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Epistemic Logic

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1 Accessible Knowledge

The antecedents of epistemic logic – the logical theory of propositions regarding belief, knowledge and, by extension, also assertion, assumption, and presupposition, go back to the Middle Ages – especially to William of Ockham (see Boh 1993). However, as a significant branch of philosophical logic, epistemic logic is an innovation of the period 1945–75, the first generation after World War II. At its center lies the relational operator *Kxp* for '*x* knows that *p*,' where *Kx* can be thought of as a parametized modality characterizing the person-relative epistemic status of a proposition. For such an operator to stand coordinate with something worthy of being called a 'logic' it is requisite to begin with a detailed analysis of the sort of 'knowledge' that is to be at issue. Construed in this way, with a focus upon knowledge (*epistêmê*) as such, epistemic logic is part of a broader project that addresses also the logic of belief, supposition, conjecture, etc. – that is, a logic of cognitive processes in general.

The conception of 'knowledge' represents clearly a flexible and internally diversified idea. In general terms, it relates to the way in which persons can be said to have access to information. This can, of course, occur in rather different ways:

- *Occurrent knowledge* This is a matter of actively paying heed or attention to accepted information. A person can say: 'I am (at this very moment) considering or attending to or otherwise taking note of the fact that hydrogen is the lightest element.' The present evidence of our senses – 'I see a cat on the mat' – is an example of this sort of thing.
- *Dispositional knowledge* This is a matter of what people would say or think if the occasion arose – of what, for example, they would say if asked. Even when *X* is reading Hamlet or, for that matter, sleeping, we would say that this individual knows (in the presently relevant dispositional manner) that Tokyo is the capital of Japan.
- *Accessible knowledge* This is a matter not of what a person *would* say if asked (= dispositional knowledge) but of what one *could* say if he is sufficiently clever about using the information that is at one's disposal occurrently or dispositionally. In other words it is what is *implied by* or *inferable from* the facts he already knows in any of these senses.

As we propose to understand it here, knowledge will be construed recursively in that third sense of what is *inferentially accessible* from one's own information. For reasons that will become increasingly clear below, our focus is upon *available* rather than *occurrent* knowledge. Accordingly, a person knows something (1) if this is known to him occurrently, or (2) if this is known to him dispositionally, or (3) if this can be derived by logical deduction or by other secure inferential means from information that is (already) known to him. It is this recursive conception of knowledge that will concern us here, the relationship Kxp, for 'x knows that p,' being understood in the specified manner. We thus immediately secure such relationships as the *Inferential Accessibility Principle*:

$$[Kxp \;\&\; (p \rightarrow q)] \rightarrow Kxq$$

as well as the Knowledge Compilation (or Conjunctivity) Principle:

$$(Kxp \;\&\; Kxq) \rightarrow Kx(p \;\&\; q)$$

(*Note*: In all such formulas, apparently free variables are to be thought of as bound by initial universal quantifiers.)

Our knowers can systematically draw appropriate conclusions and 'put two and two together.' The controlling consideration here is not that they are 'logically omniscient,' but rather that the availability-oriented sense of knowing that is at issue here provides for such inferential projection. Admittedly, to construe knowing in terms of these capabilities is to interpret the idea in a particularly generous sense. However, this approach is amply justified by the aims of the enterprise in so far as one's special interest is in the *limits* of knowledge.

Given this inclusive and generous sense of knowing, it should be noted that if p is a thesis demonstrable on logico-conceptual grounds alone, then p will be universally available since it can be deductively derived from any thesis whatsoever so that Kxp can be held to obtain for any knower x. Accordingly, given the inferential accessibility reading of K, we have

$$\mathbf{N}p \rightarrow (\forall x)Kxp$$

subject to the convention that **N** and **P** will represent logico-conceptual necessity and possibility, respectively, within the setting of modal system S5 of C. I. Lewis. This is simply an aspect of our governing supposition that logico-conceptual matters are universally accessible.

Inferential prowess notwithstanding, the 'knowers' at issue in this discussion are finite knowers. Thus while we have it that *Every knower knows something* (i.e. some truths, and specifically all necessary ones), and thus $(\forall x)(\exists p)Kxp$, we also have it that: *Every knower is ignorant of some truth*: $(\forall x)(\exists p)(p \;\&\; {\sim}Kxp)$. Moreover, any truth is a candidate for being known: whenever p is true, then $\mathbf{P}(\exists x)Kxp$.

As these remarks indicate, the present discussion will move beyond quantified modal logic (QML) to articulate principles of a qualified modal *epistemic* logic (QMEL).

2 Actual vs. Putative Knowledge

The distinction between actual and merely putative knowledge is critical for present purposes: we can and must operate with the distinction between 'our truth' and 'the truth.' Nevertheless, while we realize full well that some of the claims that we regard as true will turn out to be false, we of course cannot particularize here: 'Give me an example of a proposition that you accept as true but that really isn't' represents an absurd request. It lies in the nature of things that we see *our* truth as *the* truth in the realm of specifics.

Rational people are committed to seeing their knowledge as real knowledge – and therefore as subject to those principles which hold for genuine knowledge in general. Now in taking 'our own putative knowledge' to be true – that is, viewing it as *actual* knowledge – we accept the principle:

$Kip \rightarrow p$ (where i should be construed as 'I myself' and/or 'we ourselves').

And since we standardly credit others with the same privileges and liabilities that we claim for ourselves we can generalize the preceding principle to:

$Kxp \rightarrow p$

When claiming Kxp we take the stance that p is something that x really and truly knows to be so. That means that we take ourselves to know p to be true. On this basis, Kxp & $\sim p$ – 'x knows that p but it isn't so' – is to all intents and purposes a self-contradiction. We would not say that someone *knows* something if we thought that this were not so but would instead have to say something like 'he merely *thinks* he knows that p.' For this reason Kxp & $\sim Kip$ is also a comparable self-contradiction. To attribute knowledge of a *particular* fact to another is also to claim it for oneself. On the other hand, the generic $(\exists p)(Kxp \,\&\, \sim Kip)$ – that is 'x knows something I don't' – is a perfectly plausible proposition. It is just that one cannot concretize it to the level of specifics: particularizing existential instantiation becomes impracticable here.

The thesis $Kxp \rightarrow p$ also means that no knower ever knows that he is mistaken about something concrete that he takes himself to know. This was a commonplace among medieval logicians, who held that *Nihil scire potest nisi verum* (see Boh 1993: 48). The thesis $Kxp \,\&\, Kx(\sim Kxp)$ is thereby self-contradictory since its second conjunct entails the denial of what the first conjunct affirms. The idea at issue here is not new but was also a commonplace among medieval logicians. Thus Albert of Saxony (ca. 1325–90) argued in his treatise on *Insolubilia* that "Socrates knows that he is mistaken in believing *A*" is a self-contradictory contention. (See Kretzmann and Stump 1988: 363–4.)

3 Levels of Acceptance and Rejection

In articulating epistemological principles we must come to terms with the fact that one can distinguish three different levels or bases of assertability on which such principles can be affirmed:

1. *Conceptual truth* A thesis that holds good on logico-conceptual goals of meaning and usage alone; its denial involves one in saying things which, while perhaps understandable, are acceptable only subject to elaborate explanations and qualification and in their absence are effectively paradoxical.
2. *Contingent truth* A thesis whose acceptability cannot be substantiated by any amount of merely conceptual or verbal elucidation but whose validity roots in the cognitively discernible contingent features of the real world.
3. *Plausible truth-candidates* A thesis not clearly spoken for by the available facts but for whose substantiation cogent considerations of plausibility can be adduced and which therefore merits at least qualified endorsement and provisional acceptance.

Each of these defines a level of tenability or assertability which may be characterized as levels 1, 2, and 3, respectively. (The lower the tenability level of a principle, the more unproblematic and probatively secure it will be.)

Let $\vdash$Z indicate (as usual) that Z is an assertion of the system we are engaged in formulating. Then with respect to level-one principles we have:

If $\vdash_1$ Z then **N**Z – and therefore also, as we have seen, **N**$(\forall x)KxZ$

Since all the principles of our system are to be seen as matters of logico-conceptual necessity, the unqualified prefix $\vdash$ is to be construed as $\vdash_1$. In being matters of logico-conceptual necessity, all level-one principles are accordingly universally available in the inferential-accessibility mode of knowledge.

With respect to level two principles by contrast we merely have:

If $\vdash_2$ Z then *KiZ* and thus also (but merely) $(\exists x)KxZ$ (whence also Z)

Since *i* = we ourselves, it is unavoidable that Z be seen as representing something that we really know to be true.

Finally, with respect to level-three principles we merely have

If $\vdash_3$ Z then Z

Here we do indeed regard the thesis in question as being true but without claiming actual *knowledge* of the matter. For in general, the propositions we ourselves see as eminently plausible are accepted by us as true. (In theory something viewed as highly plausible can in fact be false, but we are of course incapable of giving a current first-hand example: 'I see *p* as deserving of acceptance, but it is false' comes close to being a contradiction in terms. Illustrations from the past or those involving others are, of course, another matter.) Here at level three we claim truth in a tentative and provisional way that falls short of actual knowledge. Accordingly, the inference

$\vdash_3$ Z then $(\exists x)KxZ$

is inappropriate – and thus *a fortiori* also the inference to *KiZ*. On the contrary, we have it that if $\vdash_3$ Z then $\sim(\exists x)KxZ$. Nobody *knows* a level-three principle (ourselves included!):

every assertion at level-three has to be seen as a truth that is not actually *known*. Such theses may be surmised or presumed, but even at best they are plausible truths that nobody *knows* to be such – such as the thesis 'There are mountains on the far side of the moon' is the cognitive state of the art of the nineteenth century.

The existence of the third level of assertion is a reminder that epistemology is broader than the theory of *knowledge*. For matters of presumption, conjecture, reasonable belief, and warranted assertability also clearly fall within its purview.

On this basis, then, all three of these modes of 'assertion' do indeed convey a commitment – an *assertion*. A claim that Z is the case obtains in every instance, but with different assertoric modalities, so to speak. For in this context we must deploy the distinction between what is known to be true and what is accepted or asserted (as true) on a weaker basis – conjuncture, plausible suppositions, or the like. The latter sort of thing is being claimed as true, alright, but in a substantially less firm and confident tone of voice. However, the tenability of level-three principles is at odds with acknowledging that someone knows the contrary. For note that when $(\forall x){\sim}Kx{\sim}Z$ is false, so that ${\sim}(\forall x){\sim}Kx{\sim}Z$ obtains, then of course we will have $(\exists x)Kx{\sim}Z$. This means that ~Z would have to obtain (at least at level 2), so that Z would not be a level-three assertion after all – contrary to our initial stipulation.

Despite the acceptability of $(\exists p)(p \,\&\, {\sim}(\exists x)Kxp)$, no *particular* proposition of the form $p_0 \,\&\, {\sim}(\exists x)Kxp_0$ is ever assertable at levels one or two. For asserting this at level one would mean accepting $\mathbf{N}p_0$ which is at odds with ${\sim}(\exists x)Kxp_0$. And asserting it at level two would involve a commitment to Kip_0 which is also at odds with ${\sim}(\exists x)Kxp_0$.

One can, of course, use some epistemic principles to deduce others; here, as elsewhere, inference from givens is a cognitively viable project. And the *epistemic* level of a conclusion derived from premises *cannot be greater than the largest index-level of the premises required for its derivation*. In point of cognitive tenability or assertability, the status of a derived thesis cannot be weaker, so to speak, than the weakest link among the premises from which it derives.

Theses that entail the negation (denial) of an assertion must themselves be denied (at the appropriate level). We shall employ the symbol $\dashv$ to indicate denial/rejection. This should be subscripted to indicate the appropriate level, subject to the convention that $\dashv$ Z obtains at a level iff $\vdash$ ~Z does so.

4 Level One Principles: Logico-Conceptual Truths

Let us consider some examples of cognitive principles at each assertion level category, beginning with the first, that of principles which inhere in the very nature of the logico-conceptual construction of 'knowledge' as *accessible* knowledge. The following seven basic principles obtain here:

K_1 *Knower capacity*
$(\forall x)(\exists p)Kxp$ and even more strongly $(\forall x)(\exists p)[Kxp \,\&\, {\sim}\mathbf{N}p]$

K_2 *Knower finitude*
$(\forall x)(\exists t){\sim}Kxt$ or equivalently ${\sim}(\exists x)(\forall t)Kxt$, where t ranges specifically over truths.

K_3 *Knowledge authenticity*
$\sim(\exists t)(\exists x)Kx\sim t$ or equivalently $(\forall t)(\forall x)\sim Kx\sim t$.

K_4 *Inferential accessibility*
$(p \rightarrow q) \rightarrow (Kxp \rightarrow Kxq)$

K_5 *Conjunctivity*
$(Kxp \ \& \ Kxq) \rightarrow Kx(p \ \& \ q)$

K_6 *Reflexivity*
$Kxp \rightarrow KxKxp$

K_7 *Truth Availability*
$(\forall t)\mathbf{P}(\exists x)Kxt$

Here $\mathbf{N}$ and $\mathbf{P}$ represent logico-conceptual necessity and possibility, respectively, and $\rightarrow$ is a strong (logico-conceptual) implication such that $p \rightarrow q$ is equivalent with $\mathbf{N}(p \rightarrow q)$. Also, the variables t, t', t'', etc. will serve to range over truths. And throughout, free variables are to be taken as tacitly bound to initial universal quantifiers.

Each of these principles merits a brief explanation.

K_1 $(\forall x)(\exists p)(Kxp \ \& \ \sim\mathbf{N}p)$ simply asserts: *Every knower knows something – and indeed some contingent (i.e. non-necessary) truth or other*. This obtains simply in virtue of the fact that we are supposed to be talking about knowers.

K_2 $(\forall x)(\exists t)\sim Kxt$ reflects the fact that we are dealing with finite knowers. In the present context of discussion, *no knower is omniscient*; none knows of all truths that they are true – not even on the present generously undemanding construal of knowledge. Since t ranges specifically over truths we have it that, for example, $(\exists t)Kxt$ comes to $(\exists p)(p \ \& \ Kxp)$.

K_3 $(\forall t)(\forall x)\sim Kx\sim t$ asserts: *Only true propositions can be known*. This thesis roots in the very nature of 'knowledge' as this concept is generally understood. For it makes no sense to say: 'x knows that p, but p is not true.' Of course, someone may *think* or *believe* that he knows something that is false. But to say that he actually knows it is to acknowledge its truth.

Let us further adopt the abbreviation Up for $\sim(\exists x)Kxp$ or equivalently $(\forall x)\sim Kxp$ – that is, for 'p is unknown.' Then the just-stated finding means that $(\forall t)U(\sim t)$. No one knows something that is false, that is: Nobody knows an *untruth* to be the case. (But of course one can know *that* it is an untruth.)

K_4 $(p \rightarrow q) \rightarrow (Kxp \rightarrow Kxq)$. *Knowers automatically know the things that follow from what they know*. This obtains because it is the tacit or implicit sense of 'knowledge' as inferentially accessible information that is at issue in our discussion.

Since in virtue of K_4 our knowers know all necessary propositions, we of course have it that every knower knows *that* any given p is true-or-false: $(\forall x)Kx(p \vee \sim p)$ or equivalently $\sim(\exists x)\sim Kx(p \vee \sim p)$. But in view of K_2 there certainly can be knowers who do not know *whether* p is true or is false: $(\exists x)(\sim Kxp \;\&\; \sim Kx\sim p)$.

K_5 $(Kxp \;\&\; Kxq) \rightarrow Kx(p \;\&\; q)$. *Knowers know conjointly and collectively anything they know distributively*. This too obtains in virtue of the generous accessibility-oriented sense of 'knowledge' that concerns us here, which supposes that knowers 'can put two and two together.'

K_6 $Kxp \rightarrow KxKxp$. *When knowers know something this very fact is cognitively accessible to them*. This again follows from the presently operative accessibility-geared sense of knowledge. For clearly, when knowledge is construed as *available* knowledge – that is, in terms of what can be inferred on the basis of what is known – then Kxp will carry $KxKxp$ in its wake. When a certain fact is known to someone, they are in a position to infer that this is so. (Observe that K_6 yields $Kx\sim p \rightarrow KxKx\sim p$, which is quite different from and emphatically does not imply $\sim Kxp \rightarrow Kx\sim Kxp$.)

K_7 $(\forall t)\mathbf{P}(\exists x)Kxt$. *Any actual truth is (in theory) knowable*. Such potential availability also inheres in our understanding of the relationship of knowers to knowledge. (Note that this principle is equivalent with $\sim(\exists t)\mathbf{N}U(t)$: no truths are *necessarily* unknown.

K_7 stipulates that any truth is a candidate for knowledge. This reflects our present understanding of **N** and **P** as logico-conceptual necessity/possibility rather than with physical necessity/possibility. It is certainly conceivable that some region of physical reality is such that its facts are inaccessible to intelligent creatures.

Could K_7 be strengthened to $(\forall t)\mathbf{P}(\forall x)Kxt$? This would preclude the prospect of 'blind spots' – bits of self-knowledge inherently unavailable to the subject himself. (On this theme see Sorensen 1988.) On this basis it seems unacceptable.

Note moreover that accepting $(\forall t)\mathbf{P}(\exists x)Kxt$ does *not* mean that any truth is knowable by some actual existent $(\forall t)(\exists x)\mathbf{P}Kxt$? The knowability at issue looks not to actual but to merely possible knowers.

5 Further Consequences

Given the principles K_1–K_7 formulated above, one can proceed to derive various further epistemic principles by purely logical means:

K_8 *Conjunctivity*
$Kx(p \;\&\; q) \rightarrow (Kxp \;\&\; Kxq)$
Knowledge of a conjunction is tantamount to knowledge of its conjuncts.
This follows from K_4 and K_5.

K_9 *Substitutivity*
$(p \rightarrow q) \rightarrow (Kxp \rightarrow Kxq)$
To know something is to know it in all of its logically equivalent guises.
This thesis pivots on K_3.

K_{10} *K-Consistency*
$Kxp \to {\sim}Kx{\sim}p$
This follows directly from K_3. (The prospect of ignorance – of having both ${\sim}Kxp$ and ${\sim}Kx{\sim}p$ obtain – means that the converse does not hold.)

K_{11} *Transmissibility*
$[Kxp \,\&\, Kx(p \to q)] \to Kxq$
This follows from K_4 and K_5.

K_{12} *Self-limitation*
$(\forall x)Kx(\exists t){\sim}Kxt$
By K_2 we have $(\forall x)(\exists t){\sim}Kxt$. And since this is a level 1 principle we will also have $(\forall y)Ky(\forall x)(\exists t){\sim}Kxt$. This entails $(\forall y)Ky(\exists t){\sim}Kyt$. Not only are individuals not omniscient, but they all know it.

In accepting that another knows a certain fact one is thereby effectively claiming that fact as part of one's own knowledge. And so, to know that another person knows some specific fact one must know this fact oneself. We thus have:

K_{13} *Knowledge cooptation*
$KxKyp \to Kxp$
To know that someone actually knows some fact to be so one must know this fact itself.

This principle can be derived from the preceding considerations by the following argument:

1.	$[Kxp \,\&\, (p \to q)] \to Kxq$	From K_3
2.	$(KxKyp \,\&\, (Kyp \to p)] \to Kxp$	From (1) by substituting Kyp/p and p/q
3.	$Kyp \to p$	From K_2
4.	$KxKyp \to Kxp$	From (2), (3)

This means that the specifically *acknowledged* knowledge of others is also knowledge. (Of course it will not be the case for unacknowledged knowledge. We certainly do *not* have: $Kxp \to Kyp$.)

Note further that we have the principle:

K_{14} *Necessity cognition*
$\mathbf{N}p \to (\forall x)Kxp$
Logico-conceptual truths are cognitively available to all.

This principle pivots on K_4 via the following proof:

1. For any x: Kxq for some suitable q, by K_1.
2. Whenever $\mathbf{N}p$, then $q \to p$, for any q, by mere logic.
3. Whenever $\mathbf{N}p$, then Kxp — rom (1) and (2) by K_4.
4. $(\forall p)(\mathbf{N}p \to Kxp)$ — from (1)–(3).
5. $(\forall p)(\mathbf{N}p \to (\forall x)Kxp)$ — from (4) since x is a free variable. Q.E.D.

Note, however, that the converse of K_{14} does *not* hold: some merely contingent fact might well be known universally.

Via the substitution $\mathbf{N}p/p$ K_{14} yields:

K_{15} *Necessity recognition*

$\mathbf{N}p \rightarrow (\forall x)Kx\mathbf{N}p$

Knowledge of necessity is universal. This principle represents a salient feature of inferentially accessible knowledge.

K_1 has it that every knower knows some truth $(\forall x)(\exists t)Kxt$. In virtue of K_{15}, we have the stronger thesis that there are truths that everyone knows $(\exists t)(\forall x)Kxt$. For any necessary truth clearly fills the bill here, given the presently operative liberal construction of K as *available* knowledge.

6 Cognitive Limitations

Let us consider somewhat more closely the matter of ignorance and unknowing, recalling that $U(p)$ comes to: $\sim(\exists x)Kxp$ or equivalently $(\forall x)\sim Kxp$.

Under what conditions on f would we have it as a general principle that $f(p)$ entails $Uf(p)$? Note that this would mean that $f(p) \rightarrow \sim(\exists x)Kxf(p)$ or equivalently $(\exists x)Kxf(p) \rightarrow \sim f(p)$. Since K_3 has it that the antecedent yields $f(p)$, it follows that there can be no principle of the indicated format as long as $f(p)$ is self-consistent. No significant feature of p is automatically unknowable.

A further important epistemic principle is represented by the thesis:

K_{16} *Cognitive myopia*

$\sim(\exists p)(\exists x)Kx(\sim Kxp \;\&\; p)$ or equivalently $(\forall x)(\forall p)\sim Kx(p \;\&\; \sim Kxp)$

Nobody ever knows of a proposition that while they do not know it, it is nevertheless true.

PROOF

1.	$Kx\sim Kxp \rightarrow \sim Kxp$	From K_2
2.	$\sim(Kx\sim Kxp \;\&\; Kxp)$	From (1)
3	$\sim Kx(\sim Kxp \;\&\; p)$	From (2), K_{14}. Q.E.D.

It is important to observe that the thesis at issue here – or equivalently $(\forall x)\sim(\exists p)Kx(p \;\&\; \sim Kxp)$ – differs significantly from $(\forall x)\sim Kx(\exists p)(p \;\&\; \sim Kxp)$ or equivalently $(\forall x)\sim Kx(\exists t)\sim Kxt$ or $(\forall x)\sim Kx\sim(\forall t)Kxt$, that is, 'For aught that anyone knows they know it all.' This latter contention is emphatically unacceptable.

A pivotal fact of the cognitive domain is:

K_{17} *Knowledge limitation*

$\sim(\forall t)(\exists x)Kxt$ or equivalently $(\exists t)(\forall x)\sim Kxt$ or $(\exists t)\sim(\exists x)Kxt$ or $(\exists t)Ut$.

There are altogether unknown truths: it is not the case that all truths are known.

This is easily established on the basis of the prior stipulations. For since K_2 assures that we are, by hypothesis, dealing with finite knowers, it transpires that for each knower x_i there is some truth that this knower does not know. Now let t^* be the conjunction of *all* these truths t_i over our (obviously finite) collection of knowers. Then in virtue of K_{12} no knower knows t^*. It follows that $(\exists t)(\forall x){\sim}Kxt$ or equivalently $(\exists t)U(t)$. Of course, any such unknown truth will have to be a non-necessary, and thus contingent, truth, given the presently operative inferential-accessibility sense of knowledge.

A different route to the same destination is that any level 3 thesis represents what must be regarded as an unknown truth. (This will be amplified below.)

In general one cannot, of course, make the transition from $(\forall x)(\exists t)f(x, t)$ to $(\exists t)(\forall x)f(x, t)$. (Thus 'For any integer there exists another that is greater' does *not* entail 'There exists an integer that is greater than any other integer.') But in the special case of $f(x, t) = {\sim}Kxt$ this inference is valid, as the preceding argumentation for K_{17} shows. And a community of finite knowers is thereby subject to substantial limitations.

One might be tempted to offer the following objection to the just-indicated implication thesis: $(\forall x)(\exists t){\sim}Kxt \rightarrow (\exists t)(\forall x){\sim}Kxt$: 'What if one divided the realm of truth T into two disjoint parts T_1 and T_2 such that x_1 knows all (but only) T_1 truths and x_2 knows all (but only) the T_2 truths. Then clearly $(\forall x)(\exists t){\sim}Kxt$ but not $(\exists t)(\forall x){\sim}Kxt$.' However, this objection is flawed. For the hypothesis that it projects cannot be realized in the circumstances of our discussion, where knowledge is inferentially transmissible in that $[Kxp \mathbin{\&} (p \rightarrow q)] \rightarrow Kxq$. Thus consider a truth $t_1 \vee t_2$ where $t_1 \in T_1$ and $t_2 \in T_2$. Then by inferential transmission this must be a known commonality for x_1 and x_2, so that the disjointness condition cannot be met. The hypothesis of truth-division runs afoul of our implicit availability construction of knowledge.

To be sure, K_{17} only assures the existence of unknown truth. To this point, we have not claimed to provide an example of this. (This awaits the discussion of level three principles in Section 8.)

What follows regarding p from $(\forall x){\sim}Kxp$ or equivalently $U(p)$? Certainly not not-p. For if we had $(\forall x){\sim}Kxp \rightarrow {\sim}p$ then it would follow that $p \rightarrow (\exists x)Kxp$ which must of course be rejected. On the other hand, poss(not-p), that is $\mathbf{P}{\sim}p$, must indeed be held to follow. For consider $(\forall x){\sim}Kxp \rightarrow \mathbf{P}{\sim}p$ which is equivalent with $\mathbf{N}p \rightarrow (\exists x)Kxp$. In view of K_{14} this must be accepted on the presently operative construction of knowledge.

7 Level Two Principles and the Consideration that Knowledge of Contingent Fact is itself Contingent

The epistemic principles to which we now turn reflect the contingent facts of life regarding the ways and means of our knowledge of things. We shall continue to use the variables τ, τ', τ'', etc. to range over the limited propositional subdomain of specifically *contingent* truths, with the variables t, t', t'', etc. ranging over truths in general.)

An elemental principle of this domain is

K_{18} $(\exists \tau)(\exists x){\sim}Kx\tau$ or equivalently ${\sim}(\forall \tau)(\forall x)Kx\tau$.
There are contingent truths that not everyone knows.

This follows from K_2 in view of the fact that necessary truths are automatically known to all (by K_9). So here we still have a level 1 principle.

Another more positive principle is:

K_{19} $(\forall x)(\exists \tau)Kx\tau$ or equivalently $\sim(\exists x)(\forall \tau)\sim Kx\tau$.
No knower is an utter ignoramus: every knower knows some contingent truth or other.

This principle projects K_1 into the contingent domain and is a more or less natural supposition relative to the liberal construction of knowledge we have taken into view. However, this new principle will obtain at level two; it does not follow from anything that precedes.

The following important principle obtains:

K_{20} *Wherever* τ *is a contingent truth, then* $Kx\tau$ *is also contingent: contingent truth is by nature cognitively contingent*. That is to say we have both $(\forall \tau)\mathbf{P}(\exists x)\sim Kx\tau$ or equivalently $\sim(\exists \tau)\mathbf{N}(\forall x)Kx\tau$ and also $(\forall \tau)\mathbf{P}(\exists x)Kx\tau$ or equivalently $\sim(\exists \tau)\mathbf{N}(\forall x)\sim Kx\tau$.

The first of the two components holds because its denial $(\exists \tau)\mathbf{N}(\forall x)Kx\tau$ falls foul of the fact that it is only for necessary truths t that $\mathbf{N}(\forall x)Kxt$, seeing that $\mathbf{N}(\forall x)Kxp \rightarrow \mathbf{N}p$ follows from $(\exists x)Kxp \rightarrow p$. And the second follows from $(\forall t)\mathbf{P}(\exists x)Kxt$ – the potential availability of truth stipulated by K_7. And so for any specifically contingent (i.e. non-necessary) truth τ we have it that $\mathbf{P}(\exists x)\pm Kx\tau$. Equivalently for no τ do we have $\mathbf{N}(\forall x)\pm Kx\tau$: neither $(\forall x)Kx\tau$ nor $(\forall x)\sim Kx\tau$ is ever necessary in the case of contingent truths. Indeed it can be shown that even $(\exists x)\pm Kx\tau$ is always contingent for contingent τ. That someone does (or does not) know a given *contingent* fact is always itself contingent.

8 Level Three Principles: Plausible Truth-Candidates

It will be recalled from the discussion of Section 4 above that any level three principle instantiates the idea of an *unknown* truth, seeing that if actual knowledge were being claimed, then the assertion in question would have to be made at a lower (deeper) level. Accordingly, the epistemic theses that will now be at issue have a standing of mere plausibility in contrast to knowability as such.

Every knower knows something. And we can actually even lay claim to a rather stronger level three principle:

K_{26} $(\exists \tau)(\forall x)Kx\tau$
There are (contingent) truths that everyone knows.

K_{26} means that we cannot accept it as a principle that only necessary truths are universally known. Thus while we have endorsed its converse (as per K_{15}), we must reject: $(\forall x)Kxp \rightarrow \mathbf{N}p$. From $(\forall x)Kxp$ – and indeed even from $(\exists x)Kxp$ – we can infer that p is true, but certainly not that it is necessary.

Let us investigate the prospect of principles of the format: If $f(p)$, then $(\forall x)Kxf(p)$. Note that

$$f(p) \to (\forall x)Kxf(p)$$

holds when $f(p) = \mathbf{N}p$ in virtue of K_{16}. On the other hand, $f(p) = (\forall x)Kxp$ leads to $(\forall x)Kxp \to (\forall y)Ky(\forall x)Kxp$. And this has some claim to plausibility. For when something is obvious enough to be known to everyone, this fact itself is presumably something about which people-in-general can secure knowledge. The principle in view thus holds at level three.

It is of interest to ask what sort of knowledge follows from ignorance. Consider a thesis of the format $\sim Kxp \to Kxf(p)$. Since $\sim Kxp$ always obtains when p is false, this would mean that $Kxf(p)$ will always obtain when p is false – as $f(p)$ must therefore also do. Thus nothing of any real interest *regarding someone's knowledge follows on general principles from his ignorance of a given fact*.

9 Knowledge of the Unknown?

Consider the contention 'I know that t_0 is an unknown truth,' symbolically $Ki(t_0 \,\&\, Ut_0)$. In view of K_8 his amounts to $Kit_0 \,\&\, KiU(t_0)$. But $KiU(t_0)$ comes to $Ki{\sim}(\exists x)Kxt_0$. This entails $\sim(\exists x)Kxt_0$ which in turn yields $\sim Kit_0$. And this produces a contradiction. There is an instructive lesson here: *We cannot concretize* $(\exists t)Ut$ *in the mode of knowledge*: That is, we cannot instantiate this thesis by advancing a particular truth t_0 which at once and the same time we claim to know to be true and also characterize as an unknown. It is perfectly true that 'There are truths I do not know' but I cannot possibly produce any concrete examples in the mode of categorical cognition. Accordingly, the reality of it is that we can only instantiate $(\exists t)Ut$ in the mode of conjecture, which is to say at the third level of assertion.

Clearly, whenever we assert a thesis Z at the third level, so that

$$\vdash_3 Z$$

we can indeed move on to the claim that Z is true (which, after all, is why we assert it), but must nevertheless acknowledge that no one actually *knows* this to be so and accordingly must ourselves refrain from claiming actual knowledge here. For if this were known, so that $(\exists x)KxZ$ then Z would obtain at the second level of assertion: $\vdash_2 Z$. And (by hypothesis) this is not the case.

'But how can you possibly maintain something that you do not actually know to be true?' The appropriate answer, clearly, is: *cautiously and tentatively*, in the decidedly guarded and hesitant tone of voice of mere conjecture. In other words, at level three.

10 Conclusion

This survey of principles of metaknowledge has not issued in one big culminating result but rather in a diversified mosaic of smaller ones. Yet in the aggregate this complex provides a unified overall picture of the epistemic situation from which some significant overall lessons emerge.

Perhaps the most important of these lessons is that we must operate a two-tier epistemology – one that looks not just to knowledge alone but also to the lesser level of epistemic commitment represented by plausible conjecture or supposition. Another lesson is that a systematic rational account of the cognitive situation is possible with 'knowledge' understood as in the sense of inferentially accessible information. Last but not least, we have seen that even under this most liberal and generous of constructions, our 'knowledge' is such that we must recognize the existence of a whole spectrum of cognitive limitations. For even as 'knowledge' in the mode of inferential accessibility is logically self-ampliating in that we have

If Kxp and $p \rightarrow q$, then Kxq

so also is ignorance, since we analogously have:

If $\sim Kxp$ and $q \rightarrow p$, then $\sim Kxq$

Both of these principles are two sides of the same coin. And the price we pay for the knowledge-amplification assured by the former principle is that ignorance-proliferation assured by its equivalent counterpart. Just as knowledge is self-ampliating, so is its lack.

In a world of finite beings even the most generous construction of 'knowledge' leaves ample scope for ignorance. And one of the ironic aspects of this topic of metaknowledge is that the very fact that our knowledge is limited inhibits our capacity to be specific about the matter by going on to specify just exactly what those limits are. Among the most difficult sorts of knowledge to achieve is detailed information about the nature of our ignorance.

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