Alternatively, the salesperson might persuade you into taking a different sound system, different wheels, an attractive stripe along the side that makes the car sportier, and so on. Effectively, what he is doing is making you reveal your actual "address" through the options you choose with perhaps the intention of also separating you from more of your money.

But how easy is it for firms to offer this type of product customization? After all, customization would seem to imply the sacrifice of economies of scale and so increased costs. You might be surprised to learn that it is becoming easier and less costly by the day. The ability to offer this type of product range is what distinguishes *flexible manufacturing systems*, defined as "production units capable of producing a range of discrete products with a minimum of manual intervention" (U.S. Office of Technology Assessment, 1984, p. 60). We discussed the properties of these types of manufacturing processes in Chapter 4, and they are becoming increasingly common. Companies such as Levi Strauss, Custom Shoe, Italian ceramic tile manufacturers, Ford, Mitsubishi, and Hitachi all operate flexible manufacturing systems. Any of you who regularly use web-retailers such as Amazon.com will have noticed that the initial page you see on entering the site changes over time to reflect your buying habits. You eventually have your very own, customized entry page resulting in your very own, customized prices.[5]

7.1 Practice Problem

Henry Shortchap is the only blacksmith in the small village of Chestnut Tree. The village is comprised of 21 households evenly distributed one-tenth of a mile apart along the main street of the town. Each such household uses at most 1 unit of smithing services per month. In addition, each household incurs a there-and-back-again transport cost of $.50 for every tenth of a mile it lives from Shortchap's smithery. The reservation price of each household for such services is $10. Henry's cost of providing smithing services is $2 per unit. However, he can operate only 1 shop at most. Where should Henry locate his shop and what price should he charge? Suppose instead that Henry could operate a mobile smithy that allowed him to offer his services at his customers' homes. However, it would cost him $.75 there-and-back-again transport costs for every tenth of a mile he has to move his smithy. Should he switch to this mobile service?

7.5 VERTICAL PRODUCT DIFFERENTIATION

The distinguishing feature of horizontal product differentiation is that consumers do not agree on what is the preferred variety of product, and so if two different varieties are offered at the same price some consumers are likely to buy one variety and other consumers will buy the other. *Vertical differentiation* is different. In this case all consumers agree on what is the preferred or best product, the next to best product and so on. If a high and a low quality good are offered at the same price, all consumers will buy the high quality good. Lower-quality goods will find a market only if they are offered at sufficiently lower prices. A Chevrolet costs much less than a Cadillac. No-frills airlines such as JetBlue and SouthWest attract customers because their flights are offered at large discounts relative to the larger carriers such as United and American. The Peninsula Hotel in New York charges much less than the Waldorf Astoria. However, while all consumers agree in their ranking of products from highest to lowest quality, they differ in their willingness to pay for quality. This may occur because consumers have very different incomes or simply because they have different attitudes regarding what quality is worth.

We would like to understand the incentives a monopoly firm has to offer different qualities of a product and the prices that it will charge for them. The analysis we use for this purpose is a simplified one. Nevertheless, it captures much of the flavor of more general treatments.[6]

7.5.1 Price and Quality Choice with Just One Product

We first consider how changes in quality change affect consumer demand when the firm offers just one product and each consumer buys at most one unit of the good. This will give

[5] A clear if brief expression of the view that e-commerce firms greatly facilitate price discrimination may be found in P. Krugman, "Reckonings: What Price Fairness," *New York Times*, October 4, 2000, p. A16.

[6] The first and classic treatment of this problem is Mussa and Rosen (1978). Unfortunately, this is also a rather complex analysis.

us some idea of how quality or product design can be used to enhance a firm's profit. Then we will examine how the firm might increase its profit by offering more than one quality of a product. The firm knows that there is some feature, or set of features, that can be used to measure the product quality valued by consumers. The firm's ability to choose these features means that it can choose the quality of the product as well as its price.

The firm knows that each consumer is willing to pay something extra to get a higher quality product, but the precise amount extra varies across consumers. Some consumers place a high value on quality and will gladly pay a considerable premium for a quality improvement. Others are less concerned with quality and, unless the accompanying price increase is minimal, such consumers would not buy a high priced better quality good. In other words, each consumer examines the price and quality of the product and the utility she obtains from consuming it. If the consumer places a value on the quality of product offered greater than the price being charged, she purchases the good—say a CD player. If not, she simply refrains from buying altogether.

The demand curve facing the monopolist will depend on precisely what is the quality of the product being marketed. This is reflected in the inverse demand function, denoted by $P = P(Q, z)$, where the market clearing price P will depend not only on how much the firm produces Q but also on the quality z of these units. To put it somewhat differently, an increase in quality z will raise the market-clearing price for any given quantity, Q. The demand curve shifts out (or up) as product quality z increases.

It is useful to distinguish between two different ways an increase in quality can shift the inverse demand curve $P(Q, z)$. Each is illustrated in Figure 7.7. To better understand this diagram note that, since consumers vary in terms of their willingness to pay for a good of given quality z, and since each consumer buys at most one unit of the good, the demand curve really reflects a ranking of consumers in terms of their reservation price for a good of a specific quality z. The reservation price of the consumer most willing to pay for the good is the intercept term. The reservation price of the consumer next most willing to pay is the next point on the demand curve as we move to the right, and so on. In both Figures 7.7(a) and 7.7(b), the initial quantity produced is Q_1 and the initial quality is z_1. The market-clearing price for this quantity–quality combination is P_1. From what we have just said, this price must be the willingness to pay or reservation price of the Q_1th consumer. At price P_1, this consumer is just indifferent between buying the good and not buying it at all given that

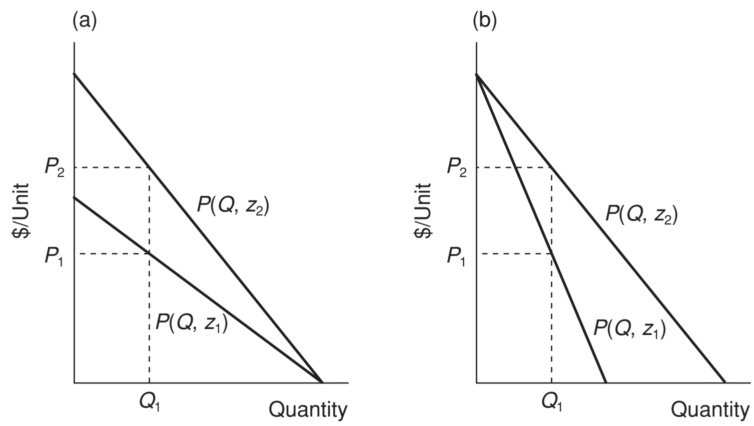

Figure 7.7 Impact of quality on demand

it is of quality z_1. This consumer is called the marginal consumer. Consumers to her left—those consumers who also buy the product—are called *infra*marginal consumers.

Figure 7.7(a) shows how the inverse demand curve shifts when there is an increase in quality that raises the willingness to pay of the inframarginal consumers by more than it raises the willingness to pay of the marginal consumer. An increase in quality from z_1 to z_2 raises the price at which the quantity Q_1 sells from P_1 to P_2. However, the increase in the reservation price is greatest for consumers who were already purchasing the product so that the demand curve shifts by "sliding along" the price axis. Figure 7.7(b) illustrates the alternative case. Here, the increase in quality from z_1 to z_2 increases the willingness to pay of the Q_1th or marginal consumer by proportionately more than it raises the reservation price of the inframarginal consumers. Once again, this quality increase will raise the market price of quantity Q_1 from P_1 to P_2. However, the demand curve now shifts by "sliding along" the quantity axis.[7]

Whether demand is described by Figure 7.7(a) or 7.7(b), we can see that for the monopolist the choice of quality really amounts to a decision as to which demand curve it will face. Increases in quality are attractive because they rotate the demand curve and so increase the firm's revenue at any given price. However, it is normally costly to increase product quality. So what the monopolist has to do is balance the benefits in increased revenue that improved quality generates against the increased costs that increased quality incurs. More precisely, the monopolist choosing quality should think through the profit-maximizing calculus in a way similar to that used when choosing output or price. For any given choice of output, the monopolist should choose the level of quality at which the marginal revenue from increasing quality equals the marginal cost of increasing quality. In other words, the monopolist who controls both quality and quantity of product has *two* profit maximizing conditions to satisfy. These are:

1. For a given choice of quality, the marginal revenue from the last unit sold should equal the marginal cost of making that unit at that quality.
2. For a given choice of quantity, the marginal revenue from increasing quality of each unit of output should equal the additional (marginal) cost of increasing the quality of that quantity of output.

Let us illustrate the quality or product design choice by assuming that demand is of the type shown in Figure 7.7(a). More specifically, let us assume that the demand function is given by the equation:

$$P = z(50 - Q) \tag{7.20}$$

Equation (7.20) says that regardless of the quality of the product, at a price of zero a total of 50 units will be sold but increased quality causes the demand function to rotate clockwise about the point $Q = 50$.

Let us also keep the example simple by assuming that the cost of improving quality is a sunk design cost so that marginal *production* cost is independent of the quality of the product. A better film or software package may require say, more expensive script or

[7] For models based on the case illustrated in Figure 7.9(b), some care must be taken to limit the ultimate size of the market. That is, quality increases cannot indefinitely expand the quantity demanded at a given price.

programming, respectively, but the actual costs of showing the film or printing the CD are independent of how good it is. To make matters even simpler, let us further assume that production costs are not only constant but also zero. Design costs, however, rise with the quality level chosen. Specifically, we shall assume that:

$$F(z) = 5z^2 \tag{7.21}$$

which implies that the marginal cost of increasing product quality is $2z$ (see inset). We can now write the firm's profit to be:

$$\pi(Q, z) = P(Q, z)Q - F(z) = z(50 - Q)Q - 5z^2 \tag{7.22}$$

Consider first the profit-maximizing choice of output. This turns out to be very simple in this case. As usual, marginal revenue has the same intercept as the demand function

Derivation Checkpoint

Optimal Choice of Output and Quality

Demand and costs are given respectively by:

$P = z(\theta - Q)$ and $C(Q, z) = \alpha z^2$

Hence the profit function is:

$$\pi(Q, z) = PQ - C(Q, z) = z(\theta - Q)Q - \alpha z^2$$

Differentiating this with respect to Q gives the first-order condition:

$$\frac{\partial \pi(Q, z)}{\partial Q} = z(\theta - 2Q) = 0$$

Solving for the optimal output Q^* then yields:

$$Q^* = \theta/2$$

Now differentiate profit with respect to quality choice z to obtain the first-order condition:

$$\frac{\partial \pi(Q, z)}{\partial z} = (\theta - Q)Q - 2\alpha z = 0$$

Substitution of Q^* for Q in this last equation then implies:

$$2\alpha z = \theta^2/4$$

Solving this last expression for the optimal quality z^*, then yields:

$$z^* = \theta^2/8\alpha$$

In the text example, $\theta = 50$ and $\alpha = 5$. Hence, $Q^* = 25$ and $z^* = 62.5$.

but twice the slope. So with the demand function $P = 50z - zQ$ we know that marginal revenue is $MR = 50z - 2zQ = z(50 - 2Q)$. Equating this with marginal cost gives the profit-maximizing output condition: $z(50 - 2Q) = 0$. Hence, the profit-maximizing output is: $Q^* = 50/2 = 25$.

In this simple example the monopolist's choice regarding the quantity is independent of the choice of quality. The profit-maximizing output remains constant at $Q^* = 25$ no matter the choice of quality. Going back to the demand function, the profit-maximizing price is given by $P^* = z(50 - Q^*)$ so that the optimal price is: $P^* = 50z/2$. Unlike output, the profit-maximizing price is affected by the choice of quality. Moreover, the quality choice will also affect the firm's design costs. A higher quality design z permits the monopolist to raise price and earn more revenue but it also raises the firm's costs. This is the tradeoff that the monopolist must evaluate.

To choose the profit-maximizing quality the monopolist must compare the additional revenue resulting from an increase in z with the increase in design cost that the higher quality requires. Because the quantity sold is constant at $Q^* = 25$, the additional revenue of an increase in quality is just this output level times the difference in price that can be charged following the rise in quality. In our example, we can see from equation (7.20) that the firm's revenue PQ at product quality z when it charges the profit-maximizing price $P^* = 50z/2$, is:

$$P^*Q^* = \frac{50z}{2} \cdot \frac{50}{2} = \frac{2{,}500}{4}z = 625z \tag{7.23}$$

Increasing product quality by one "unit" increases revenue by \$625, which is therefore the marginal revenue from increased quality. We know also from equation (7.21) that the marginal cost of increased quality is $10z$. Equating marginal revenue with marginal cost then gives the profit-maximizing quality choice:

$$z^* = 625/10 = 62.5 \tag{7.24}$$

An interesting question that arises in connection with the monopoly firm's choice of quality is how that choice compares with the socially optimal one. Does the monopolist produce too high or too low a quality of good? Looking at Figure 7.7(a) you might be able to work out that in this case the monopolist's quality choice will be too low. The reason is straightforward. An increase in z rotates the demand curve upward and increases the total surplus earned from the 25 units that are always sold. The social optimum requires that quality be increased so long as this gain in the total surplus exceeds the extra design cost. However, the monopolist only gets to keep the increase in producer surplus that a quality increase generates. As a result, profit maximization will lead the monopoly firm to increase quality only so long as the extra producer surplus covers the additional design cost. Since the producer surplus is less than the total surplus, the monopolist will stop short of producing the socially optimal quality. Of course, the monopolist holds quantity below the optimal amount, too.[8]

Our primary objective is not to determine whether firms with monopoly power choose to market products of either too low or too much quality. The main point is to show that for such firms the quality choice matters. By carefully choosing product quality jointly with product price, the monopolist can again extract further surplus from the market.

[8] Our results regarding the monopolist's quality choice might have been different had we instead assumed that quality affects demand as in Figure 7.9(b).

7.2 **Practice Problem**

Will Barret is the only lawyer in the small country town of Percyville. The weekly demand for his legal services depends on the quality of service he provides as reflected by his inverse demand curve: $P = 4 - Q/z$. Here, P is the price per case; Q is the number of cases or clients; and z is the quality of service Will provides. Will's costs are independent of how many cases he actually takes. But they do rise with quality. More specifically, Will's costs are given by: $C = z^2$.

a. Draw Will's demand curve for a given quality, z. How do increases in z affect the demand curve?
b. Consider the three options: $z = 1$, $z = 2$, and $z = 3$. Derive the profit maximizing output for each of these choices.
c. Compute the market price and profit—net of quality costs—for each of the three choices above. Which quality choice leads to the highest profits?

Reality Checkpoint

Room Service? We'd Like a Baby and a Bottle of Your Best Champagne!

Although hospitals are always under pressure to rein in costs, they are also always on the lookout for ways to raise quality—at least for those who are willing to pay for it. Nowhere is this more apparent than in the recent splurge in spending on deluxe maternity wards. Moms-to-be who are willing to pay a little more can get private suites, whirlpool baths, Internet access, and top culinary food. For an extra fee, St. Vincent's Hospital in Indianapolis throws in a massage and the services of a professional photographer. Robert Wood Johnson Hospital in New Jersey allows patients to order from a restaurant-style menu and serves a "high tea" daily at 3:00 p.m. Offering such services is part of trend that started some years ago as a means to attract customers, especially wealthy ones. Matilda Hospital in Hong Kong is one of a number of hospitals that particularly courts those interested in a luxury hospital stay. Their three-day maternity package includes four-star cuisine meal service with champagne while staying in a beautiful private room with molding-trimmed high ceilings, cherry finished wood floors, and balcony that overlooks the sea far down below. Talk about vertical differentiation!

Sources: J. Barshay, "Luxury Rooms Are Latest Fads for Private Hospitals in Asia," *Wall Street Journal*, January 26, 2001, p. A1; and P. Davies "Hospitals Build Deluxe Wings for New Moms," *Wall Street Journal*, February 8, 2005.

7.5.2 Offering More Than One Product in a Vertically Differentiated Market

Now that we have worked through the basics of how product quality can affect market demand, let's consider a multi-product strategy. To make it simple, suppose that monopolist knows there are only two types of consumer, distinguished by their willingness to pay for quality. Each consumer type buys at most one unit of the firm's product per period. In deciding which quality of product to buy the consumer buys the quality of product yielding the greatest consumer surplus. For consumer type i the indirect utility obtained from consuming a product of quality z at price p is:

$$V_i = \theta_i(z - \underline{z}_i) - p \qquad (i = 1, 2) \tag{7.25}$$

In this equation θ_i is a measure of the value that consumers type i places on quality and $\underline{z}_i$ is the lower bound on quality below which a consumer type i would not buy a product . . . We assume that $\theta_1 > \theta_2$. That is, type 1 consumers place a higher value on quality than type 2 consumers, perhaps because type 1 consumers have higher incomes than type 2 consumers or more generally because they have more intense preferences for quality. We also assume that $\underline{z}_1 > \underline{z}_2 = 0$. In other words, type 1 consumers will not buy the monopolist's product unless it is at least of quality $\underline{z}_1$. These are consumers who "wouldn't be seen dead" flying in coach, eating in fast-food outlets or shopping in discount stores. By contrast, type 2 consumers are willing to buy the monopolist's product of any quality provided, of course, that consumer surplus is non-negative.

Unfortunately for the firm, while it knows that these different consumer types exist it has no objective measure by which it can distinguish the different types. Similar to second-degree price discrimination, the monopolist would like to choose a product line that makes the consumers reveal their true types. The monopolist would like to induce type 1 consumers to buy a product of high quality z_1 at a high price while simultaneously inducing type 2 consumers to purchase a product of low quality z_2 at a lower price. As in second-degree price discrimination, the type 2 consumers will be charged a price that is equal to their willingness to pay for the low quality product.

Suppose that the firm is able to produce any quality in the quality range $[\underline{z}, \bar{z}]$. For simplicity we also assume that the marginal costs of production are constant and identical across all qualities of product and are equal to zero.[9] Finally, we make the following important assumption (we explain why below):

Assumption 1: $\bar{z} > \dfrac{\theta_1 \underline{z}_1}{(\theta_1 - \theta_2)}$.

Note that Assumption 1 is most easily satisfied when the difference between θ_1 and θ_2 is relatively large.

Let's look first at a consumer of type 2, the one with a low willingness to pay for quality. What the firm will do is charge this consumer a price that is just low enough for her to be willing to purchase the low quality product. From equation (7.25), and given that $\underline{z}_2 = 0$, consumer type 2 will buy z_2 if:

$$p_2 = \theta_2 z_2 \tag{7.26}$$

Now consider a consumer of type 1 with a stronger preference for quality. This consumer can, of course, buy the low quality product. So, in pricing the high quality product the firm faces the same type of *incentive compatibility constraint* that we met when discussing second-degree price discrimination. (There is also the incentive compatibility constraint, of course,

[9] It might be, for example, that the majority of the firm's costs are set-up costs and that crimping higher quality products makes lower quality products. Relaxing this assumption doesn't change much. Having it makes the analysis a bit easier.

that the type 2 consumers do, indeed, buy the low rather than the high quality product. We return to this below.) For a type 1 consumer to buy the high quality product it is necessary that:

$$\begin{aligned} &\theta_1(z_1 - \underline{z}_1) - p_1 \geq \theta_1(z_2 - \underline{z}_1) - p_2 \\ &\theta_1(z_1 - \underline{z}_1) - p_1 \geq 0 \end{aligned} \tag{7.27}$$

The expressions in (7.27) say that the consumer surplus that a type 1 consumer obtains from buying the high quality product must be non-negative and greater than or equal to the consumer surplus that could be obtained if the type 1 consumer bought the low quality good. Substituting $p_2 = \theta_2 z_2$ from (7.26) into the first expression in (7.27) we find that:

$$p_1 \leq \theta_1 z_1 - (\theta_1 - \theta_2)z_2 \tag{7.28}$$

Equation (7.28) says that the maximum price p_1 that can be charged for the high quality product is $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$. The price is greater the higher are the values θ_1 and θ_2 that the two types of consumers place on quality, and the higher is the quality differential between z_1 and z_2. That is, quality can be priced more highly when it is valued more highly by all consumers. And because by offering two products of different qualities the monopolist is effectively competing with itself, increasing the quality differential between the products, the monopolist makes the two goods more differentiated. The monopolist thereby weakens the competition between products and allows the firm to increase the price of its high quality product. Note that when $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$ the condition that consumers of type 1 receive non-negative surplus when they buy the high quality good can now be written as $\theta_1(z_1 - \underline{z}_1) - p_1 \geq 0 \Rightarrow (\theta_1 - \theta_2)z_2 - \theta_1\underline{z}_1 \geq 0$. This explains why we need Assumption 1 $\bar{z} > \dfrac{\theta_1 \underline{z}_1}{(\theta_1 - \theta_2)}$. Given that this assumption holds, the condition that type 1 consumers have non negative surplus can always be satisfied by some $z_2 \leq \bar{z}$.

It is easy to check that the incentive compatibility constraint is always satisfied for type 2 consumers. For this type of consumer *not* to want to buy the high quality product it must be the case that $\theta_2 z_{\underline{1}} - p_1 < 0$ which given that $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$ implies $-(\theta_1 - \theta_2)z_1 + (\theta_1 - \theta_2)z_2 < 0$. Since $z_1 > z_2$ and $\theta_1 > \theta_2$ this must be true. In other words, the prices given by (7.26) and (7.28) guarantee that type 1 consumers buy the high quality product and type 2 consumers buy the low quality product.

Now assume that there are N_i consumers of each type. Furthermore suppose that variable costs of production do not depend on quality and so for simplicity we set the unit production costs of each good $c_1 = c_2 = 0$. Again for simplicity assume that there are no fixed costs as well. Given that $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$ and $p_2 = \theta_2 z_2$ the firm's total profit is:

$$\Pi = N_1 p_1 + N_2 p_2 = N_1\theta_1 z_1 - (N_1\theta_1 - (N_1 + N_2)\theta_2)z_2 \tag{7.29}$$

The issue that we want to address now is what quality of goods, z_1 and z_2, will maximize the firm's profit.

In this respect, it is clear from equation (7.29) that the coefficient on z_1 is the positive, given by $N_1\theta_1$. That is, profit rises as z_1 rises. As a result, the firm should set z_1 as high as possible; that is:

$$z_1^* = \bar{z} \tag{7.30}$$

The firm should set the quality of its highest quality product at the maximum quality level possible.

For z_2 matters are not quite as straightforward. Equation (7.29) tells us that the impact of z_2 upon the monopolist's profit depends upon the sign of the coefficient, $N_1\theta_1 - (N_1 + N_2)\theta_2$. When this term is positive, the monopolist's profit decreases as z_2 increases. When it is negative profit increases as z_2 increases. We need to examine these two cases separately.

Case 1: $N_1\theta_1 > (N_1 + N_2)\theta_2$

From (7.29) it is clear that in this case profit is decreasing in z_2 and so the firm has an incentive to offer two qualities that are as differentiated as possible. The firm will choose $z_1^* = \bar{z}$, and set z_2 as low as is feasible. This does not mean, however, that z_2 can be reduced to its minimum of z. Remember that we must also satisfy the constraint that type 1 consumers receive nonnegative consumer surplus from buying the high quality good $\bar{z}$. That is, $\theta_1(z_1 - \underline{z}_1) - p_1 \geq 0 \Rightarrow (\theta_1 - \theta_2)z_2 - \theta_1\underline{z}_1 \geq 0 \Rightarrow z_2 \geq \dfrac{\theta_1\underline{z}_1}{\theta_1 - \theta_2}$. It follows that the monopolist will choose:

$$z_2^* = \frac{\theta_1\underline{z}_1}{\theta_1 - \theta_2} \tag{7.31}$$

It is here that we see the impact of the assumption that the monopolist cannot distinguish the two consumer types. The monopolist would like to set z_2 even lower than implied by (7.31) but cannot do so if she is to offer products that make the consumers self-select into their true types.

We can now work out the profit-maximizing prices for the two goods. Substituting (7.31) in (7.26) we find $p_2^* = \dfrac{\theta_2\theta_1\underline{z}_1}{\theta_1 - \theta_2}$. Similarly substituting (7.31) and $z_1^* = \bar{z}$ into $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$, we find that $p_1 = \theta_1(\bar{z} - \underline{z}_1)$. In other words, type 1 consumers are charged their maximum willingness to pay for the highest quality possible, $\bar{z}$, and type 2 consumers are charged their maximum willingness to pay for the lower quality $\dfrac{\theta_1\underline{z}_1}{\theta_1 - \theta_2}$. Aggregate profit is:

$$\Pi = N_1(\bar{z} - \underline{z}_1)\theta_1 + N_2\frac{\theta_2\theta_1\underline{z}_1}{\theta_1 - \theta_2} \tag{7.32}$$

Case 2: $N_1\theta_1 < (N_1 + N_2)\theta_2$

If $N_1\theta_1 < (N_1 + N_2)\theta_2$ then it follows from (7.29) that profit is increasing in z_2. In this case the firm should set $z_2^* = z_1^* = \bar{z}$. In other words, the firm should offer only one product and that product should be of the highest possible quality.

It is, perhaps, easier to see the intuition behind this result by rewriting the inequality $N_1\theta_1 < (N_1 + N_2)\theta_2$ as

$$\frac{N_1}{N_1 + N_2} < \frac{\theta_2}{\theta_1} < 1 \tag{7.33}$$

Thus, what we are saying is that if there are not too many type 1 consumers, those who really like quality, or if their willingness to pay for quality is very high relative to that of type 2 consumers, then the firm should only offer one type of good. The intuition is that whenever the monopolist offers two products, the low quality product tends to cannibalize sales from the high quality product.

To be precise, we can see from equation (7.28) that offering both a high quality and a low quality product costs the firm $(\theta_1 - \theta_2)z_2$ in foregone revenue from each type 1 consumer. Accordingly, when there are a lot of type 1 consumers and/or when the difference between θ_1 and θ_2 is large, the firm will want to minimize this cost by offering a second good that is very low in quality so that it does not compete too vigorously with the high quality product. On the other hand when there are roughly equal numbers of consumers of the two types and/or when preferences do not differ greatly across different consumer types the monopolist should offer a single high quality good.

There remains the question of whether the firm in this case should price the high product to sell to both types of consumer or price it to sell only to type 1 consumers.

Selling to both types of consumer requires that the product be priced low at $p = \theta_1\bar{z}$. Assumption 1 then ensures that both consumer types receive non-negative surplus from purchasing the high quality good $\bar{z}$. The firm earns a total profit of $(N_1 + N_2)\theta_2\bar{z}$. On the other hand, selling the high quality good $\bar{z}$ to only type 1 consumers means that the product can be priced at $\theta_1(\bar{z}_1 - \underline{z}_1)$ giving the firm a total profit of $N_1\theta_1(\bar{z}_1 - \underline{z}_1)$. Comparing these two profit levels reveals that selling to both types of consumer is more profitable if:

$$N_1\theta_1(\bar{z} - \underline{z}_1) < (N_1 + N_2)\theta_2\bar{z} \Rightarrow N_1\theta_1 < (N_1 + N_2)\theta_2 \frac{\bar{z}}{(\bar{z} - \underline{z}_1)} \tag{7.34}$$

A close look at equation (7.34) reveals that for this second case in which we have assumed that $N_1\theta_1 < (N_1 + N_2)\theta_2$ the condition in the equation must hold true. Therefore, the monopolist has an incentive in this case to price the product to sell to both consumer types.[10]

7.3 **Practice Problem**

General Foods is a monopolist and knows that its market for Bran Flakes contains two types of consumers. Type-A consumers have indirect utility functions $V_a = 20(z - \underline{z}_1)$ while type-B consumers have indirect utility functions $V_b = 10z$. In each case z is a measure of product quality, which can be chosen from the interval [0,2]. There are N consumers in the market, of which General Foods knows that a fraction η is of type A and the remainder is of type B. Assume marginal cost is $C = 0$.

a. Suppose that General Foods can tell the different consumer types apart and so can charge them different prices for the same quality of breakfast cereal. What is the profit maximizing strategy for General Foods?

Now suppose that General Foods does not know which type of consumer is which.

b. Show how its profit maximizing strategy is determined by η.
c. What is the profit maximizing strategy when $\underline{z}_1 = e$?

[10] Unlike the result in section 7.5.1, the monopolist here chooses the highest quality because there is no cost to increasing z.

7.6 EMPIRICAL APPLICATION
Price Discrimination, Product Variety, and Monopoly versus Competition

We have set our discussion of price discrimination and product variety over the last few chapters in the framework of a monopolized market. Nevertheless the strategies that we have described such as tying and quantity discounts are often practiced by firms that are far from a perfect monopoly. Competitive pressure and price discrimination often go hand-in-hand for at least two reasons. We will show formally in Chapter 14 that when rival firms practice price discrimination it tends to intensify the price competition between them. Moreover, imperfectly competitive firms also have an incentive to pursue discriminatory pricing strategies, as Borenstein (1985) was one of the first to emphasize.

Consider our two-store model from section 7.2. Let us now modify that example in a couple of significant ways. First, we will assume that while consumers have specific locations, they do not have transport costs, i.e., $t = 0$. Second, we will assume that there are two types of consumers both of which number N in total. For each group, the maximum willingness to pay is V. The difference is that the first type of consumers always shops at the store that is closest, no matter what the price at the alternative store. The second group is just the opposite. These consumers always shop wherever the price is lowest.

A little economic reasoning should convince you that a monopolist serving this market will set a price of V and serve all the $2N$ customers, half at each store. Clearly the price cannot be higher than this or no customer will buy the product. However, there is no need to reduce the price. The first set of consumers will not consider the price at any location other than the closest. The second group will consider alternative prices but, since the price is V at each spot, this group also splits evenly between the two stores. The monopolist would then make a total profit of $2(V - c)N$ divided evenly between the two stores. No amount of price discrimination can increase this value.

Now consider what would happen if the two stores were instead owned by two different firms, firm 1 and firm 2. If there were just the first type of consumers who are totally brand loyal to the nearest store, then each of these firms could again charge a price of V. Imagine that they are doing this. Now consider the second group of consumers who always shop where the price is cheapest. At the current price V, these consumers would also be split evenly between the two shops. Each firm would then earn a profit of $(V - c)N$ just as did each of the monopolist's two stores. However, this outcome cannot be an equilibrium. A slight cut in firm 1's price would lose very little profit from its existing customers. Yet because its price would now be less than firm 2's price, it would gain all of the $N/2$ consumers currently at firm 2 who always shop where the price is lowest. Of course, if firm 1 cuts its price, firm 2 will respond with price cut, too. Unfortunately, this brings the price down to all consumers, including the brand loyal ones who do not care about the price. Indeed, at any common price $p_1 = p_2 > c$, each firm will have a strong incentive to cut price to attract the $N/2$ price-sensitive consumer currently shopping at the rival's store. The result will be that prices are driven very close to cost marginal cost c and each store's profit will be very low. In such a setting, we can easily see the incentives that each firm has to implement price discriminatory policies that permit it to cut the price to price-sensitive consumers while still charging V or at least a high price to the brand loyal ones.

The foregoing simple example provides the basic intuition as to why we might observe price discrimination in more price competitive markets. Price discrimination permits competing actively for those consumers who perceive alternatives to the firm's product while still charging a high price to those that do not. If there was only one other brand only a few customers might be tempted to try it. As more brands are available, however, more of the firm's customers are likely to consider an alternative product. Being able to fight for these customers without lowering the price to the cadre of brand loyal buyers then becomes all the more valuable.

The foregoing insight lies at the heart of the paper by Stavins (2001), which examines the influence of competition on the use of discriminatory pricing in the airline industry. For this purpose, she looked at price and other information for 5,804 tickets over 12 different routes on a specific Thursday in September, 1995. Selecting a single day is useful because it eliminates any price differentials due to flying on other days of the week, especially weekend days. A September choice also avoids both peak summer and winter demand periods. The key characteristics that Stavins (2001) looks at are: 1) whether a Saturday night stay-over was required; and 2) whether a 14-day advance purchase was required.

As discussed in Chapter 5, each of these restrictions serves as a means for airlines to identify and separate customers based on how they value their time and their need for flexibility. Prices for tickets requiring a Saturday night stay over or that had to be purchased 14 days prior to departure should sell for less than other tickets. The hypothesis to be tested is that the price discount on these restricted tickets gets bigger the more competition there is on the route. Stavins (2001) constructs a Herfindahl–Hirschman Index *HHI* for each route to serve as a rough measure of that market's competitive pressure.

To test this hypothesis, Stavins (2001) runs two sets of regressions. The first of these serves to confirm that ticket restrictions do indeed translated into discriminatory price differentials. It takes the basic form:

$$p_{ijk} = \beta_0 + \beta_1 R_{ijk} + \beta_2 HHI_i + \beta_3 S_{ij} + \beta_4 First_{ijk} + \beta_5 Days_{ijk} + \beta_6 Z_i + \varepsilon_{ijk} \tag{7.35}$$

Here, p_{ijk} is the (log of) the price of the kth ticket sold by airline j in city-pair market i. R_{ijk} is a dummy variable equal to 1 if there was a restriction on the flight (Saturday night stay-over or pre-purchase requirement) and 0 otherwise. HHI_i is the Herfindahl–Hirschman Index for the ith market. S_{ij} is the market share of airline i in market j. $First_{ijk}$ is a dummy variable equal to 1 if the ticket was for first-class fare and 0 otherwise. $Days_{ijk}$ is the number of days prior to departure that the fare for that ticket was last offered. Z_i is a vector of other market i characteristics such as average income and population. The error term ε_{ijk} is assumed to be normally distributed with a mean of 0.

If ticket restrictions serve as a means of implementing price discrimination then the coefficient β_1 on R_{ijk} should be negative. This would imply that passengers flying the same flight on the same airline paid lower prices if they accepted a requirement that they stay over Saturday night or that they purchase the ticket in advance.

However, simply finding that β_1 is negative only shows that price discrimination occurs. It does not tell us if there is any connection between the extent of such discrimination and the degree of competition in the market. To test this hypothesis, Stavins (2001) runs regressions of the basic form:

$$\begin{aligned} p_{ijk} = \beta_0 &+ \beta_1 R_{ijk} + \beta_2 HHI_i + \beta_3 (HHI_i x R_{ijk}) + \beta_4 S_{ij} + \beta_5 First_{ijk} + \beta_6 Days_{ijk} \\ &+ \beta_7 Z_i + \varepsilon_{ijk} \end{aligned} \tag{7.36}$$

This is exactly the same as the previous regression except that it now includes the interactive term ($HHI_i \times R_{ijk}$), the product of the concentration index and the restricted travel

Table 7.1 Ticket restrictions and air fares

Variable	*Coefficient*	*t-Statistic*	*Coefficient*	*t-Statistic*	*Coefficient*	*t-Statistic*	*Coefficient*	*t-Statistic*
Saturday night stay-over required	−0.249	−2.50	—	—	−0.408	−4.05	—	—
Saturday night stay-over × HHI	—	—	—	—	0.792	3.39	—	—
Advance purchase requirement	—	—	−0.007	−2.16	—	—	−0.023	−5.53
Advance purchase requirement × HHI	—	—	—	—	—	—	0.098	8.38

variables. If the coefficient on this term is positive, it says that the discount associated with, say, a Saturday night stay-over requirement declines as the level of concentration rises. Stavins' (2001) results are shown in Table 7.1.

The first four columns indicate that passengers do indeed pay different prices depending on the restrictions applied to their tickets. These effects are both statistically significant and economically substantial. For example, passengers who accepted the requirement that they not return until after Saturday night paid 25 percent less on average than those who did not accept this restriction even though they were otherwise getting the same flight service.

However, the real issue is how these discounts vary as the extent of competition in the market as measured by *HHI* varies. This is where the next four columns become relevant as they show what happens when the term interacting competition or concentration and ticket restrictions is included. In both cases, the estimated coefficient on the interaction term is positive. This indicates that while ticket restrictions still lead to price reductions, this effect diminishes as the airline route becomes less competitive or has high concentration.

Given the range of *HHI* values observed over the 12 routes Stavins (2001) studies, she estimates that in the most competitive markets, a Saturday night stay-over requirement led to a price reduction of about \$253, whereas in the least competitive ones the same restriction led to a price reduction of only \$165. Likewise, an advance purchase requirement was associated with a price reduction of \$111 in the most competitive markets but a cut of only \$41 in the least competitive markets.

Summary

This chapter has investigated product-differentiated strategies that a monopolist may use when it sells to consumers with diverse tastes. By offering a line of products the firm can better appropriate consumer surplus and increase profit. First we considered horizontal product differentiation. In this scenario, consumers differ in their preferences for specific product characteristics. Some prefer yellow, some black, some soft, some hard, some sweet and some sour. By selling different varieties of the product, the monopolist expands its market and simultaneously enhances its ability to charge customers higher prices in return for selling a variety of product that is close to their most preferred flavor, color, or design.

A feature of this kind of market is that the monopolist tends to offer too much variety—a prediction for which there is a good bit of supportive casual evidence. However, the monopolist's incentive to over-supply variety is mitigated if the firm is able to price discriminate. Indeed, perfect or first-degree price discrimination encourages the firm to offer consumers the socially optimal product variety.

We also investigated the monopolist's strategy when products are vertically differentiated. In this case, all consumers agree that more quality is better, where quality is measured by some feature, or set of features of the product. Consumers however differ in their willingness to pay for quality. In the case where the monopolist offers only one type of product and quality is costly we found that the monopolist may choose too low a quality. The monopolist may also have an incentive in the vertically differentiated case to offer a range of different qualities in order to exploit the differences in consumers' preferences. In doing so however, the firm faces an incentive compatibility constraint and must choose quality and price such that the different types of consumer will purchase the quality targeted to their type. These incentive compatibility constraints are similar to those that affect second-degree price discrimination schemes, as discussed in the previous chapter.

Finally, while we have cast our theoretical discussion mainly in the context of monopoly, the practices we have described are often employed by firms that are far from a perfect monopolist without any significant rivals. This reflects the fact that imperfectly competitive firms often have a very powerful incentive to price discriminate as well. Such discrimination permits them to compete fiercely for price-sensitive consumers who are not brand loyal and who may easily choose an alternative brand while maintaining a high price to those less likely to buy from a competitor. Stavins (2001) offers empirical evidence of this phenomenon in the airline industry.

Problems

1. A monopolist faces the following inverse demand curve: $P = (36 - 2Q)z$; where P is price; Q is her total output; and z is the quality of product she sells. Quality z can take on only one of two values. The monopolist can choose to market a low quality product for which $z = 1$. Alternatively, she can choose to market a high quality product for which $z = 2$. Marginal cost is independent of quality and is constant at zero. Fixed cost, however, depends on the product design and increases with the quality chosen. Specifically, fixed cost is equal to $65z^2$.
   a. Find the monopolist's profits if she maximizes profits *and* chooses a low quality design.
   b. Find the monopolist's profits when she maximizes profits *and* chooses a high quality design.
   c. Comparing your answers to 9a and 9b, what quality choice should the monopolist make?

2. In the early 1970s, the six largest manufacturers of ready-to-eat breakfast cereals shared 95 percent of the market. Over the preceding twenty years, these same manufacturers introduced over eighty new varieties of cereals. How would you evaluate this strategy from the viewpoint of the Hotelling spatial model described earlier in the chapter?

3. Crepe Creations is considering franchising its unique brand of crepes to stall-holders on Hermoza Beach, which is five miles long. CC estimates that on an average day there are 1,000 sunbathers evenly spread along the beach and that each sunbather will buy one crepe per day provided that the price plus any disutility cost does not exceed \$5. Each sunbather incurs a disutility cost of getting up from resting to get a crepe and returning to their beach spot of 25 cents for every $^1/_4$ mile the sunbather has to walk to get to the CC stall. Each crepe costs \$0.50 to make and CC incurs a \$40 overhead cost per day to operate a stall. How many franchises should CC award given that it determines the prices the stall-holders can charge and that it will have a profit-sharing royalty scheme with the stall-holders? What will be the price of a crepe at each stall?

4. Return to problem 3 above. Suppose now that CC requires that each stall-holder deliver the crepes in its own designated territory. How many franchises should now be awarded, if we make the standard assumption that the effort costs of the stall-holders are the same as those of the sunbathers? How would your answer change if the stall holders instead incurred effort costs half as much as those of the sun bathers, i.e., if their costs were 12.5 cents for every quarter mile of distance?

5. Imagine that Dell is considering two versions of a new laptop. One version will meet high performance standards. The other will only meet medium performance standards. To make the second, Sony use cheaper materials and then crimp the keyboard of the high performance machine with the result that the marginal cost of the each product is an identical \$500. There are two types of consumers for the new laptop. "Techies" have the (indirect) utility function $V_t = 2000(z - 1)$. "Norms" have the (indirect) utility function: $V_n = 1000z$, where z is a measure of product quality. Dell can choose the quality level z for each machine from the interval (1,3), subject only to the restriction that the medium performance machine have a lower z quality than the high performance machine. If Dell knows that there are N_t Techies and N_n Norms and if Dell also can identify which type any consumer is, what is its optimum price and product quality strategy?
6. Return to problem 5, above. Assume now that Dell cannot identify each customer but only knows the number of each type. Show that Dell's profit-maximizing strategy is determined by the relative numbers of each type.
7. A monopolist faces an inverse demand curve given by: $P = 22 - Q/100z$, where z is an index of quality. The monopolist incurs a cost per unit of: $c = 2 + z^2$.
   a. How do increases in product quality z affect demand?
   b. Imagine that the firm must choose one of three quality levels: $z = 1$; $z = 2$; and $z = 3$. Which quality choice will maximize the firm's profit? What profit-maximizing output and price are associated with this profit-maximizing quality level?
8. Return to problem 6, above. What is the quality choice that will maximize the social welfare? If the monopolist were constrained to produce this socially optimal quality, what price would she charge?

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Appendix A

Location Choice with Two Shops

We assume in this case that the shops have identical costs. Therefore, the monopolist locates them symmetrically, some distance d from each end of the market.

1. $d \leq 1/4$: Suppose that d is less than 1/4. Then the maximum full price that can be charged if all consumers are to be served is determined by the consumers at the market center.

These consumers can be charged a price p such that when transport costs are included, they pay a full price equal to their reservation price V. In other words, since a consumer located at $x = 1/2$ dictates the highest price that can be charged for any value $d \leq 1/4$, the maximum price the firm can charge and still serve all consumers is:

$$p(d) + t\left(\frac{1}{2} - d\right) = V \quad \Rightarrow \quad p(d) = V - t\left(\frac{1}{2} - d\right) \tag{7.10A1}$$

Aggregate profit at this price is also a function of d and is given by:

$$\pi(d) = [p(d) - c]N = \left(V + td - \frac{t}{2} - c\right)N \tag{7.10A2}$$

This profit increases as d gets larger. Thus, if d is less than 1/4, the firm can increase its profit by making d larger until $d = 1/4$. Hence, d should never be less than 1/4.

2. $d > 1/4$. Suppose now that d is greater than 1/4. Then the maximum price that can be charged if all consumers are to be served is determined by consumers at the endpoints of the market. They will pay a full price equal to their reservation price, so that the maximum price the monopolist can charge is now determined by

$$p(d) + td = V \qquad \Rightarrow \qquad p(d) = V - td \tag{7.10A3}$$

Aggregate profit is now

$$\pi(d) = [p(d) - c]N - (V - td - c)N \tag{7.10A4}$$

This is decreasing in d. If d is greater than 1/4, then the firm can raise profit by making d smaller until $d = 1/4$. Hence, d should never exceed 1/4. Since we have already shown that d should also never be less than 1/4, it follows that profit maximization implies that $d = 1/4$ exactly. The shops should be located symmetrically 1/4 of the total length from each market endpoint. In this way, no consumer will ever travel more than 1/4 of that length to reach a store.

Appendix B

The Monopolist's Choice of Price When Her Shops Have Different Costs

Assume that shop 1 has marginal cost c_1 and shop 2 has marginal cost c_2. Once again, we have to distinguish between the case in which the monopolist chooses to supply only part of the market and the case in which she chooses to supply the entire market.

1. Supply only part of the market

Consider shop 1. At a price p_1 this shop will supply a fraction x_1 of the market, where x_1 is given by $p_1 + tx_1 + V$, which implies $x_1 = (V - p_1)/t$. Profit to this shop is then

$$\pi_1 = (p_1 - c_1)x_1N = (p_1 - c_1)(V - p_1)N/t \qquad (17.11B1)$$

Differentiating with respect to p_1 gives the first-order condition

$$(V - 2p_1 + c_1)N/t = 0 \qquad (17.11B2)$$

The profit-maximizing price for shop 1 is therefore

$$p_1^* = (V + c_1)/2 \qquad (17.11B3)$$

It follows immediately that the profit-maximizing price for shop 2 is

$$p_2^* = (V + c_2)/2 \qquad (17.11B4)$$

2. Supply the entire market

Consider the marginal consumer—the consumer who is indifferent between buying from shop 1 and shop 2. If the entire market is to be served, then this marginal consumer will be charged a full price just equal to his reservation price. If prices are set any lower, profits can always be increased by increasing the price at both shops since there will be no loss of sales. So we know that if the marginal consumer is distance x' from shop 1, $p_1 + tx' = V$, which implies that $x' = (V - p_1)/t$. Also, $p_2 + t(1 - x') = V$. Substituting for x' gives

$$p_2 = V - t(1 - x') = V = t[1 - (V - p_1)/t - 2V - t - p_1] \qquad (17.11B5)$$

In other words, the prices at the two shops are connected. Increasing the price at shop 1 requires that the price at shop 2 be reduced if the entire market is to be served and the price charged to the marginal consumer is to be as high as possible.

Profit to the monopolist is

$$\pi = (p_1 - c_1)x'N + (p_2 - c_2)(1 - x')N \qquad (17.11B6)$$

Substituting for x' and p_2 gives

$$\pi(p_1) = N\left[(p_1 - c_1)\frac{(V - p_1)}{t} + (2V - p_1 - t - c_2)\left(1 - \frac{(V - p_1)}{t}\right)\right] \qquad (17.11B7)$$

Since the monopolist must coordinate the prices at the two shops, profits are fully determined by the price at shop 1. Differentiating profit with respect to p_1 gives

$$\frac{\partial \pi}{\partial p_1} = N\left[\frac{1}{t}(V - 2p_1 + c_1) - \left(1 - \frac{V - p_1}{t}\right) + \frac{1}{t}(2V - p_1 - t - c_2)\right] \qquad (17.11B8)$$

Collecting terms gives the first-order condition

$$\frac{N}{t}(4V - 4p_1 + c_1 - c_2 - 2t) = 0 \qquad (17.11B9)$$

This gives the optimal price at shop 1:

$$p_1^{**} = V + \frac{(c_1 - c_2 - 2t)}{4} \tag{17.11B10}$$

You can confirm that this is the solution in the text when there are two shops and $c_1 = c_2$. Substituting into the equation for p_2 gives the price at shop 2:

$$p_2^{**} = V + \frac{(c_2 - c_1 - 2t)}{4} \tag{17.11B11}$$

The final question is under what circumstances it is better for the monopolist to serve only part of the market. For this to hold, it must be the case that, for example, $p_1^* > p_1^{**}$. This requires that

$$\frac{V + c_1}{2} < V + \frac{(c_1 - c_2 - 2t)}{4} \quad \Rightarrow \quad V < \frac{c_1 + c_2}{2} + t = \bar{c} + t \tag{17.11B12}$$

where $\bar{c}$ is the mean of the marginal production costs of the two shops. So we have a rule that is nearly identical to that in the text:

1. Serve the entire market if the consumer reservation price is greater than the sum of transport costs plus the mean of the shops' marginal costs.
2. Serve only part of the market if the consumer reservation price is less than the sum of transport costs plus the mean of the shops' marginal costs.

The intuition behind this result is exactly the same as that presented in the main body of the chapter.