8 Moments

Introduction

When you use a spanner to tighten or loosen a nut, you are applying a *moment*. A moment is a turning effect and is related to the concept of leverage: if you use a screwdriver to prise open the lid on a can of paint, or a bottle opener to open a bottle of beer, or a crowbar to lift a manhole cover, you are applying leverage and hence you are applying a moment.

Designed by British architect Sir Norman Foster, the modern glass dome shown in Fig. 8.1 is the historic German parliament building's crowning glory.

Fig. 8.1 Reichstag dome, Berlin.

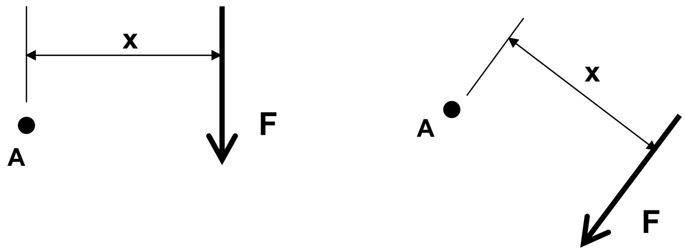

Fig. 8.2 Moments illustrated.

What is a moment?

A moment is a turning effect. A moment always acts about a given point and is either clockwise or anticlockwise in nature. The moment about a point A caused by a particular force F is defined as the force F multiplied by the perpendicular distance from the force's line of action to the point.

Units of moment are kN.m or N.mm Note: This follows because a moment is a force multiplied by a distance, therefore its units are the units of force (kN or N) multiplied by distance (m or mm) – hence kN.m or N.mm. The units of moment are ***never*** kN or kN/m.

In both the cases illustrated in Fig. 8.2, if M is the moment about point A, then $M = F.x$.

Practical examples of moments

See-saw

A see-saw is a piece of equipment often found in children's playgrounds. It comprises a long plank of wood with a seat at each end. The plank of wood is supported at its centre point. The support is pivoted, so is free to rotate. A child sits on the seat at each end of the see-saw and uses the pivoting characteristic of the see-saw to move up and down. A series of modern see-saws is shown in Fig. 8.3.

Imagine two young children sitting at opposite ends of a see-saw, as shown in Fig. 8.4 (a). If the two children are of equal weight and sitting at equal distances from the see-saw's pivot point, there will be no movement because the clockwise moment about the pivot due to the child at the right-hand end ($F.x$) is equal to the anticlockwise moment about the pivot point due to the child at the left-hand end ($F.x$). Therefore the two moments cancel each other out.

If the child at the left-hand end was replaced by an adult or a much larger child, as shown in Fig. 8.4 (b), the child at the right-hand end would move rapidly upwards. This is because the (anticlockwise) moment due to the larger person at the left-hand end (Large Force × Distance) is greater than the (clockwise) moment due to the small child at the right-hand end (Small Force × Distance). The overall moment is thus anticlockwise, causing upward movement of the small child.

Fig. 8.3 See-saws.

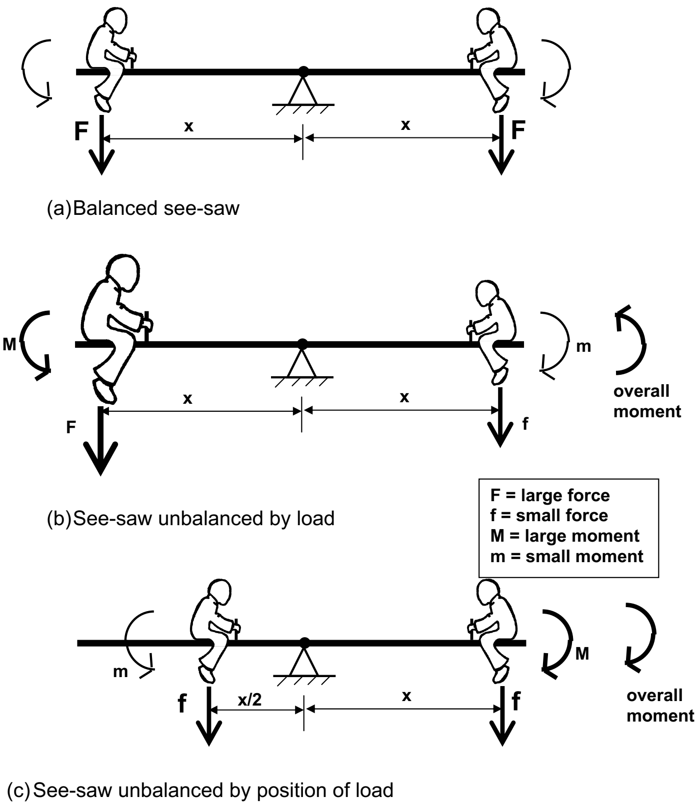

Fig. 8.4 Forces on a see-saw.

Let's return to the original situation, with two young children at opposite ends of the see-saw. But suppose now that the left-hand child moved closer to the pivot point, as shown in Fig.8.4 (c). As a result the right-hand child would move downwards. This is because the anticlockwise moment due to the left-hand child (Force × Small Distance) is smaller than the clockwise moment due to the right-hand child (Force × Large Distance). The overall movement is thus clockwise, causing downward movement of the right-hand child.

Spanners, nuts and bolts

The reader will know from experience that it is much easier to undo a seized-up nut or bolt if a long spanner is used rather than a short spanner. This is because, although the force used may be the same, the 'lever arm' distance is longer, thus causing a greater turning effect or moment to be applied. Practical problems using 'leverage' also illustrate this principle, such as the examples of prising open paint cans, beer bottles and manhole covers already mentioned.

Numerical problems involving moments

It can be seen from the above that it is important to distinguish between clockwise and anticlockwise moments. After all, turning a spanner clockwise (tightening a nut) has a very different effect from turning the spanner anticlockwise (loosening a nut). In this book:

- clockwise moments are regarded as positive (+)
- anticlockwise moments are regarded as negative (–).

It is, of course, quite possible for a given pivot point to experience several moments simultaneously, some of which may be clockwise (+) while others may be anticlockwise (–). In these cases, moments must be added algebraically to obtain a total (net) moment.

Some simple worked examples of moment calculation

In each of the following examples, involving simple beams, we are going to calculate the net moment about point A (remember, clockwise is +, anticlockwise is –).

Example 1 (see Fig. 8.5 (a))
By inspection, the 4 kN force is trying to turn clockwise about A, therefore the moment will be positive (+). The 2 metre distance is measured horizontally from the (vertical) line of action of the 4 kN force; in other words, the distance given is measured perpendicular (i.e. at right angles or 90 degrees) to the line of action of the force, as required.

Remember, a moment is a force multiplied by a distance. If we use the symbol M to represent moment, then in this case:

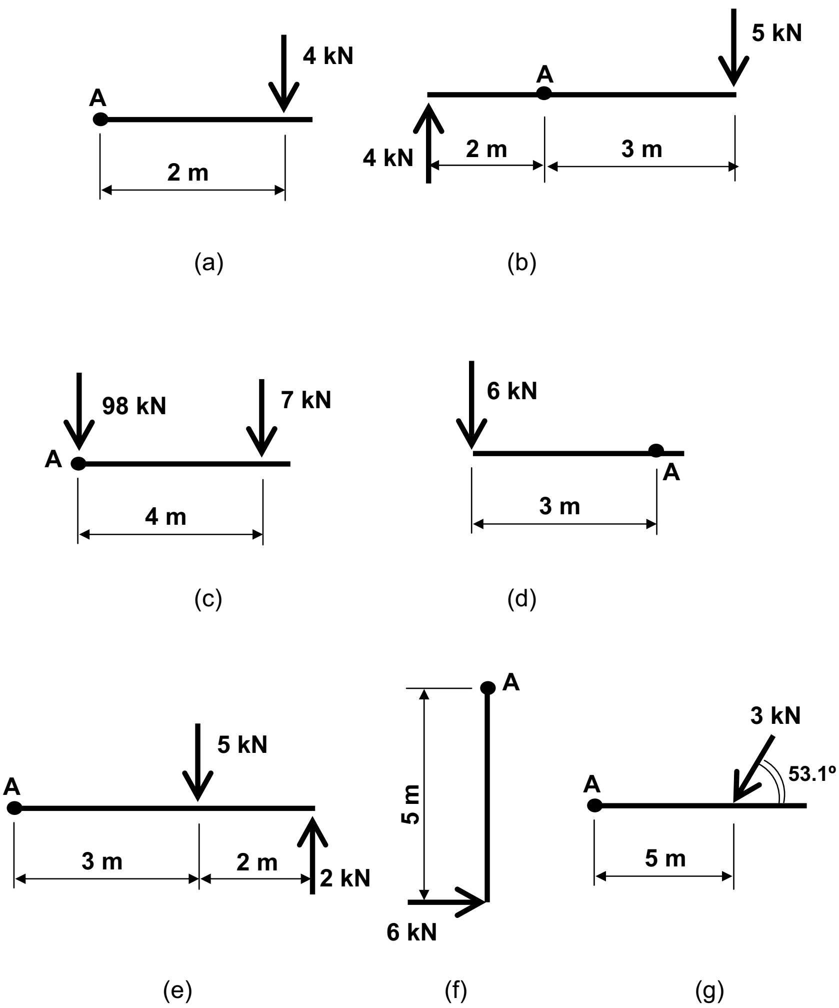

Fig. 8.5 Moment worked examples.

$M = + (4 \text{ kN} \times 2 \text{ m}) = + 8 \text{ kN.m}$

Example 2 (see Fig. 8.5 (b))

This time there are two forces, supplying two moments. A common mistake with this example is to assume that since the two forces are in opposite directions (i.e. one upwards, one downwards), the moments must also oppose each other. In fact, a closer inspection will reveal that the moments about A generated by the two forces are both clockwise (+). So the moment about A for each force is calculated, and the two added together, as follows:

$M = + (5 \text{ kN} \times 3 \text{ m}) + (4 \text{ kN} \times 2 \text{ m})$

$= +\ 15\ \text{kN.m} + 8\ \text{kN.m}$
$= +\ 23\ \text{kN.m}$

(If you attempted this example and obtained an answer of + 7 kN.m, you fell into the trap mentioned above!)

Example 3 (see Fig. 8.5 (c))
Once again there are two forces, supplying two moments. The 7 kN force clearly gives rise to a clockwise moment about A. The 98 kN force, however, passes straight through the pivot point A; in other words, its line of action is zero distance from A. Since a moment is always a force multiplied by a distance, if the distance is zero then it follows that the moment must be zero (since multiplying any number by zero gives a product of zero). So, in this example:

$$M = +\ (7\ \text{kN} \times 4\ \text{m}) + (98\ \text{kN} \times 0\ \text{m})$$
$$= +\ 28\ \text{kN.m} + 0\ \text{kN.m}$$
$$= +\ 28\ \text{kN.m}$$

The lesson to be learned from this example is: if a force passes through a certain point, then the moment of that force about that point is zero.

Example 4 (see Fig. 8.5 (d))
The 6 kN force is turning anticlockwise about A, so the resulting moment will be negative (–).

$$M = -(6\ \text{kN} \times 3\ \text{m}) = -18\ \text{kN.m}$$

Example 5 (see Fig. 8.5 (e))
The 5 kN force is trying to turn clockwise about A, therefore will give rise to a clockwise (+) moment. By contrast, the 2 kN force is trying to turn anticlockwise about A, therefore will produce an anticlockwise (–) moment.

$$M = +(5\ \text{kN} \times 3\ \text{m}) - (2\ \text{kN} \times 5\ \text{m})$$
$$= +\ 15\ \text{kN.m} - 10\ \text{kN.m}$$
$$= +\ 5\ \text{kN.m}$$

Example 6 (see Fig. 8.5 (f))
Not all forces are vertical! But the same rules apply.

$$M = -\ (6\ \text{kN} \times 5\ \text{m}) = -30\ \text{kN.m}$$

Example 7 (see Fig. 8.5 (g))
This slightly harder example will confuse readers who haven't yet grasped the fact that a moment is a force multiplied by a ***perpendicular*** (or 'lever arm') distance. There are two ways of solving this problem – see Fig. 8.6.

1 By using trigonometry to find the perpendicular distance. Figure 8.6 (a) will remind you of the definitions of sines, cosines and tangents in terms of the lengths of the sides of a right-angled triangle. Applying

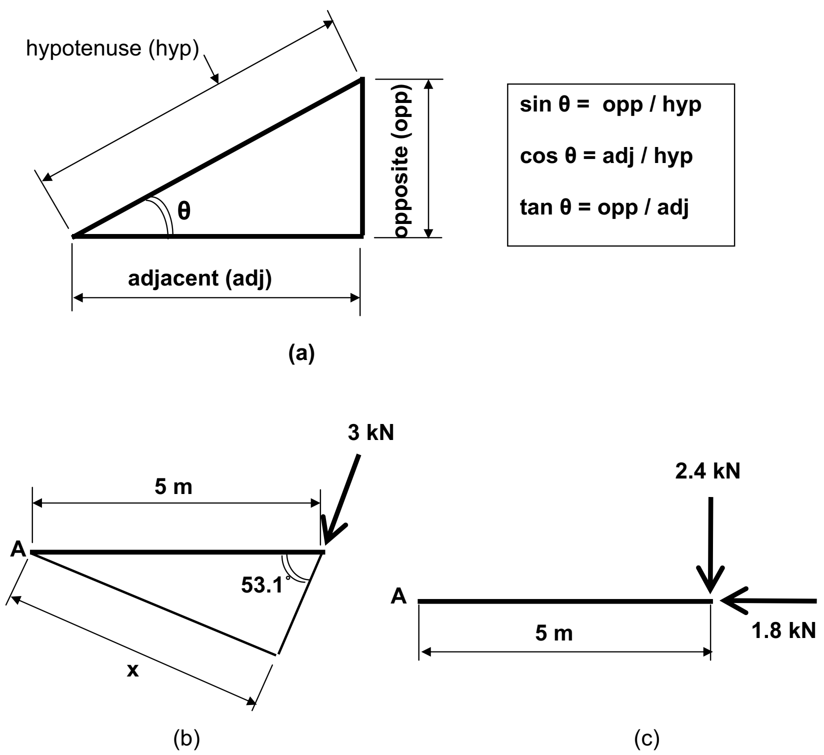

Fig. 8.6 Moments and resolution of forces.

this to the current problem, we find from Fig. 8.6 (b) that the perpendicular distance (x) in this case is 4 metres. So $M = + (3 \text{ kN} \times 4 \text{ m}) = +$ 12 kN.m.

2 By resolving the 3 kN force into vertical and horizontal components. In Chapter 7 we learned that any force can be expressed as the product of two components, one horizontal and one vertical. For any force F acting at an angle of θ to the horizontal axis, it can be shown that:
- the horizontal component is always $F \times \cos \theta$, and
- the vertical component is always $F \times \sin \theta$ (so sin acts upwards: 'Sign Up').

In this problem the 3 kN force acts at an angle of 53.1 degrees to the horizontal. So its vertical component = 3 × sin 53.1° = 2.4 kN ↓

And its horizontal component = 3 × cos 53.1° = 1.8 kN. ←

The problem can now be expressed as shown in Fig. 8.6 (c).

Note that since the 1.8 kN force (extended) passes through point A, the moment of that force about point A will be zero.

$$M = + (2.4 \text{ kN} \times 5 \text{ m}) + (1.8 \text{ kN} \times 0)$$
$$= + 12 \text{ kN.m}$$

(Obviously, this is the same answer as that obtained in part (a)!)

There are some further examples at the end of the chapter for you to try.

Notes on moment calculations

From the above discussion and examples come the following notes and observations:

(1) Always consider whether a given moment is clockwise (+) or anticlockwise (–).
(2) If a force F passes through a point A, then the moment of force F about point A is zero (as illustrated in Example 3 above).
(3) It may be necessary to resolve forces into components in order to calculate moments (as shown in Example 6 above).

Moment equilibrium

Imagine that you are doing some mechanical work on a car and you have a spanner fitted onto a particular bolt in the car's engine compartment. You are trying to tighten the nut and hence you are turning it clockwise. In other words, you are applying a clockwise moment. Now imagine that a friend (in the loosest possible sense of the word!) has another spanner fitted to the same nut and is turning it anticlockwise. This has the effect of loosening the nut.

If the anticlockwise moment that your friend is applying is the same as the clockwise moment that you are applying (regardless of the fact that the two spanners might be of different lengths), you can imagine that the two effects would cancel each other out – in other words, the nut would not move. This applies to any object subjected to equal turning moments in opposite directions: the object would not move.

So if the total clockwise moment about a point equals the total anticlockwise moment about the point, no movement can take place. Conversely, if there is no movement (as is usually the case with a building or any part of a building), then clockwise and anticlockwise moments must be balanced. This is the principle of moment equilibrium and can be used in conjunction with the rules of force equilibrium (discussed earlier) to solve structural problems.

To summarise: if any object (such as a building or any point within a building) is stationary, the net moment at the point will be zero. In other words, clockwise moments about the point will be cancelled out by equal and opposite anticlockwise moments.

Equilibrium revisited

As discussed in Chapter 6, if an object, or a point within a structure, is stationary, we know that forces must balance, as follows:

$\sum V = 0$, i.e. Total Upward Force = Total Downward Force ($\uparrow = \downarrow$)

$\sum H = 0$, i.e. Total Force to Left = Total Force to Right (← = →)

From our newly acquired knowledge of moments, we can add a third rule of equilibrium, as follows:

$\sum M = 0$, i.e. Total Clockwise Moment = Total Anticlockwise Moment

We can use these three rules of equilibrium to solve structural problems, specifically the calculation of end reactions, as discussed in the next chapter.

What you should remember from this chapter

- A moment is one of the most important concepts in structural mechanics.
- A moment is a turning effect, either clockwise or anticlockwise, about a given point.
- If a force passes through the point about which moments are being taken, then the moment of that force about the point concerned is zero. (It is very important to remember this concept as it crops up several times in the solution of problems later in this book.)
- It may be necessary to resolve forces into components in order to calculate moments.

Tutorial examples

In each of the examples shown in Fig. 8.7, calculate the net moment, in kN.m units, about point A.

Tutorial answers

(1) $M = +90$ kN.m
(2) $M = -40$ kN.m
(3) $M = +50$ kN.m
(4) $M = +90$ kN.m
(5) $M = +1$ kN.m
(6) $M = +63$ kN.m

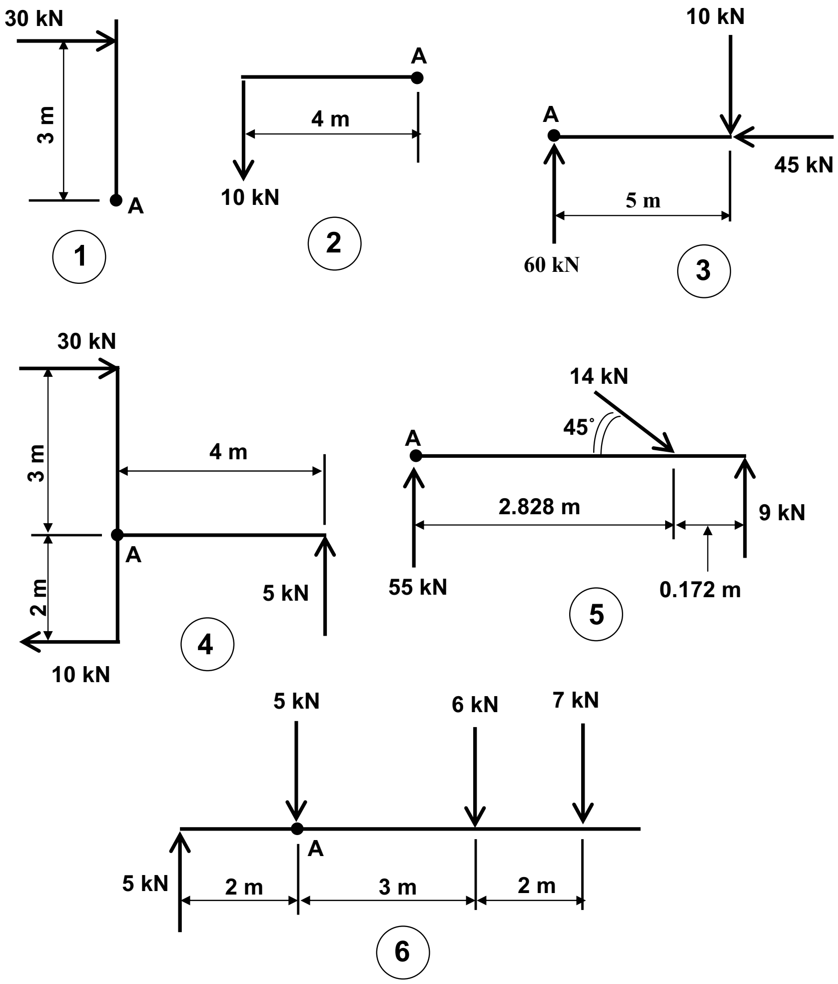

Fig. 8.7 Moment tutorial examples.